Discretization of Deformable Interfaces

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For problems of solid mechanics the Lagrangian description of the material is often considered more natural while for fluid mechanics the Eulerian description tends to be preferred. Historically this distinction has been particularly clear for computational simulations. However, engineering applications are increasingly concerned with problems where both regimes are present in a single domain and coupling mechanisms have become quite important. Because much expertise exists for each regime in isolation and well established simulation codes are in existence for both, an attractive methodology is to couple already existing tools.

We consider a compressible fluid flowing past an elastic solid. The fluid description is given by the compressible Navier-Stokes equations while the solid is described by the equations of elasticity, or elasto-plasticity, together with heat conduction. A Lagrangian formulation is taken for the elastic body and an Eulerian formulation for the fluid equations. At the interface between the fluid and the elastic object we enforce continuity of normal stresses, velocity, temperature, and heat conduction. Several possibilities for the numerical discretization of the coupling problem occur depending on whether the solid/fluid interface is aligned with the computational grids used on the various subdomains or not. One option is that the coupling conditions are imposed directly at the solid/fluid interface. A second possibility is that the conditions are imposed through the use of ghost points on each component grid. A final option is the use of a third membrane grid where the actual coupling conditions are imposed. Interpolation is then used to communicate back the interface quantities to the various solvers as needed. We will discuss advantages and disadvantages of the different options.

We will present analysis in one space dimension which shows that the physically natural solid/fluid interface conditions are well-posed. Furthermore, the analysis is easily modified to show stability for discretizations using summation-by-parts operators near the interface. We will use such a stable method to solve a one dimensional test problem consisting of an elastic rod falling in a compressible fluid. These numerical experiments are used to show the influence of order of accuracy, time integration, and boundary conditions used at the outer boundary of the computational domain. For general coupling of fluid and solid solvers we also consider the case when staggered grid approximations are coupled to non-staggered approximations, or when node based solvers are coupled to solvers using cell averages. In these cases we introduce interpolation operators to transfer values between nodes and cell centers on the component grids, before imposing the interface conditions. We will show some of the properties of the above methods by numerical experiments in two space dimensions, using moving overset grids.

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