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UPDATE OF DEPTH OF FOCUS CONSIDERATIONS FOR RED LIGHT INCIDENT ON A THICK CCD IN A FAST OPTICAL SYSTEM

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Note: This discussion was slightly updated on 11 July 2002 to include Fig. 1. The main conclusion is that depth-of-focus problems are very much mitigated by the high index of refraction of silicon, $n \approx 3.63$, which reduces all incident light angles by $1/n$.

For a CCD at $-120°$ C, the absorption length of light (ℓ) at 950 nm is 120 μm, and at 1000 nm it is nearly 400 μm. At $-100°$ C these absorption lengths are about 100 μm and 325 nm. Since e-h pairs are made with probability decreasing exponentially from the back surface, there are some depth-of-focus questions in a fast optical system—the light cannot be in focus at all depths at which the light interacts.*

The geometry is shown in Fig. 1. The central ray strikes the CCD normally; the extreme ray strikes it at an angle $\approx 1/2f$, where f is the focal ratio of the instrument.

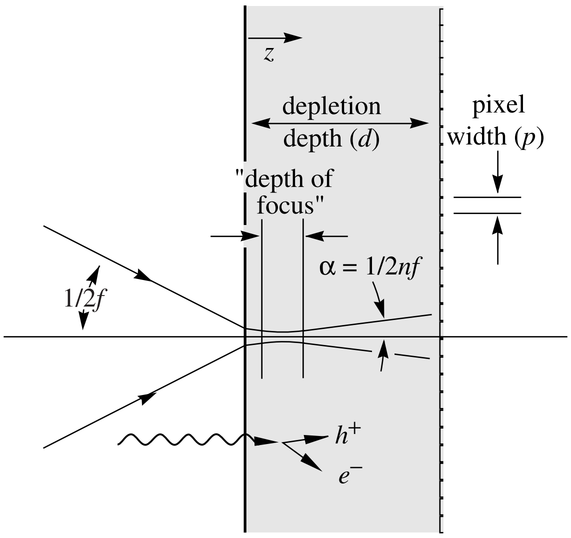

Figure 1: Geometry for defining depth of focus. The incident light has maximal (and therefore nearly typical) angle $\tan^{-1}(1/2f) \approx 1/2f$, where f is the focal ratio of the instrument, in this case $f1$.

We have estimated the resulting image degradation as follows: The out-of-focus image PSF is modeled as a Gaussian stellar image with rms width σ_* convoluted with a uniform circle whose radius R is set by geometric optics. The MTF for the marginal distribution is given by $\phi(u) = 2\exp(-\sigma^2u^2/2)\,J_1(Ru)/Ru$. The variance of this distribution, obtained by evaluating the 2nd derivative at $u = 0$, is $\sigma_*^2 + R^2/4$.

* We are ignoring e-h production by light reflected from the front surface or reflected multiple times. In addition to producing fringes, it can produce a halo as the reflected light reaches greater and greater distances from the central ray.

If the system has a focal ratio f, the divergence (or convergence) angle of the most extreme ray inside the silicon is given by $\alpha = 1/2nf$. Here n is the index of refraction of silicon, which is close to 3.63 in the red and near-infrared. The spot radius as a function of distance away from the best-focus point z_0 is given by $R = \alpha|z - z_0|$. The number of e-h pairs made at z is proportional to $\exp(-z/\ell)$. Averaging the variance by this factor, we obtain

$$\langle \text{Variance} \rangle = \sigma_*^2 + \tfrac{1}{4}\alpha^2\ell^2 + \tfrac{1}{4}\alpha^2\left(\ell - z_0\right)^2 \ . \tag{1}$$

The best focus is at $z_0 = \ell$, where

$$\langle \text{Variance} \rangle|_{\text{min}} = \sigma_*^2 + \tfrac{1}{4}\alpha^2\ell^2 \ . \tag{2}$$

As a function of distance z from the CCD backside, the variance is given by

$$\langle \text{Variance} \rangle = \sigma_*^2 + \tfrac{1}{4}\alpha^2\ell^2 + \tfrac{1}{4}\alpha^2\left(\ell - z\right)^2 \ . \tag{3}$$

Examples of the dependence of the focus spot for several values of σ_* (taking the full width at half maximum, the usual measure of seeing, as multiples of pixel size 15 μm) are shown in Fig. 2.

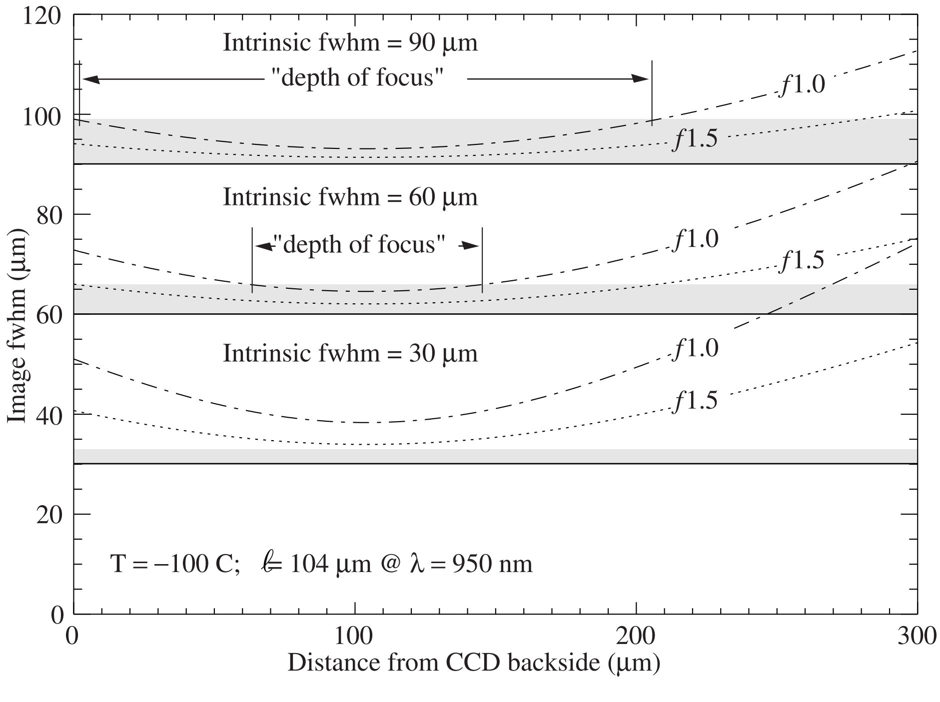

Figure 2: Image fwhm, in parabolic approximation, for 950 nm light incident on silicon at -150 C. At this temperature the absorption length of light in silicon is about 100 μm. Gray areas indicate the "allowed" deviation from best focus (up to 10%) in the absense of depth effects. Curves for $f1.0$ and $f1.5$ systems are shown for several intrinsic image sizes. There is no allowed focus range in the case of 30 μm intrinsic image size, although the $f1.5$ nearly touches the upper bound. The allowed range is just over 80 μm for $f1.0$ and fwhm $= 60$ μm, and conditions are very relaxed in the $f1.5$ case. In all of these cases the depth of focus in silicon is increased by the refractive index of silicon from that in air.

A reasonable criterion[†] might be that the square root of this variance does not exceed σ_* by more than 10%, or $\sigma_*^2 + \frac{1}{4}\alpha^2\ell^2 < (1.1)^2\sigma_*^2$. This reduces to $(0.35\,\ell/\mathrm{fwhm}_* < f)$, where we have introduced the more conventional full width at half-maximum as a measure of resolution. We see from Fig. 2 that the inequality is not satisfied for systems faster than about $f1.5$ if the intrinsic stellar fwhm is less than 30 μm—there is no acceptable focus. If it is satsisfied, then the focus is acceptable over the range

$$\text{depth of focus} = 2\sqrt{(1.1^2-1)\,(4fn\times(\mathrm{fwhm}_*/2.3548))^2-\ell^2}\;. \tag{4}$$

For fast optical systems (small f) and/or large absorption length ℓ, the surd can be less than zero and there is no focus that meets the criterion of a stellar image with a fwhm no more than 10% larger than the best possible with small ℓ.

With our assumption of a uniform circle of light convoluted with a Gaussian PSF, the distribution is flatter than a Gaussian. In a real telescope with a central obstruction there is even likely to be a "hole" in the center. Averaging the distribution over the exponential attenuation tends to make the distribution more "pointy," partly compensating for the flatness. We have also ignored pixel size—if the system is seriously undersampled, then the pixel size dominates the effective stellar image size. A better treatment is beyond the scope of the present discussion.

[†] As per discussion with Sandy Faber.