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An Evaluation of the Beam Width at CDF

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Abstract

Using inclusive jet data, we evaluate the Tevatron beam width at CDF using the observed vertex distribution from the vxprim module. We take into account the z-dependence of the beam width and fit it to the functional form:

$$\sigma_{beam} = \sqrt{\epsilon(\beta^* + (z - z_0)^2/\beta^*)}.$$

We also investigate the stability of the beam width over the course of a store. Using select high luminosity runs, we find the following results for the beam width fit parameters averaged over 6 selected runs:

$$\epsilon = (1.49 \pm 0.03) \times 10^{-7}\ \text{cm}, \beta^* = 37.1 \pm 1.1\ \text{cm}, z_0 = 12.1 \pm 0.7\ \text{cm for } x,$$

$$\epsilon = (1.49 \pm 0.02) \times 10^{-7}\ \text{cm}, \beta^* = 38.0 \pm 1.0\ \text{cm}, z_0 = -8.2 \pm 0.5\ \text{cm for } y.$$

These values are quoted assuming that the Tevatron beam width parameters did not change over the course of these 6 runs. The uncertainties given are statistical only.

We also find that the average beam widths in x and y are:

$$\sigma_x^{avg} = 29.6 \pm 0.4\ \mu\text{m}\ , \sigma_y^{avg} = 30.4 \pm 0.4\ \mu\text{m}.$$

1 The Observed Beam Width Distribution

The methodology we use to determine the Tevatron beam width at CDF is similar to the method outlined in Ref. [1]. We use vxprim [2] to determine the primary vertex position in the xy plane and ZVertexModule to estimate the z-vertex position. We use G stream data from the dataset *gjet08* that was processed with CDF software release 4.8.4. The analysis code used release 4.9.1hpt1, with the beamline from history code 19. Using history code 19 beamlines takes into account changes in the beamline over the course of a given store. The track refitter is also used [3]. We used vertices with at least four tracks from outside-in algorithms, with these tracks having at least 4 silicon $r - \phi$ hits, 25 axial COT hits, and 25 stereo COT hits.

We then obtained an observed distribution Δx and Δy of the beam width by taking the difference between the event vertex position and the nominal beam position:

$$\Delta x = x_{pvtx} - x_{beam} \qquad \Delta y = y_{pvtx} - y_{beam}, \tag{1}$$

where x_{beam}, y_{beam} is obtained from the fitted beamline at the same value of z as x_{pvtx}, y_{pvtx}. We select vertices that have a vxprim uncertainty below 40 μm for both x and y. We divide these observed distributions in 10 cm bins of z with medians $z = 0, \pm 10, \pm 20, \pm 30, \pm 40$ cm. We do not consider vertices with $z > 45$ cm. Examples of these observed width distributions are shown in Figures 1 and 2.

We fit these observed distributions with a Gaussian distribution to obtain the observed beam width. Only the central region of the $\Delta x, y$ distributions is included in the fit. This central region varies by z-bin, and its values are listed in Table 1.

2 Investigations of the vxprim Uncertainty

The observed beam widths do not represent an accurate estimation of the Tevatron beam width at CDF. This is because each vertex coordinate has an uncertainty, which increases the width of the observed distribution. Thus we need to subtract the mean uncertainty for the primary vertices in quadrature with the observed beam width. Furthermore, as we shall show, the uncertainties that vxprim reports are underestimated, and we estimate a scale factor to correct for this. The beam width that we will report will be calculated by the following:

$$\sigma_{beam} = \sqrt{\sigma^2_{observed} - \kappa^2 \bar{\sigma}^2_{vtx}}, \tag{2}$$

z-bin (cm)	0	± 10	± 20	± 30	± 40
Central Region (μm)	$\Delta_{x,y} < 110$	$\Delta_{x,y} < 110$	$\Delta_{x,y} < 125$	$\Delta_{x,y} < 130$	$\Delta_{x,y} < 140$

Table 1: Central regions for $\Delta x, y$ for the vertex to be included in the Gaussian fit.

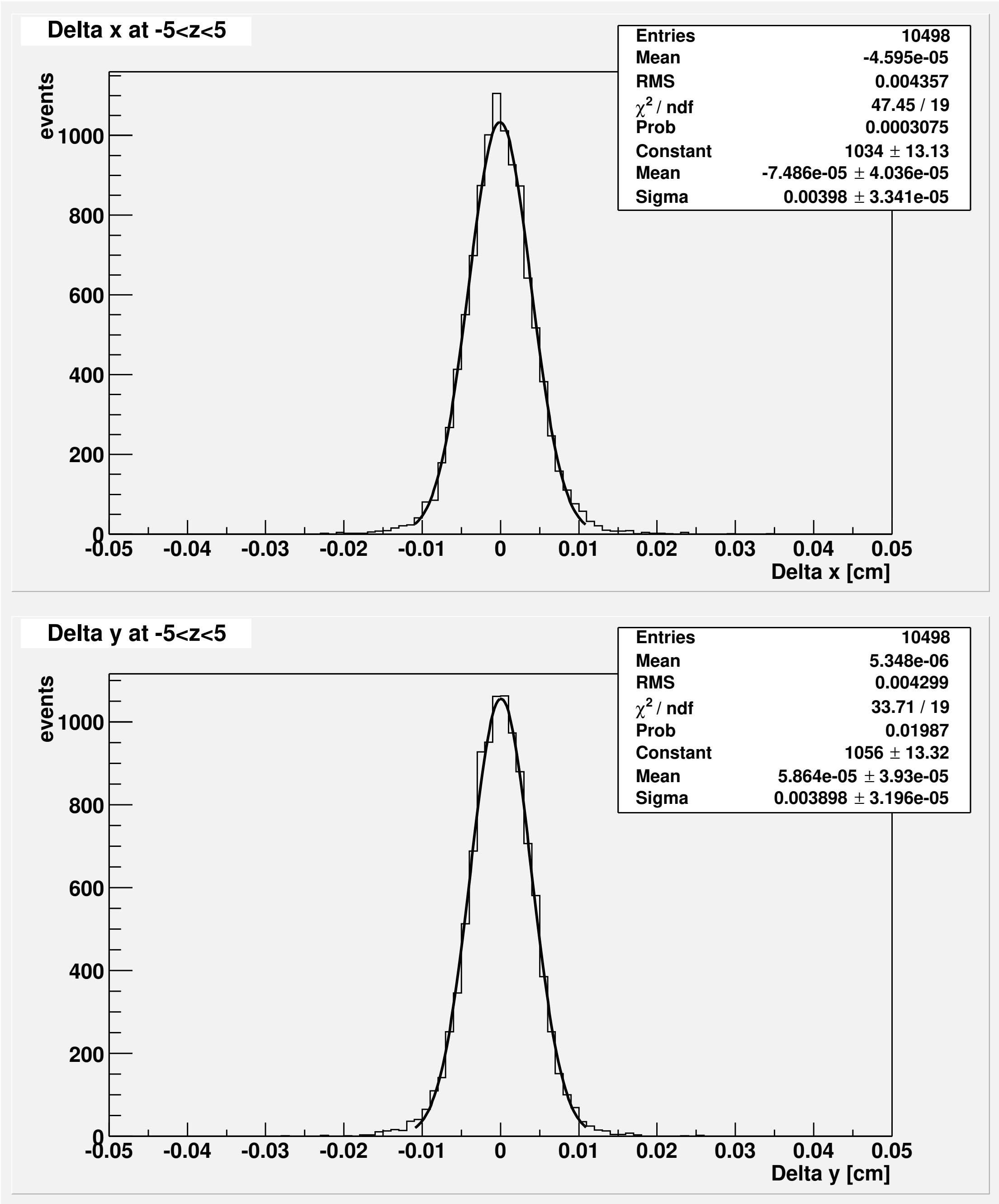

Figure 1: The observed width $\Delta x = x_{pvtx} - x_{beam}$ and $\Delta y = y_{pvtx} - y_{beam}$ for run 155130 in the $z = 0$ cm bin. ($-5 < z < 5$ cm)

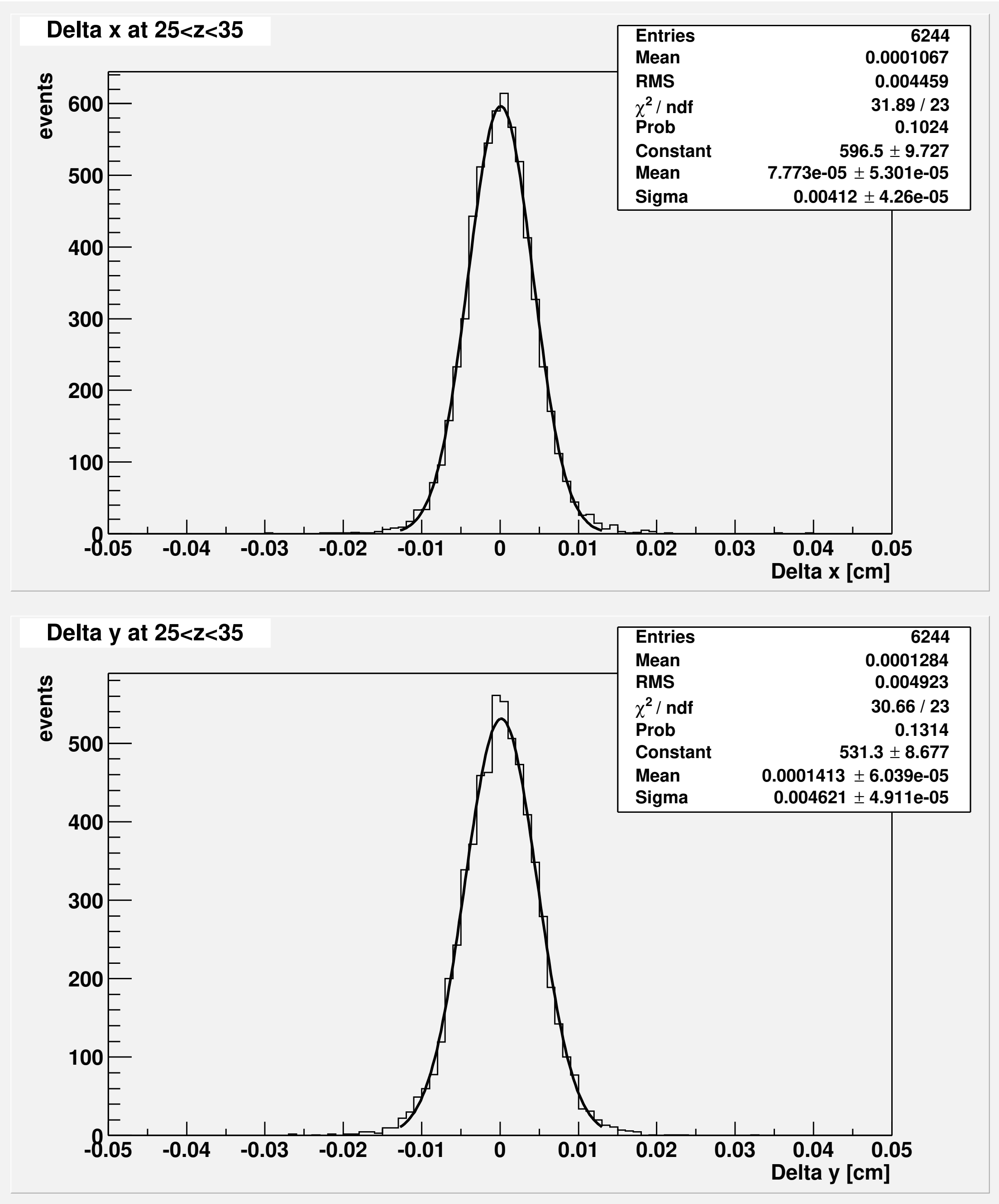

Figure 2: The observed width $\Delta x = x_{pvtx} - x_{beam}$ and $\Delta y = y_{pvtx} - y_{beam}$ for run 155130 in the $z = 30$ cm bin. ($25 < z < 35$ cm)

Store	Run Number	Date	Luminosity (nb^{-1})	κ_x	κ_y
1574	148824	Jul 23/02	580	1.90 ± 0.11	1.85 ± 0.10
1622	149387	Aug 3/02	571	1.94 ± 0.11	1.88 ± 0.11
1918	153416	Oct 28/02	565	1.80 ± 0.10	1.86 ± 0.11
1981	154452	Nov 20/02	696	1.85 ± 0.11	1.92 ± 0.11
1999	154654	Nov 26/02	1127	1.85 ± 0.10	1.82 ± 0.10
2045	155129	Dec 7/02	537	1.85 ± 0.11	1.83 ± 0.10
2045	155130	Dec 7/02	812	1.83 ± 0.10	1.86 ± 0.11
Combined				1.86 ± 0.05	1.86 ± 0.04

Table 2: Scale factors on the vxprim reported uncertainty for selected high luminosity runs.

where κ is the scale factor that corrects the reported vxprim uncertainty σ_{vtx}. This calculation will be done independently for x and y for each z-bin.

Examples of the distribution of the vxprim uncertainty appear in Figure 3 for Run 154452. The mean uncertainty in each z-bin is shown in Figure 4. Note that the highest reported vxprim uncertainties occur at $z = \pm 20$ cm. This bin is where the bulkheads of the different silicon barrels meet (± 16 cm), resulting in a higher average vertex uncertainty. As expected, uncertainties in x and y seem to be correlated.

Monte Carlo studies have shown that the vxprim uncertainty is an underestimate of the true primary vertex uncertainty. To estimate the scale factor by which the vertex uncertainty is underestimated in the data, we take the primary vertices found by vxprim and bin them by vxprim reported uncertainty. We plot the square of the observed width distributions σ^2_{obs} vs. the square of the vxprim uncertainty σ^2_{vtx}. Using Equation 2, we fit a straight line to this and take the slope as κ^2. Incidentally, the y-intercept of this plot gives a measure of the intrinsic beam width σ^2_{beam} (averaged over z) and provides a good cross-check for our beam width determination. An example for Run 154452 of this plot and straight line fit is shown in Figure 5. Here we determine that $\kappa_x = 1.85 \pm 0.11$ and $\kappa_y = 1.92 \pm 0.11$. The y-intercept of the plots give an average beam width of 30.7 ± 1.9 μm in x and 29.1 ± 2.2 μm.

To ensure that the scale factor is independent of z, we compute the scale factor as above, but independently for each z-bin. The results are shown for Run 154452 in Figure 6. The deviations of the scale factor are within the statistical uncertainties, and we conclude that there is no z-dependence on the vxprim scale factor. We check the independence of the scale factor from z in other runs as well, and come to the same conclusion. Therefore, we obtain one scale factor κ averaged over all z-coordinates of the primary vertex.

We obtain scale factors for 7 high luminosity runs. These are shown in Table 2. Since these scale factors are consistent within their statistical precision, we use the average scale factor from these runs, with the uncertainty derived from the standard deviation of these 7 measurements. We get $\kappa_x = 1.86 \pm 0.05$ and $\kappa_y = 1.86 \pm 0.04$.

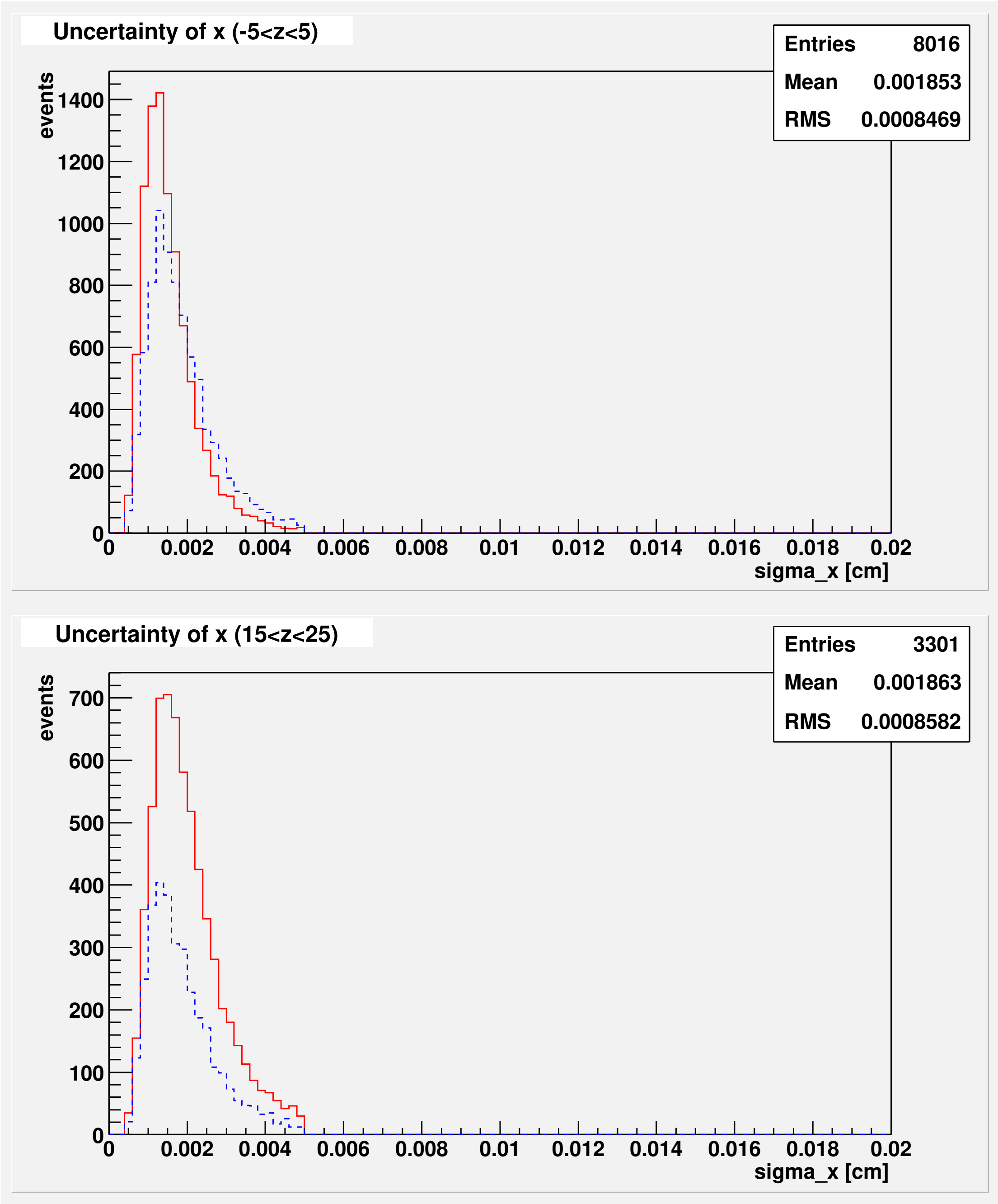

Figure 3: Vertex uncertainty distribution for the x-coordinate in Run 154452 as reported by vxprim. The top histogram shows the distribution of σ_{vtx} at the $z = 0$ cm bin in solid line, and the $z = 10$ cm bin in dashed line. The bottom histogram shows the uncertainty distribution for $z = 20$ cm in solid line and $z = 40$ cm in dashed line.

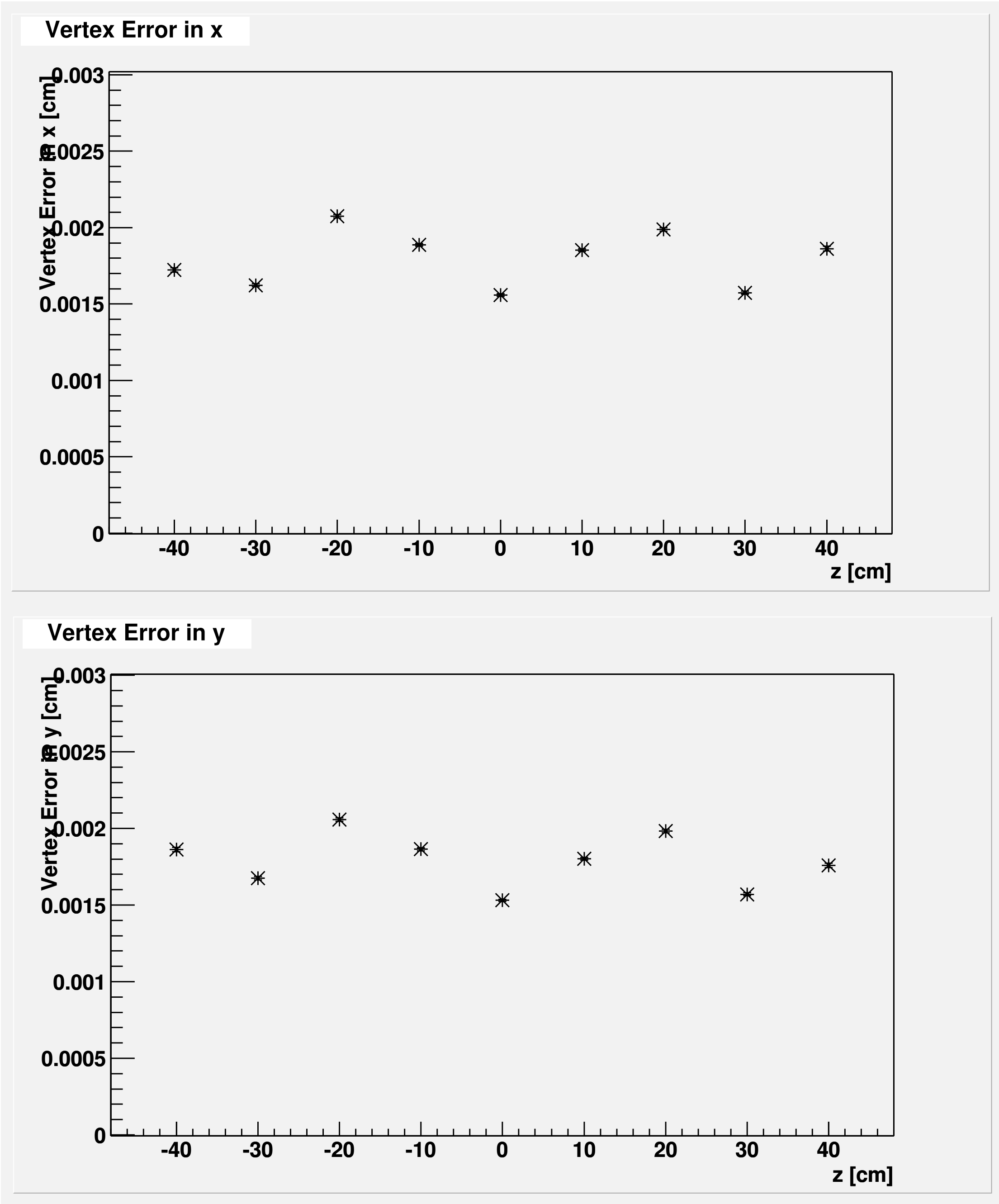

Figure 4: The mean vertex uncertainty as reported from vxprim for x and y is shown as a function of z-bin for Run 154452. (These uncertainties have not been corrected by the scale factor and thus are underestimated.)

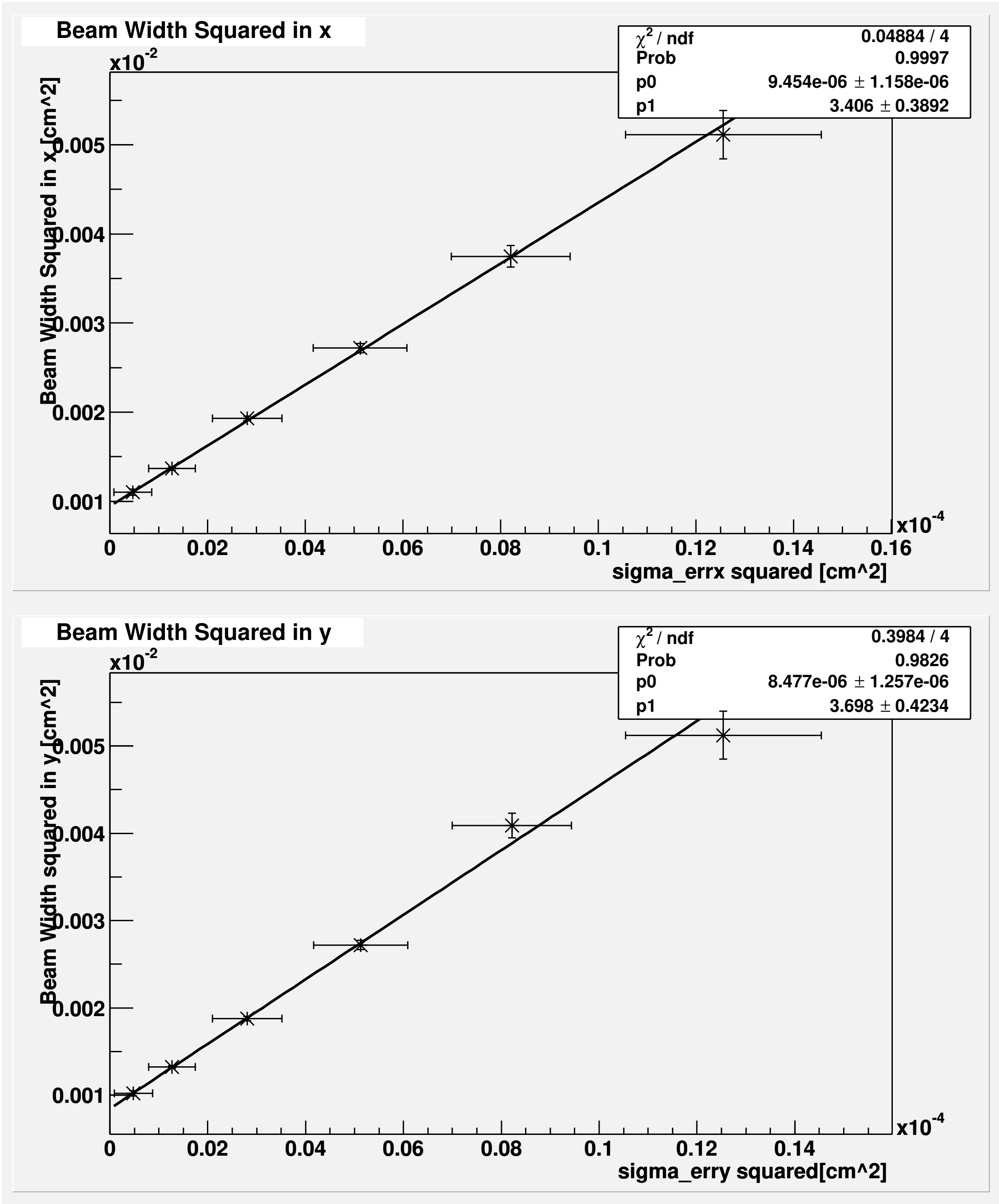

Figure 5: A plot of σ^2_{obs} vs. σ_{vtx} for Run 154452 for x and y. We fit a straight line to this and interpret the slope as the vxprim scale factor κ^2.

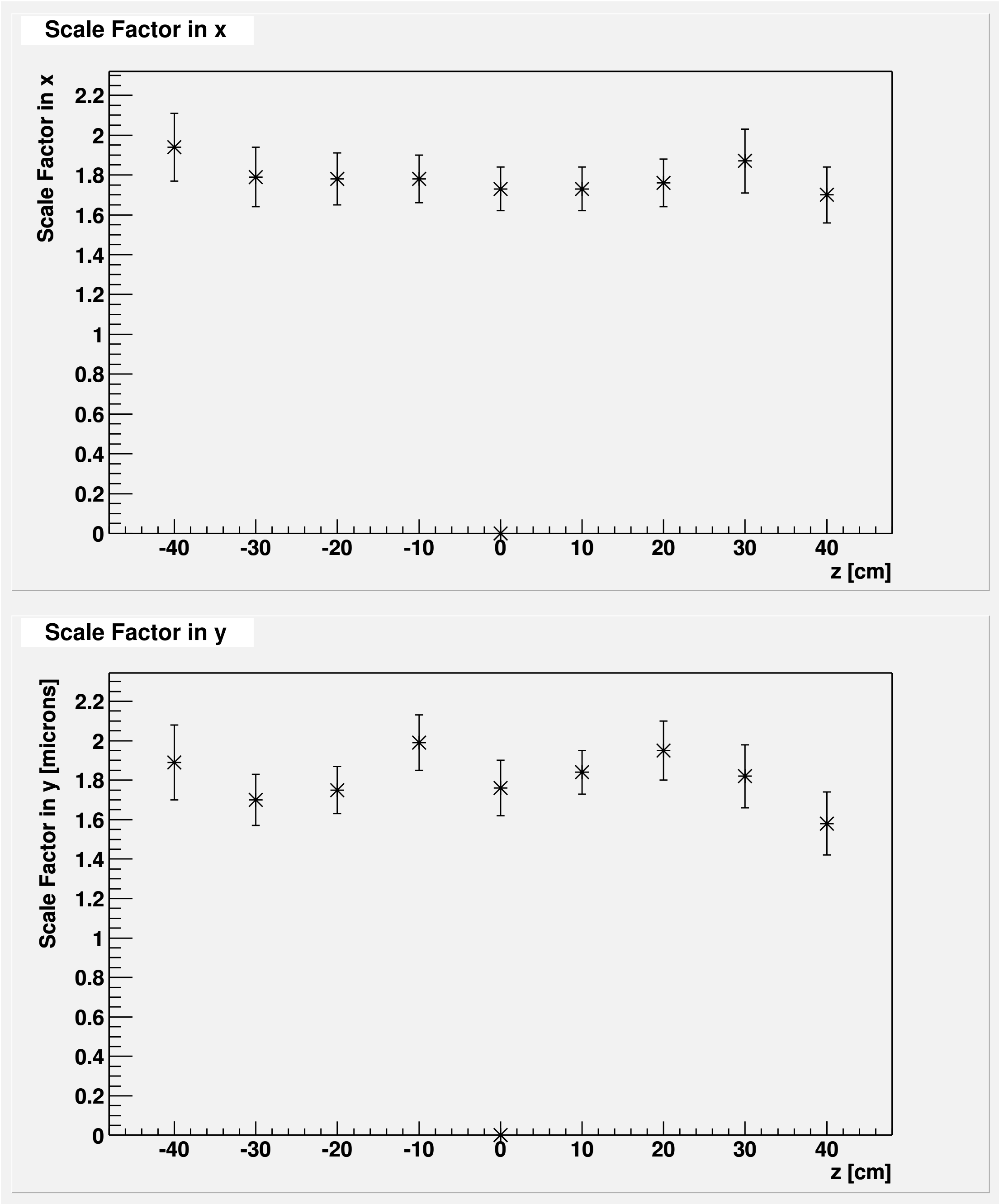

Figure 6: The z-dependence of the vxprim uncertainty scale factor κ for Run 154452 in x and y. The scale factor appears to be independent of z.

Run Number	$\epsilon * 10^{-7}$ (cm)	β^* (cm)	z_0 (cm)
148824	1.69 ± 0.07	41.0 ± 2.6	13.8 ± 1.7
149387	1.41 ± 0.06	35.3 ± 2.4	12.6 ± 1.6
153416	1.30 ± 0.05	30.6 ± 2.6	9.6 ± 1.6
154452	1.53 ± 0.07	41.7 ± 3.0	12.3 ± 1.8
154654	1.69 ± 0.07	43.5 ± 2.9	13.7 ± 1.8
155129	1.45 ± 0.06	36.5 ± 2.8	11.1 ± 1.7
155130	1.73 ± 0.06	37.3 ± 2.3	13.2 ± 1.5
Combined	1.49 ± 0.03	37.1 ± 1.1	12.1 ± 0.7

Table 3: Beam width parameters for x in selected high luminosity runs. The combined result assumes that the parameters are stable from store to store. Run 148824 is excluded in the combined result.

3 Evaluation of the Beam Width

Using the scale factors κ_x, κ_y for the vxprim vertex uncertainty, we can now evaluate the beam width using the equation

$$\sigma_{beam} = \sqrt{\sigma^2_{observed} - \kappa^2 \bar{\sigma}^2_{vtx}}. \tag{3}$$

We do this for each z-bin independently, and arrive at beam widths for different values of z. Once we have this, we can fit the z-dependence of the beam width to

$$\sigma_{beam} = \sqrt{\epsilon(\beta^* + (z - z_0)^2/\beta^*)}, \tag{4}$$

where ϵ is the tranverse emittance of the beam, β^* is the amplitude function at the interaction point, and z_0 is the location of the beam width minimum. Examples of this z-dependence fit for various runs are shown in Figures 7-10.

We perform fits for ϵ, β^*, and z_0 on all 7 runs mentioned previously. The results are shown in Tables 3 and 4.

Assuming that the beam width parameters do not change appreciably from store to store, we perform a weighted average of all runs together to get a combined result. However, we exclude Run 148824 since there is evidence of significant drifting of the beam during the course of this run, along with other anomolous beam conditions. The results are: $\epsilon = (1.49 \pm 0.03) \times 10^{-7}$ cm, $\beta^* = 37.1 \pm 1.1$ cm, and $z_0 = 12.1 \pm 0.7$ cm for x, and $\epsilon = (1.49 \pm 0.02) \times 10^{-7}$ cm, $\beta^* = 38.0 \pm 1.0$ cm, and $z_0 = -8.2 \pm 0.5$ cm for y, where the uncertainties quoted are statistical only. This implies that the beam width minimum at the point z_0 is 23.5 microns in x and 23.8 microns in y.

4 Discussion of the Result

The nominal Tevatron accelerator parameters for the beam width are $\epsilon = 1.40 * 10^{-7}$ cm for the tranverse emittance [4], $\beta^* = 35$ cm for the interaction point amplitude

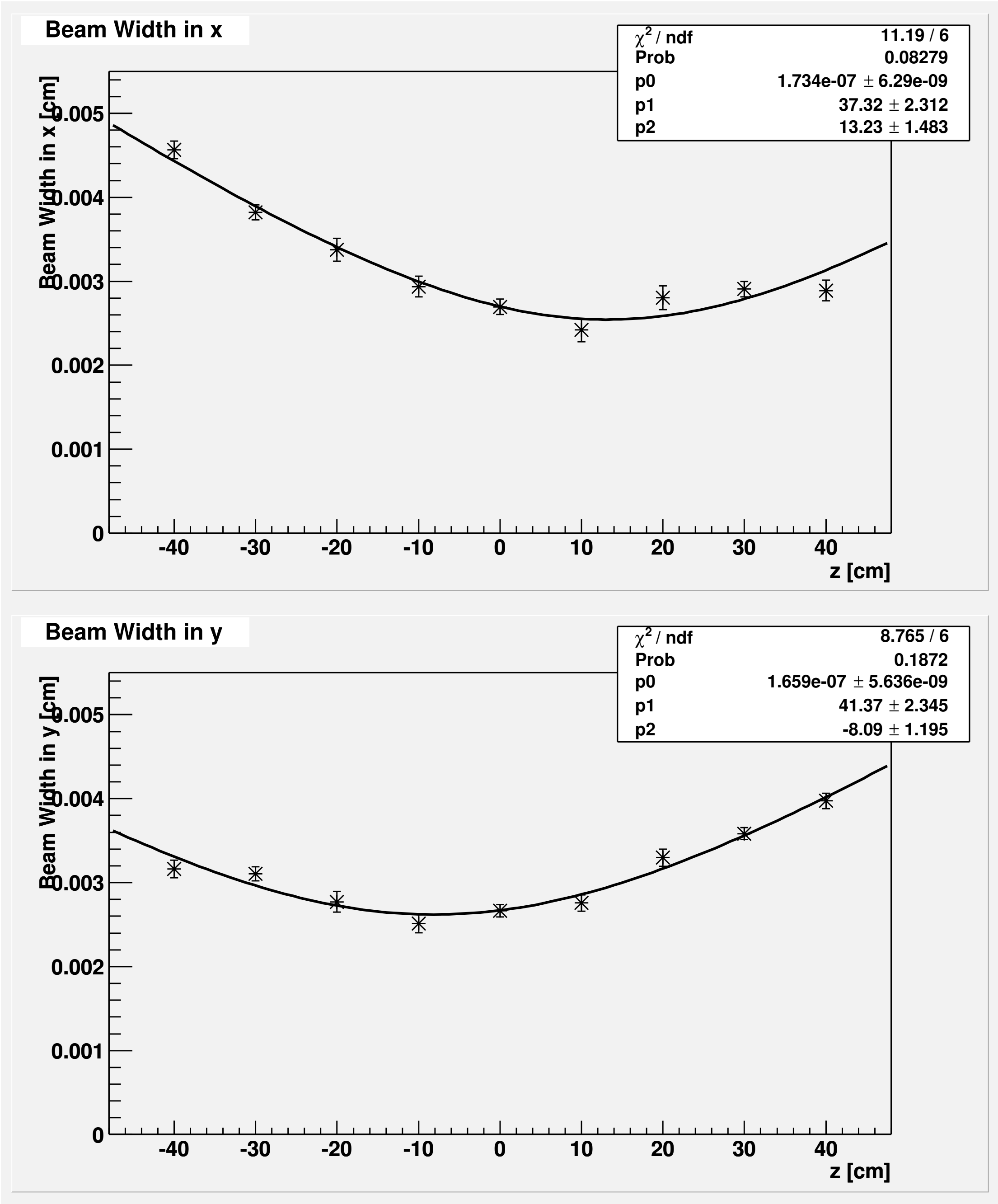

Figure 7: Run 155130 beam width z-fit.

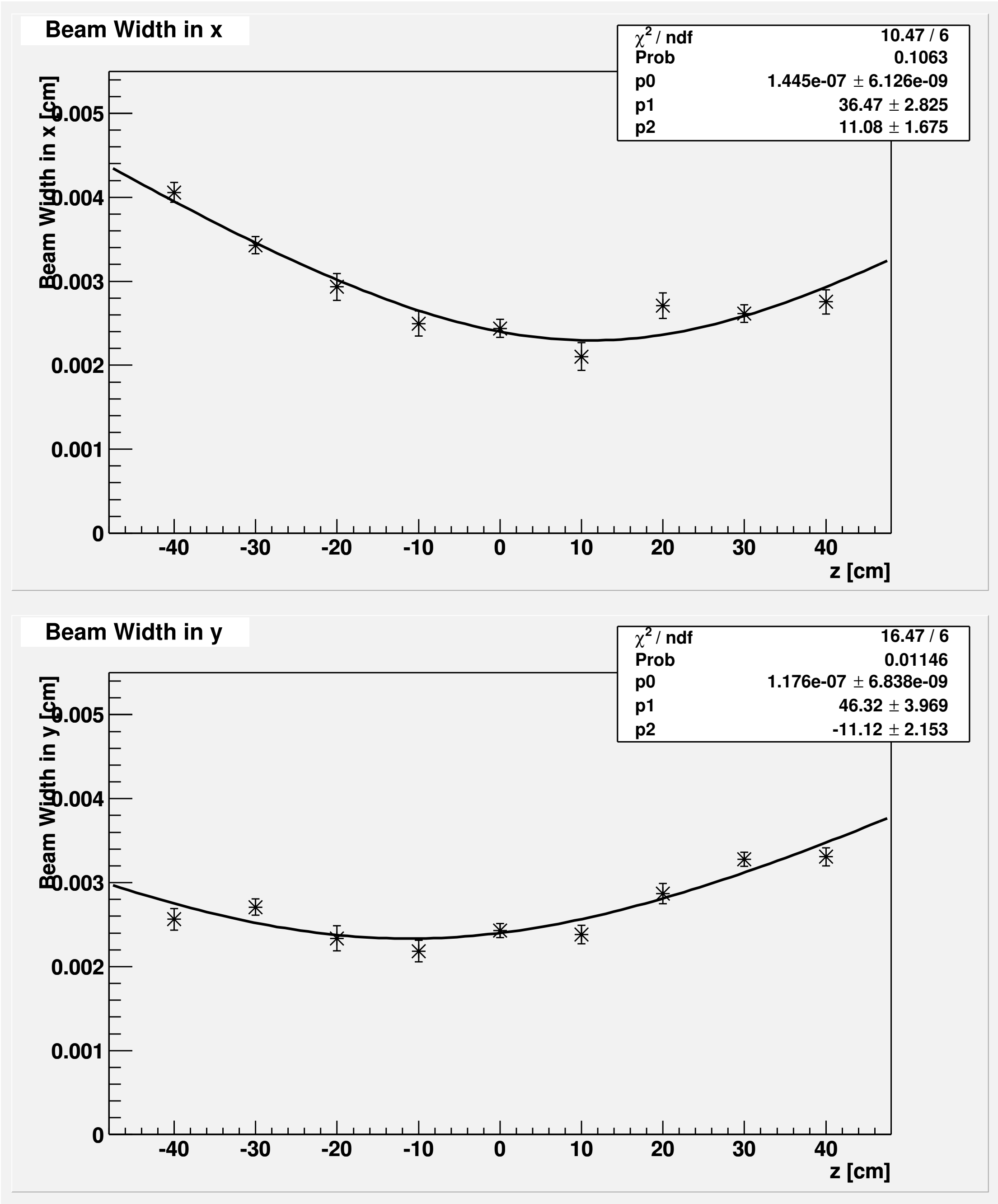

Figure 8: Run 155129 beam width z-fit.

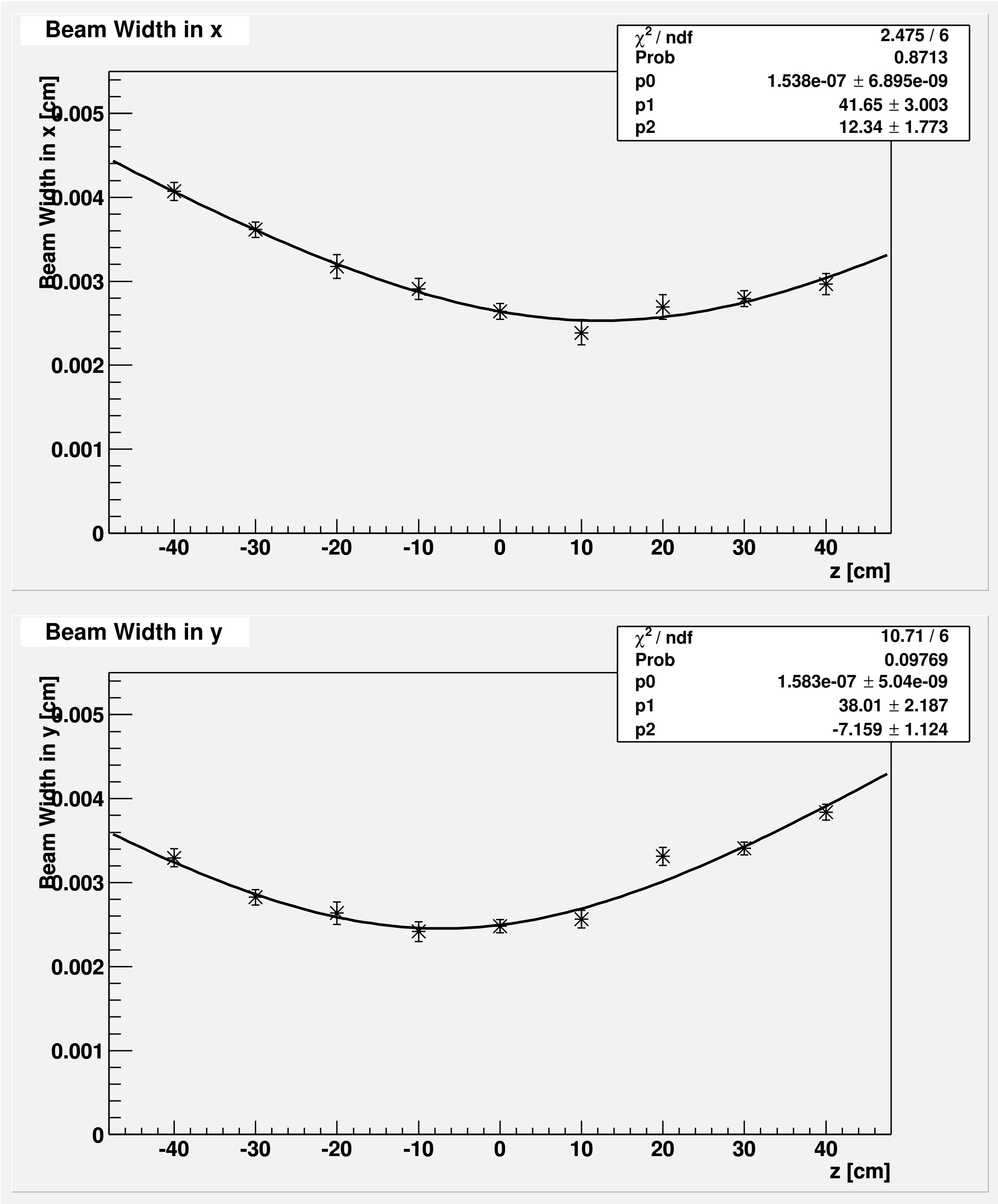

Figure 9: Run 154452 beam width z-fit.

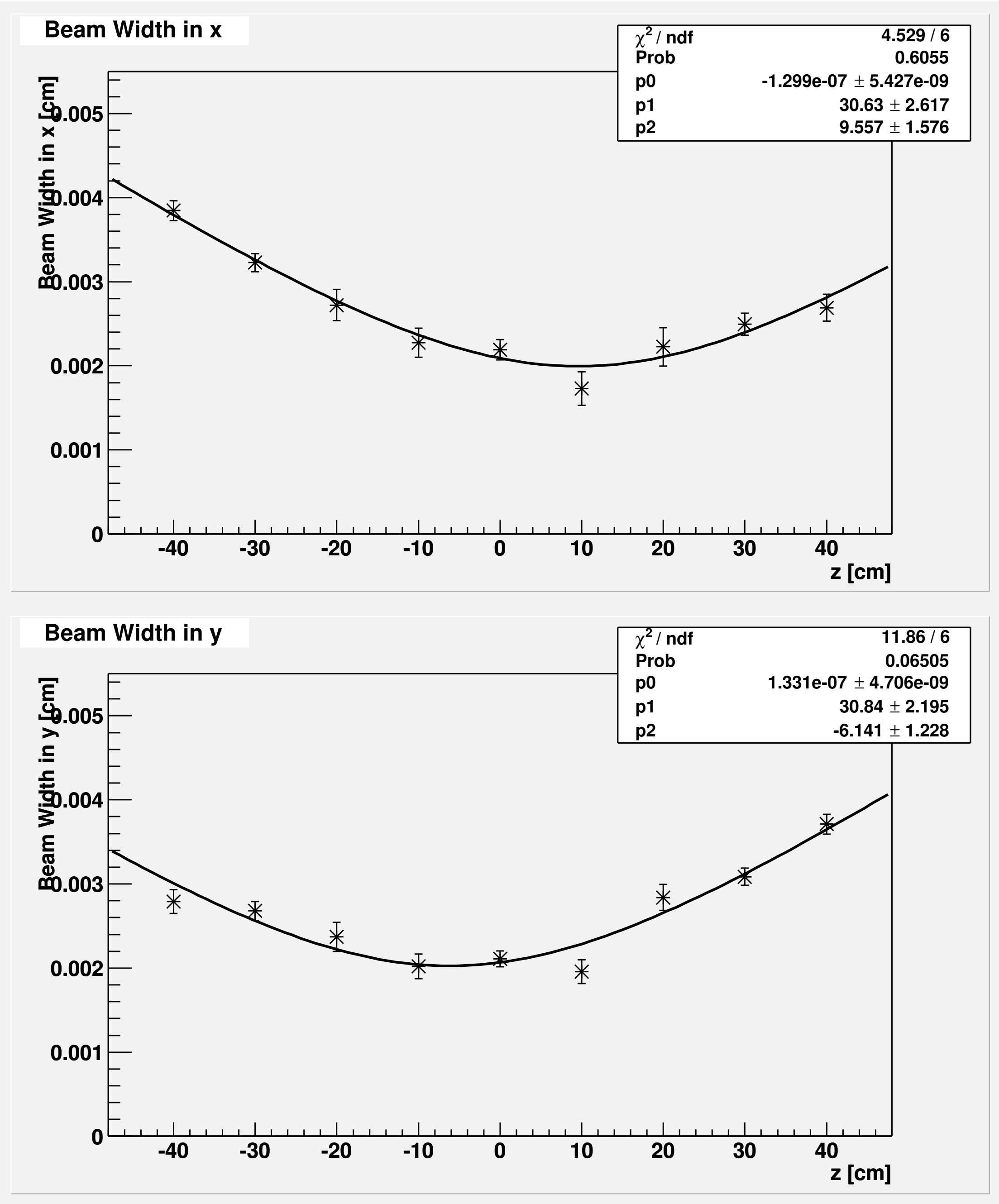

Figure 10: Run 153416 beam width z-fit.

Run Number	$\epsilon * 10^{-7}$ (cm)	β^* (cm)	z_0 (cm)
148824	2.31 ± 0.11	54.3 ± 3.0	-20.7 ± 2.2
149387	1.36 ± 0.05	36.9 ± 2.3	-9.1 ± 1.3
153416	1.33 ± 0.05	30.8 ± 2.2	-6.1 ± 1.2
154452	1.58 ± 0.05	38.0 ± 2.2	-7.2 ± 1.1
154654	1.66 ± 0.05	40.7 ± 2.2	-9.7 ± 1.2
155129	1.18 ± 0.07	46.3 ± 4.0	-11.1 ± 2.2
155130	1.66 ± 0.05	41.4 ± 2.3	-8.1 ± 1.2
Combined	1.49 ± 0.02	38.0 ± 1.0	-8.2 ± 0.5

Table 4: Beam width parameters for y in selected high luminosity runs. The combined result assumes that the parameters are stable from store to store. Run 148824 is excluded in the combined result.

function, and $z_0 = 0$ [5]. It should be noted that the tranverse emittance can vary from store to store, while β^* and z_0 should be more stable. Our results confirm that the emittance ϵ and the amplitude function β^* are slightly larger than the nominal Tevatron design parameters.

The main contributions to the uncertainty on the beam width come from the statistical uncertainty from the Gaussian fit to vertices in each z-bin, and from the uncertainty on the vxprim scale factor κ. Though we make no attempt to obtain a systematic uncertainty on the beam width or on κ, we would like to discuss the implications of a systematic bias of κ on the beam width. Using the equation

$$\sigma_{beam}^2 = \epsilon(\beta^* + (z - z_0)^2/\beta^*) = \epsilon/\beta^*(\beta^{*2} + (z - z_0)^2), \tag{5}$$

we see that the curvature of the functional form is given by the ratio ϵ/β^*, while the size of the beam width scales with β^*, assuming the ratio of tranverse emittance to amplitude function remains constant. Assuming that the shape of the beam width dependence on z is correct, a systematic bias of 5% on κ would result in a systematic effect of order 10% on β^* ($\sim$ 3 cm) and ϵ. Another possible systematic uncertainty on the beam width may arise from alignment effects, which would be independent of the uncertainty on the vxprim scale factor κ.

It is also interesting to note that the minimum of the beam width in x is consistently found to be in the $z > 0$ cm region, while for the beam width in y, it is found to be at $z < 0$ cm, suggesting the x and y foci for the beam are at different spots along z.

The correlation coefficients for the beam width parameters for x and y are shown in Tables 5, 6 for Run 154654. We see that ϵ and β^* are anticorrelated to a large degree, while $|z_0|$ is anticorrelated with ϵ and correlated with β^*.

We can also cross check our evaluation of the beam width by looking at pull

Parameter	ϵ	β^*	z_0
ϵ	1.00	-0.804	-0.806
β^*	-0.804	1.00	0.677
z_0	-0.806	0.677	1.00

Table 5: Correlation Matrix for beam width parameters in Run 154654 in x.

Parameter	ϵ	β^*	z_0
ϵ	1.00	-0.743	0.682
β^*	0.682	1.00	-0.525
z_0	-0.806	-0.525	1.00

Table 6: Correlation Matrix for beam width parameters in Run 154654 in y.

distributions of vxprim vertices. Defining the pull as

$$\frac{(x_{vtx} - x_{beam})}{\sqrt{\sigma_{beam}^2 + \kappa^2 \sigma_{vtx}^2}}$$

we plot the pull distribution of vxprim vertices in Figure 11. We obtain pull widths of 1.026 ± 0.003 in x and 1.022 ± 0.003 in y, confirming that our evaluation of the beam width and scale factor is correct to $\sim 2\%$.

5 Other Considerations and Future Plans

Our parametrization of the beam width can be used to obtain a more precise value for the pimary vertex uncertainty in physics analyses. For some physics analyses, there may not be a need to use the full functional form of the Tevatron beam width at CDF. We can quote an average value for the beam width over all values of z. To do this, we perform a Gaussian fit to all vertices that satisfy our previous criteria without separating them by z-coordinate. We also subtract in quadrature the vxprim error enhanced by the scale factor that we found previously. The average beam widths for the 7 runs are shown in Table 7. Assuming that the average beam widths do not change appreciably for these runs, we take the weighted average of these widths, we find that the average beam width in x is $\sigma_x^{avg} = 29.6 \pm 0.4$ μm, and in y is $\sigma_y^{avg} = 30.4 \pm 0.4$ μm. As above, we have excluded Run 148824 in the determination of this combined result.

We also investigate the possibility of a change in beam width over the course of a run. We take runs 149387, 153416, and 155130 and break them up into 3 sections. We plot the widths separately for these runs. These are shown in Figures 12-14. Note that there is no significant difference between the widths in the different sections of these runs.

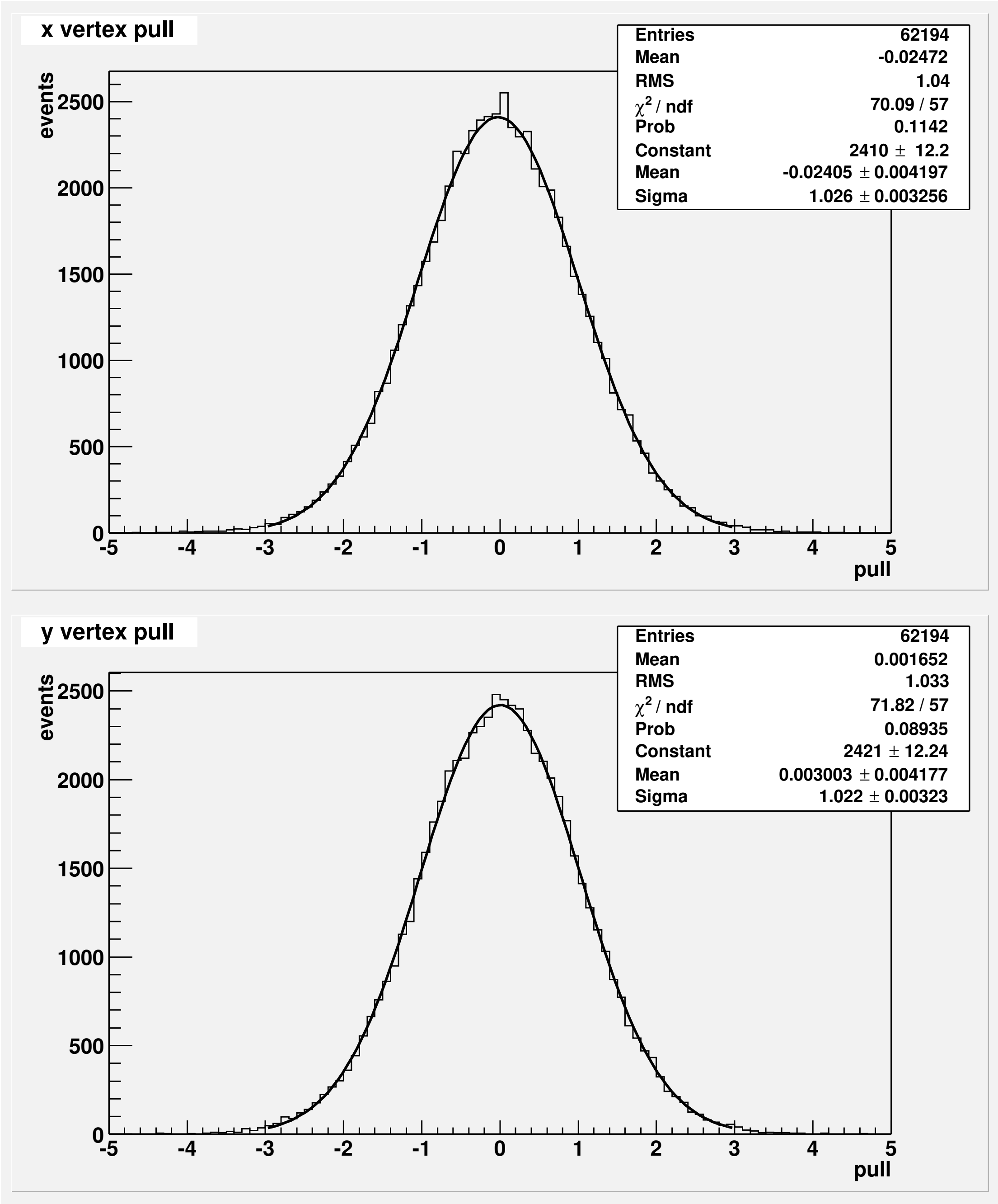

Figure 11: The pull distribution for vxprim vertices in Run 155130.)

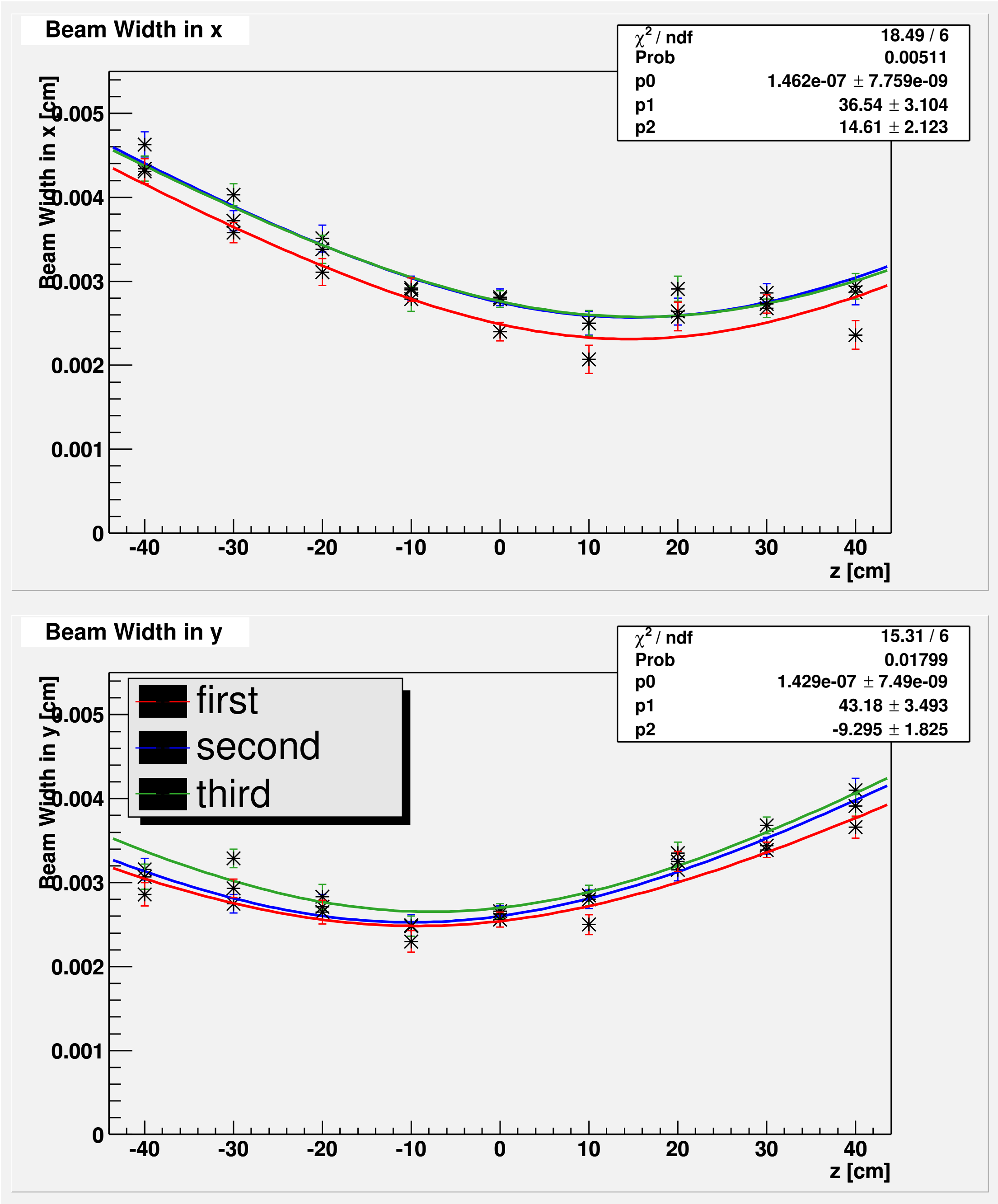

Figure 12: Run 155130 beam widths for different portions of the run.

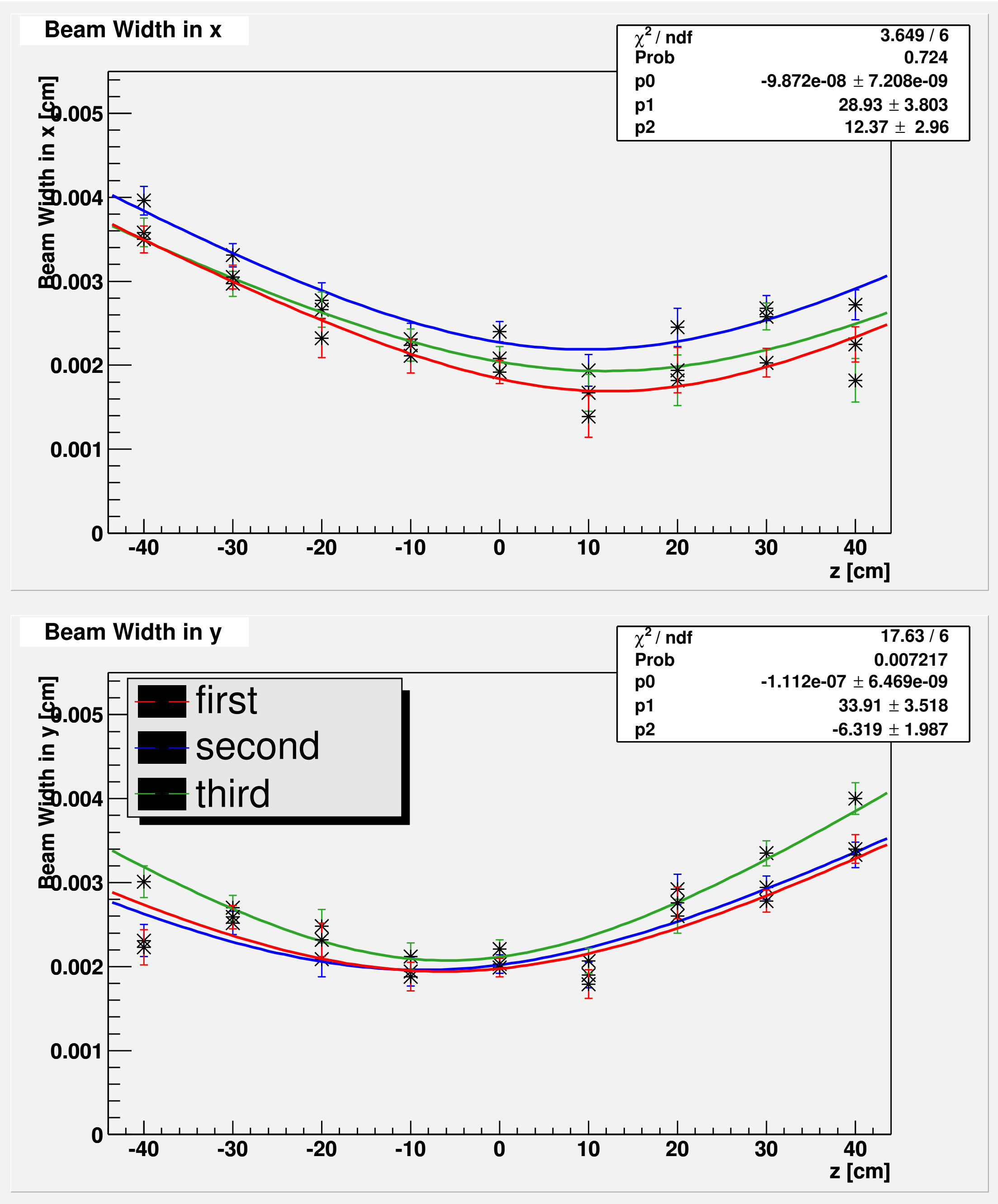

Figure 13: Run 153416 beam widths for different portions of the run.

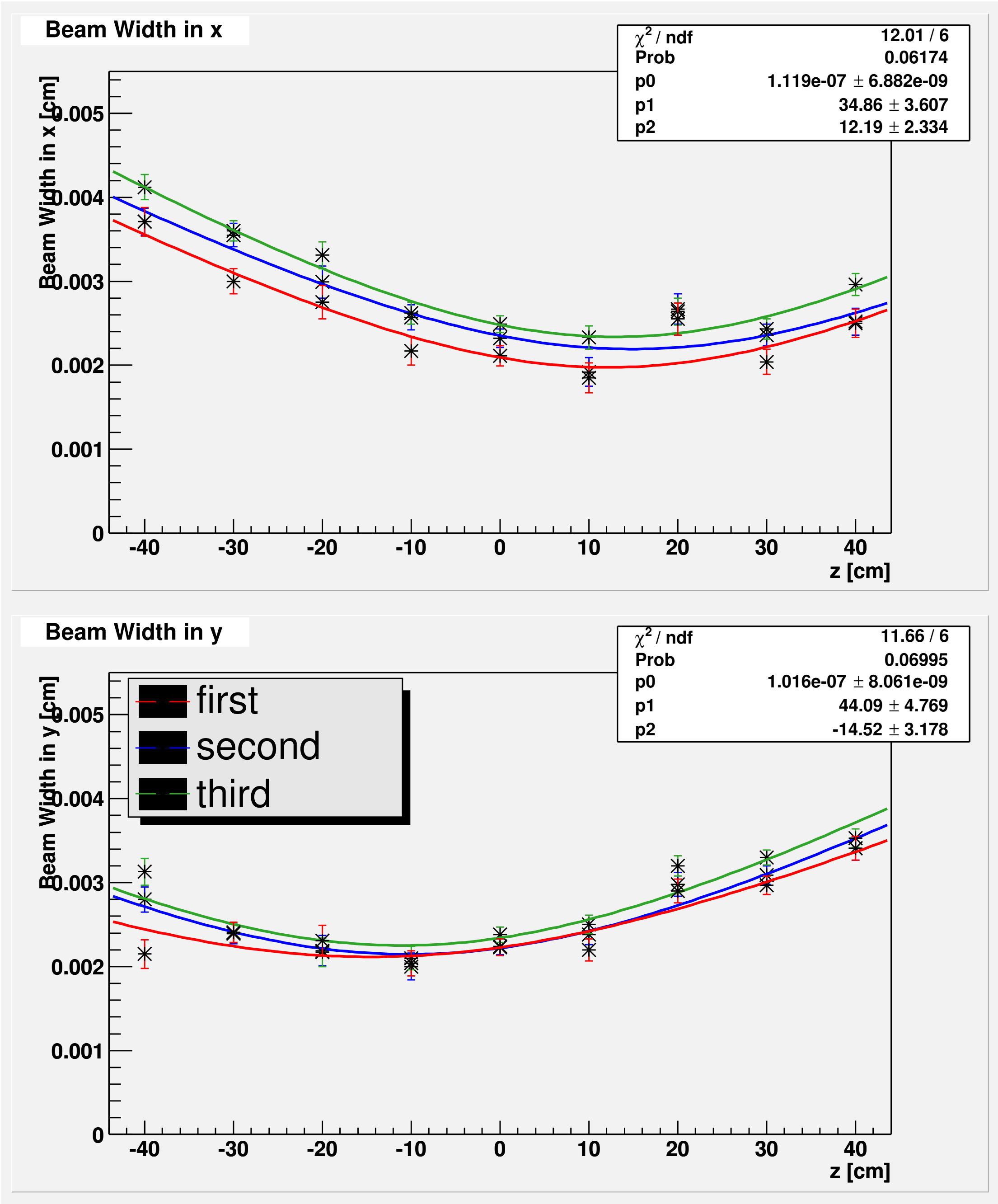

Figure 14: Run 149387 beam widths for different portions of the run.

Run Number	$\sigma_x^{avg}(\mu\text{m})$	$\sigma_y^{avg}(\mu\text{m})$
148824	31.3 ± 1.0	41.0 ± 0.6
149387	28.3 ± 1.1	36.4 ± 0.9
153416	25.9 ± 1.3	25.6 ± 1.1
154452	30.1 ± 1.0	29.7 ± 0.8
154654	32.0 ± 1.0	31.0 ± 0.8
155129	28.3 ± 1.1	27.2 ± 0.9
155130	31.2 ± 1.0	30.9 ± 0.8
Combined	29.6 ± 0.4	30.4 ± 0.4

Table 7: The average widths for x and y for high luminosity runs. The combined result assumes that the average beam width is stable from store to store. The combined result excludes Run 148824.

For the future, we are working on automating the beam width evaluation process, so that each run gets assigned a beam width with fit parameters ϵ, β^*, and z_0 for x and y as well as the averaged beam width σ_x^{avg} and σ_y^{avg}. This information will be stored in the Alignment database so that it can be accessed easily for analyses requiring the beam width.

References

[1] Wolfgang Wagner, "An Estimate of the Average Beam Width at CDF", CDF Note 6104 (2002).

[2] Hartmut Stadie *et al.*, "Vxprim in Run II", CDF Note 6047 (2002).

[3] Without using TrackRefitModule, the primary vertices that are found will be incorrect. TrackRefitModule is found in TrackingUserMods/src/

[4] The transverse emittance ϵ is related to the 95% normalized emittance $\epsilon_{95\%}^N$ by $\epsilon = \frac{\epsilon_{95\%}^N}{6\pi(\beta\gamma)}$.

[5] Fermilab Beams Division, "Run II Handbook", http://www-bd.fnal.gov/runII/index.html