LUNAR LASER TESTS OF GRAVITATIONAL PHYSICS

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Analyses of laser ranges from Earth to Moon yield several tests of interest to gravitational physics. Examined are the equivalence principle, geodetic precession, PPN β and γ based on the point-mass interaction, and invariance of the gravitational constant G. Ranges extend into year 2000 and the weighted rms range residual for the past four years is 1.8 cm after fitting parameters associated with the Earth, Moon, and orbit. The improved range accuracy, the longer data span, and improved modeling result in solution uncertainties for gravitational physics parameters which are reduced by factors of two or more compared to 1996 results. The rate for G improves seven fold. The lunar data has moderate sensitivity to solar oblateness.

1 Lunar Laser Ranging

There is a three decade span of accurate laser ranges from three observatories on the Earth to four retroreflectors on the Moon. Participating observatories are McDonald, Haleakala, and Observatoire de la Côte d'Azur/CERGA. The lunar retroreflectors are at the Apollo 11, 14, and 15 sites plus the Lunakhod 2 rover. The 13,985 laser ranges span March 1970 to April 2000.

For the last four years the weighted rms scatter after the fits is 1.8 cm. This scatter is 0.5×10^{-10} relative to the 385,000 km mean distance of the Moon. For the motion of the Earth-Moon system about the Sun, $v^2/c^2 \approx GM/rc^2 \approx 10^{-8}$. The relative range accuracy is small enough to test for major relativistic effects.

A review of Lunar Laser Ranging (LLR) [1] includes results for gravitational physics. A second publication [2] is devoted to gravitational physics. This paper updates those results.

2 Solutions

All fits of the ranges involve a number of standard solution parameters for the Earth, Moon, and orbit. In this paper five separate least-squares solutions are presented. Each involves the standard solution parameters plus one or more parameters for testing gravitational physics:

- Equivalence principle
- PPN β and γ
- Geodetic precession
- Changing gravitational "constant" G
- Solar J_2

Background discussion of the first four tests is available. [2]

Least-squares solutions require partial derivatives of range with respect to all solution parameters. Partial derivatives of orbit and lunar rotation variations are generated by numerical integration.

3 Equivalence Principle

3.1 Solution

The equivalence principle test depends on the relative acceleration of the Earth and Moon in the gravitational field of the Sun. A failure of the equivalence principle would polarize the lunar orbit along the Earth-Sun line. [3,4,5] The principal range signature has the 29.53 day synodic period of the new-full-new Moon cycle. The solution parameter is the ratio of gravitational to inertial mass M_G/M_I. The test is sensitive to the difference in M_G/M_I between the Earth and Moon.

The solution gives

$$\left(\frac{M_G}{M_I}\right)_{EARTH} - \left(\frac{M_G}{M_I}\right)_{MOON} = (-1 \pm 2) \times 10^{-13}. \tag{1}$$

This solution is equivalent to a range variation of 2 ± 5 mm at the 29.53 day synodic period. The solution includes fit parameters for the GM of the Earth-Moon system and solid-body tidal displacements on the Moon. GM correlates with the equivalence principle by 0.34. The nonuniform distribution due to a lack of ranges at new and full Moon causes the correlation. Tides on the Moon also contribute to the uncertainty.

3.2 Implications

The LLR test is sensitive to equivalence principle violations, whether due to composition or self energy. A University of Washington equivalence principle laboratory experiment [6] is designed to simulate the compositional differences of the Earth and Moon. That acceleration uncertainty is 3.2×10^{-13}. The combination with the Lunar Laser result, in order to extract the self-energy contribution to the equivalence principle, gives $(-1 \pm 4) \times 10^{-13}$ for the self-energy part of M_G/M_I. To convert to the linear combination of the PPN parameters β and γ divide by -4.45×10^{-10} to get

$$4\beta - \gamma - 3 = 0.0002 \pm 0.0009. \tag{2}$$

In order to extract β, the equivalence principle result of eq. (2) for $4\beta - \gamma - 3$ can be combined with the radiometric and VLBI determinations of γ. The published uncertainty for the Viking gravitational time delay test of γ is 0.002. [7] Published uncertainties from VLBI are 0.002 [8] and 0.0022. [9] These γ uncertainties dominate the resulting uncertainty for β,

$$|\beta - 1| \leq 0.0006. \tag{3}$$

Earlier LLR equivalence principle tests are available in publications [1,2,10,11], and earlier references therein. A contemporary LLR test includes a more extensive discussion of the equivalence principle test and the implications for PPN parameters. [12]

4 PPN β and γ

This test of parameterized post-Newtonian β and γ is distinct from the β and γ sensitivity through the equivalence principle. Here, β and γ are parameterized through the N-body point mass interaction. In addition to the dynamical partial derivatives from numerical integration, γ also contributes through the gravitational time delay.

The solution gives

$$\beta - 1 = -0.001 \pm 0.004 \tag{4}$$

$$\gamma - 1 = 0.002 \pm 0.004, \tag{5}$$

where β and γ are highly correlated at -0.95. Other substantial correlations exist with plate motion in longitude and lunar J_2. The sum of β and γ is better determined than each separately.

$$\beta + \gamma - 2 = 0.0007 \pm 0.0014 \tag{6}$$

The results of this section and eq. 2 of the previous section are derived from the same data. While the physical effects are different and the solutions are separate, a joint solution would be needed to combine all of the LLR information on the PPN parameters.

5 Geodetic Precession

For the Earth-Moon system orbiting the Sun, the expected geodetic precession of the lunar orbit is 19.2 milliarcseconds/yr (mas/yr). The geodetic precession is implicit in the previous section's solution for γ. Here the solution parameter K_{GP} measures the departure from General Relativity of a term in the differential equations of motion related to the geodetic precession. [2] In the first approximation the resulting precession matches the geodetic precession, $(1 + K_{GP})$ 19.2 mas/yr for General Relativity plus correction.

The solution gives

$$K_{GP} = -0.0026 \pm 0.0035. \tag{7}$$

The lunar J_2 is also a solution parameter and it influences the result for geodetic precession. They are correlated 0.67.

6 Rate of Change for G

Any change in G directly affects Kepler's third law for both the lunar orbit and the orbit of the Earth-Moon system about the Sun. The latter also affects the solar perturbation terms in the lunar distance.

The solution yields

$$\frac{\dot{G}}{G} = (0.0 \pm 1.1) \times 10^{-12}/yr. \tag{8}$$

There are correlations of 0.4 and 0.3 with Earth tidal dissipation parameters. Both tidal dissipation and a rate for G change the size of the lunar orbit, but the modifications to Kepler's third law and perturbation terms are not the same for the two effects.

7 Solar J_2

A test of the Sun's second-degree gravitational harmonic J_2 gives the value

$$J_2 = (-5 \pm 10) \times 10^{-7}. \tag{9}$$

This dynamical result for the solar J_2 may be compared to the conventional value of 2×10^{-7}. The LLR sensitivity does not indicate a very large value of the solar oblateness, but the accuracy is not yet at the level of the conventional value.

8 Changes in Data, Solutions and Models

Since the 1996 JPL results [2] the uncertainties in gravitational physics parameters have reduced by a factor of two or more. This decrease is from improved range accuracy, increased data span, and improved modeling and solutions.

The changes in range data include: a 6 yr increase in span, the accuracy of the best data has improved from 3 to 1.8 cm, and the number of ranges has increased from 8,427 to 13,985.

Modern solutions have additional solution parameters, e. g., more complete plate motion plus solid-body tides on the Moon. The former influences PPN β and γ while the latter influences the equivalence principle. The lunar orbit, the heliocentric orbit of the Earth-Moon system, and the lunar rotation are improved and the Earth's orientation is better known. Solar J_2 is an optional solution parameter.

Major additions were made to modeling energy dissipation in the Moon. Dissipation at a lunar fluid-core/solid-mantle boundary is now in the dynamical model. Solution parameters were added for both core dissipation and frequency dependent tidal dissipation. [13] These dissipation improvements remove former limitations on the G rate test. [2]

9 Summary

Uncertainties in the Lunar Laser tests of gravitational physics shrink due to improved range accuracy, longer data span, and improved modeling and solutions. Compared to the 1996 results, [2] the tests for the equivalence principle, geodetic precession, and PPN β and γ improve by a factor of two. The rate for G improves by a factor of seven. Additional lunar range data, improved range accuracy, and improved modeling will contribute to future advances in tests of gravitational physics.

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