PROBABILITY EXERCISES

Things to Remember:

- The probability of any outcome is the number of times that outcome can occur divided by the total number of outcomes.
- If there are "n" equally likely outcomes to one event, then the probability of each outcome is 1 divided by n, 1/n.
- To determine the probability that any of several outcomes will occur for one event, such as the draw of <u>one</u> card, <u>one</u> spin of the roulette wheel, etc., <u>add</u> the separate probabilities together.
- To determine the probability of two separate events occurring at the same time, <u>multiply</u> the separate probabilities together.

Conversions:

Fraction to decimal

3/5 = 3 ÷ 5 = .6
3/20 = 3 ÷ 20 = .15

Decimal to fraction

.1 = 1/10	1 in ten
.01 = 1/100	1 in a hundred
.001 = 1/1000	1 in a thousand

Percent to probability

50% = 50/100 = .50 probability

Probability to percent

.50 probability = .50 x 100 = 50%
.35 probability = .35 x 100 = 35%

<u>Conversion exercises</u>

Convert to a decimal.

a) 1/2 = *1 ÷ 2 = .5*

b) 1/3 = *1 ÷ 3 = .33*

c) 3/4 = *3 ÷ 4 = .75*

d) 2/3 = *2 ÷ 3 = .67*

Convert to a fraction.

a) .25 *.25 x 100 = 25* *.25 = 25/100*

b) .60 *.60 x 100 = 60* *.60 = 60/100*

c) .934 *.934 x 1000 = 934* *.934 = 934/1000*

Convert from percent to probability.

a) 25% *25/100 = .25 probability*

b) 33% *33/100 = .33 probability*

c) 75% *75/100 = .75 probability*

Convert from probability to percent.

a) .2 *.2 x 100 = 20%*

b) .64 *.64 x 100 = 64%*

c) .934 *.934 x 100 = 93.4%*

Exercises

1. In flipping a coin six times, the following sequence was observed: H,T,T,T,T,T. What is the probability that on the seventh flip the coin will come up tails?

   Regardless of what happened on previous flips, the probability is .50 every time the coin is flipped.

2. Draw one card from an ordinary deck of cards. (Express answers as fractions and decimals.)

   a) What is the probability that it is the Queen of Hearts?

      1/52, 1 in 52 or .019

   b) What is the probability that it is either the King or the Queen of Hearts?

      1/52 + 1/52 = 2/52 = 1/26 or .038

3. Draw two cards from an ordinary deck. What is the probability of getting both the King and Queen of Hearts?

   For the first card drawn, there is a 2 in 52 chance of getting one of the two cards. For the second card, there is a 1 in 51 chance. The probability of drawing both the Queen and King of Hearts on two draws is (2/52) x (1/51) = (2x1)/(52x51) = 2/2652 = 1/1326 = .00075

4. Draw five cards. What is the probability of drawing the A,K,Q,J,10 of Hearts? What is the probability of drawing any royal flush?

   There are 5 cards we want to choose from the deck. For the first card, there is a 5 in 52 chance (5/52) of drawing one of the cards we want. For the second card, there are 4 cards left in the 51 remaining cards in the deck, so a 4 in 51 chance (4/51) for the second card. Using the same method, there is a 3 in 50 chance for the third card, a 2 in 49 chance for the fourth card, and a 1 in 48 chance for the fifth card. So, the probability of drawing that hand is 5/52 x 4/51 x 3/50 x 2/49 x 1/48 = (5x4x3x2x1)/(52x51x50x49x48) = 120/311,875,200 = .000000384. This is the probability of drawing one royal flush. Since there are four possible royal flushes, one for each suit, we need to multiply this probability by 4, giving us a .00000154 probability for drawing a royal flush or approximately 1.5 in a million.

5. Two basketball teams of equal skill are involved in a four-game tournament. What is the probability of one of the teams winning the tournament in four straight games?

   Each team has a one in two chance of winning each game. Therefore, the probability of winning four straight games is (1/2)x(1/2)x(1/2)x(1/2) = 1/16 = .0625 or approximately 6%.

PROBABILITY EXERCISES
Challenge Level

1. A typical roulette wheel has 38 slots that are numbered 1, 2, 3,..., 34, 35, 36, 0, and 00. The 0 and 00 slots are green. Of the remaining slots, half are red and half are black. Also half of the integers from 1 to 36 are even and half are odd. 0 and 00 are defined as neither even or odd. A ball is rolled around the wheel and ends up in one of the slots. We assume that each slot has an equal chance.

   a) What is the probability of each slot?

   There are 38 slots, each with an equal chance. Therefore, each slot has a 1/38 probability.

   b) What is the probability of the ball landing in a green slot? A red slot? A black slot?

   2 of the 38 slots are green, so, the probability of the ball landing in a green slot is 2/38 or 1/19. Of the 36 remaining slots, half are red and half are black. So, there are 18 red and 18 black slots. The probability of red = the probability of black = 18/38 or 9/19.

   c) What is the probability of the ball landing on an even number?

   Half of the numbers from 1 to 36 are even, so, there are 36/2 = 18 even slots. Therefore, the probability of getting an even number is 18/38 or 9/19.

   d) What is the probability of getting a 1, 12, 24, or 36?

   The probability for each of these numbers is 1/38 so the probability of getting one of these four is 1/38 + 1/38 + 1/38 + 1/38 = 4/38 or 2/19.

For the following questions, to calculate the “expected” value of an event multiply the consequence (profit or loss) under each outcome by the probability of the outcome and add them together. For example, if you bet $1.00 on the flip of a coin, there is a .50 probability that you win and a .50 probability that you lose. The expected value of this game is .50($1.00) + .50(0) = $.50 + $0 = $.50

2. In a particular lottery 2,000,000 tickets are sold each week for $.50 each. Each week there are 12,009 tickets drawn and awarded prizes: 12,000 people receive $25; 6 people win $10,000; 2 people win $50,000; and 1 person wins $200,000.

   a) Determine the probability of winning each prize.

Prize	*# of prizes awarded*	*Probability of each prize*
$25	12,000	12,000/2,000,000 = .006
$10,000	6	6/2,000,000 = .000003
$50,000	2	2/2,000,000 = .000001
$200,000	1	1/2,000,000 = .0000005

b) If you play this game, what is your "expected" payoff?

Prize x Probability	=	
25(.006)	=	*.15*
10,000(.000003)	=	*.03*
50,000(.000001)	=	*.05*
200,000(.0000005)	=	*.10*
Expected winnings	=	*$.33*

But, don't forget you spent $.50 on the ticket.
Adjusted "winnings" $.33 - .50 = - $ -.17

3. Suppose you must choose between two products to sell in your shop. Your choice depends on what the economy is going to do. If the economy goes up, you will make a profit of $100,000 on (product A) or $60,000 on (product B). If the economy stays the same, you will earn a profit of $50,000 on (product A) and $40,000 on (product B). And if the economy goes down, you will lose $20,000 on (product A) but can still earn $10,000 on (product B).

You don't know for sure what the economy is going to do, but you might know the probabilities of these things happening. Suppose the probability of the economy going up is .4, the probability of it staying the same is .4, and the probability of it going down is .2.

Determine the expected profit for each product. Which product would you choose and why?

Product A: .4(100,000) + .4(50,000) + .2(-20,000) =
40,000 + 20,000 - 4,000 = 56,000

Product B: .4(60,000) + .4(40,000) + .2(10,000) =
24,000 + 16,000 + 2,000 = 42,000

The average business professional would choose (product A). However, a very conservative or cautious business professional would be willing to sacrifice some of the profit in order to avoid any risk of losing money. So, he or she would choose (product B).