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Implications of $\mu\nu\gamma\gamma$ Production on Precision Measurements of the W Mass *

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ABSTRACT

The process $p\bar{p} \to \mu^- \bar{\nu}\gamma\gamma$ is calculated including finite lepton mass effects. Implications for precision measurements of the W mass at the Tevatron are discussed.

I. INTRODUCTION

A precise measurement of the W boson mass (M_W) is of fundamental importance in testing the standard model (SM) and in constraining new physics. Future experiments at LEP2 [1] and the Tevatron [2, 3] aim at a precision of about 40 MeV and 20 MeV for M_W, respectively. In order to be able to achieve a precision of $\mathcal{O}(10$ MeV) for M_W in hadron collider experiments, it will be necessary to control not only the higher order QCD corrections, but also the electroweak radiative corrections.

At $\mathcal{O}(\alpha^n)$, W decay with collinear emission of photons from a final state charged lepton of mass m_ℓ gives rise to terms which are proportional to $(\alpha/\pi)^n \log^n(M_W^2/m_\ell^2)$, in n-photon exclusive rates. These terms can have significant effects in determining M_W. At $\mathcal{O}(\alpha)$, photon emission shifts the W mass by [4]

$$\Delta M_W \approx -\frac{\pi\beta}{8}\,\Gamma_W\,, \tag{1}$$

where Γ_W is the total width of the W boson, and

$$\beta = \frac{\alpha}{\pi}\left(\log\frac{M_W^2}{m_\ell^2} - 1\right). \tag{2}$$

When finite detector resolution effects are included, Γ_W is replaced by the uncorrected experimental width. For CDF, the resulting shift is -168 MeV (-65 MeV) in the muon (electron) channel [5]. Similar results are found in the DØ W mass analysis [6]. In the electron case, understanding the exact amount of energy lost in photon bremsstrahlung is also important to determine the E/p distribution, which is used by CDF to determine the energy scale of the central electromagnetic calorimeter [5]. When aiming for a precision of $\mathcal{O}(10$ MeV) in M_W, it is therefore important to control the effects of multiple photon radiation in W events.

Here, we present a calculation of two photon radiation in the $W \to \mu\nu$ channel at the tree level, including the finite mass of the muon which regulates the collinear singularity originating from photons radiated off the final state muon. Initial and final state bremsstrahlung, together with finite W width effects are taken into account in our calculation.

II. CALCULATIONAL DETAILS

The cross section for $p\bar{p} \to \ell^- \bar{\nu}\gamma\gamma$ has been calculated in the limit of massless fermions in Ref. [7] and [8], using the helicity amplitude technique of Ref. [9]. To evaluate the matrix elements for a massive final state charged lepton, we use the MADGRAPH [10] package, which automatically generates the SM matrix elements in HELAS format [11].

However, in order to maintain electromagnetic gauge invariance in presence of finite W width effects, the W propagator and the $WW\gamma$ and $WW\gamma\gamma$ vertex functions in the amplitudes generated by MADGRAPH have to be modified [8, 12]. Finite width effects are included by resumming the imaginary part of the W vacuum polarization, $\Pi_W(q^2)$. The transverse part of $\Pi_W(q^2)$ receives an imaginary contribution

$$\mathrm{Im}\,\Pi_W^T(q^2) = q^2\frac{\Gamma_W}{M_W} \tag{3}$$

while the imaginary part of the longitudinal piece vanishes. The W propagator is thus given by

$$D_W^{\mu\nu}(q) = \frac{-i}{q^2 - M_W^2 + iq^2\gamma_W}\left[g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}(1 + i\gamma_W)\right], \tag{4}$$

with

$$\gamma_W = \frac{\Gamma_W}{M_W}. \tag{5}$$

A gauge invariant expression for the amplitude is then obtained by attaching the final state photons to all charged particle propagators, including those in the fermion loops which contribute to

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$\Pi_W(q^2)$. As a result, the triple and quartic gauge vertices ($\Gamma_0^{\alpha\beta\mu}$ and $\Gamma_0^{\alpha\beta\mu\rho}$) are modified [8, 12] to

$$\Gamma^{\alpha\beta\mu} = \Gamma_0^{\alpha\beta\mu}(1+i\gamma_W), \quad (6)$$

$$\Gamma^{\alpha\beta\mu\rho} = \Gamma_0^{\alpha\beta\mu\rho}(1+i\gamma_W). \quad (7)$$

Due to the mass singular terms which arise from the $\mu\gamma$ collinear region, it is nontrivial to numerically compute the $\mu\nu\gamma\gamma$ cross section. To optimize the Monte Carlo integration, we choose the logarithms of the invariant masses of the $\mu\gamma_1$, the $\mu\gamma_2$, and the $\mu\gamma_1\gamma_2$ system as integration variables:

$$dPS_4 \sim d\hat{s}\, d\log m(\mu\gamma_1\gamma_2)\, d\log m(\mu\gamma_1)\, d\log m(\mu\gamma_2). \quad (8)$$

The resulting cross section is checked for gauge invariance, and is numerically stable when the photons are collinear with the muon.

For $p\bar{p} \to e\nu\gamma\gamma$, due to the smaller electron mass, the singularities for $both$ photons and the electron being collinear, are much more severe than in the muon case. The method used in $p\bar{p} \to \mu\nu\gamma\gamma$ to perform the phase space integration is not sufficient to obtain a stable numerical answer for $p\bar{p} \to e\nu\gamma\gamma$.

III. RESULTS AND DISCUSSION

For our numerical simulations we use the MRSA set of parton distributions [13], $M_W = 80.22$ GeV, and $\alpha = 1/128$. The factorization scale is fixed to the parton center of mass energy, $\sqrt{\hat{s}}$. To simulate detector response, we impose the following acceptance cuts on transverse momenta and pseudorapidities:

$$p_T(\mu) > 25\ \mathrm{GeV}, \quad \not{p}_T > 25\ \mathrm{GeV}, \quad (9)$$

$$|\eta(\mu)| < 1.0, \quad |\eta(\gamma)| < 3.6. \quad (10)$$

If the two photons have a separation smaller than a critical value $\Delta R(\gamma,\gamma) = R_c$, they cannot be discriminated in the detector, and thus will be treated as one photon. Taking the typical cell size of an electromagnetic calorimeter as a criterion [14], we choose $R_c = 0.14$ in the following. The results of our simulations are listed in Table I.

When no separation requirement is imposed on the muon and the photon, approximately 13% (5.7%) of all $W \to \mu\nu$ events contain one photon with a minimum photon transverse energy of $E_T^0(\gamma) = 0.1$ GeV ($E_T^0(\gamma) = 1$ GeV) [12] at $\mathcal{O}(\alpha)$. About 0.8% (0.13%) of all events contain two isolated photons with transverse energies larger than $E_T^0(\gamma) = 0.1$ GeV ($E_T^0(\gamma) = 1$ GeV). If photons are required to be isolated from the muon ($\Delta R(\mu,\gamma) > 0.7$), about 3.8% of the $W \to \mu\nu$ events contain one photon with $E_T(\gamma) > 0.1$ GeV, but only 0.07% have two photons for the same $E_T(\gamma)$ threshold (see Table Ib). For larger photon E_T thresholds, the fraction of two photon events drops very rapidly.

Although the two-photon tree level calculation presented here is not a full next-to-next to leading order calculation of $W^\pm$ production, we can still gain some important information on multiphoton radiation. Knowing the cross section for the lowest order process, σ_0, and for $\mathcal{O}(\alpha)$ one photon emission, σ_1, one can estimate the probability, $P(n)$, for radiating n photons. Neglecting the dependence of the soft photon emission rate on the (hard) quark and lepton configuration, the probability is given by [15]

$$P(n) = \frac{<n>^n}{n!} e^{-<n>}, \quad (11)$$

where $<n> = \sigma_1/\sigma_0$. Comparison with Table I shows that Eq. (11) provides a useful guideline for multiple photon radiation in W events if the minimum photon E_T threshold is smaller than about 3 GeV.

In summary, we have calculated two photon bremsstrahlung in $W \to \mu\nu$ events at hadron colliders. Our calculation takes both initial and final state photon radiation, and finite W width effects into account. We also incorporate finite muon mass effects, which makes our calculation valid over the entire phase space region. Approximately 0.8% of all $W \to \mu\nu$ events are found to contain two isolated photons with transverse energy $E_T(\gamma) > 0.1$ GeV, which roughly corresponds to the tower threshold of the electromagnetic calorimeter of CDF and DØ [14]. Multiple photon bremsstrahlung thus is expected to have a non-negligible effect on the W mass extracted from experiment. Unfortunately, our calculation at present does not produce stable numerical results in the $W \to e\nu$ case, due to the more severe nature of the collinear singularities in the electron case. It, therefore, should only be viewed as a very first step towards a more complete understanding of multiple photon radiation in W boson events.

Table I: Fraction of $W \to \mu\nu$ events (in percent) containing one or two photons at the Tevatron ($\sqrt{s} = 1.8$ TeV), (a) with no isolation cut between the photons and the muon imposed, (b) requiring $\Delta R(\mu,\gamma) > 0.7$. In the two photon case, the two photons are required to fulfill the condition $\Delta R(\gamma,\gamma) > 0.14$. Fractions are obtained by normalization with respect to $\sigma_0 = 0.346$ nb.

(a) no isolation cut

$E_T^0(\gamma)$	$W \to \mu\nu\gamma$	$W \to \mu\nu\gamma\gamma$
0.1	13.4	0.77
0.3	9.60	0.39
1.0	5.67	0.13
3.0	2.62	0.024
10.0	0.32	7.6×10^{-4}
30.0	0.013	1.8×10^{-5}

(b) $\Delta R(\mu,\gamma) > 0.7$

$E_T^0(\gamma)$	$W \to \mu\nu\gamma$	$W \to \mu\nu\gamma\gamma$
0.1	3.76	0.071
0.3	2.60	0.037
1.0	1.46	0.012
3.0	0.65	2.9×10^{-3}
10.0	0.13	2.0×10^{-4}
30.0	0.012	7.4×10^{-6}

IV. REFERENCES

[1] *A. Ballestrero et al.*, in Proceedings of the Workshop on Physics

at LEP2, G. Altarelli,T. Sjostrand and F. Zwirner (eds.), CERN Yellow Report CERN-96-01 (1996).

[2] *D. Amidei et al.*, in *"Future Electroweak Physics at the Fermilab Tevatron: Report of the tev_2000 Study Group"*, eds. D Amidei and R. Brock, FERMILAB-Pub/96-082 (1996), p. 13.

[3] *U. Baur and M. Demarteau*, these proceedings.

[4] *W. Hollik and D. Wackeroth*, FERMILAB-Pub-96/094-T (preprint); *U. Baur, S. Keller and D. Wackeroth*, in preparation.

[5] *F. Abe et al.* (CDF Collaboration), Phys. Rev. **D52**, 4784 (1995); Phys. Rev. Lett. **75**, 11 (1995).

[6] *S. Abachi et al.* (DØ Collaboration) Phys. Rev. Lett. **77**, 3309 (1996) and FERMILAB-Conf/96-251-E, submitted to the *"28$^{\rm th}$ International Conference on High Energy Physics"*, Warsaw, Poland, 25 – 31 July 1996.

[7] *T. Han and R. Sobey*, Phys. Rev. **D52**, 6302 (1995).

[8] *U. Baur, T. Han, N. Kauer, R. Sobey and D. Zeppenfeld*, in preparation.

[9] *K. Hagiwara and D. Zeppenfeld*, Nucl. Phys. **B274**, 1 (1986); Nucl. Phys. **B313**, 560 (1989).

[10] *T. Stelzer and W. F. Long*, Comput. Phys. Commun. **81**, 357 (1994).

[11] *H. Murayama, I. Watanabe, and K. Hagiwara*, KEK Report 91-11 (1992).

[12] *U. Baur and D. Zeppenfeld*, Phys. Rev. Lett. **75**, 1002 (1995).

[13] *A.D. Martin, R.G. Roberts and W.J. Stirling*, Phys. Rev. **D50**, 6734 (1994).

[14] *F. Abe et al.*, (CDF Collaboration) Nucl. Instrum. Meth. **A217**, 387 (1988); *S. Abachi et al.*, (DØ Collaboration) Nucl. Instrum. Meth. **A338**, 185 (1994).

[15] *F. Bloch and A. Nordsieck*, Phys. Rev. **52**, 54 (1937); *D.R. Yennie, S.C. Frautschi, and H. Suura*, Ann. Phys. (New York) **13**, 379 (1961).