University of California, Berkeley
Physics H7B Spring 1999 (*Strovink*)

PROBLEM SET 8

1. (Taylor and Wheeler problem 27)
The clock paradox, version 1.
On their twenty-first birthday, Peter leaves his twin Paul behind on the earth and goes off in the x direction for seven years of *his* time at 24/25 the speed of light, then reverses direction and in another seven years of *his* time returns at the same speed. [In this most elementary version of the problem, we assume that the necessary periods of acceleration are infinitesimal in duration, requiring Peter's acceleration to be infinite. Nonetheless, our plucky twin remains uninjured.]
(a.)
Make a spacetime diagram (ct vs. x) showing Peter's motion. Indicate on it the x and ct coordinates of the turn-around point and the point of reunion. For simplicity idealize the earth as an inertial frame, adopt this inertial frame in the construction of the diagram, and take the origin to be the event of departure.
(b.)
How old is Paul at the moment of reunion?

2. Prove that

$$\tanh(\eta_1 + \eta_2) = \frac{\tanh\eta_1 + \tanh\eta_2}{1 + \tanh\eta_1 \tanh\eta_2} \; .$$

Using this relation, deduce Einstein's law for the addition of velocities.

3. The thermonuclear "deuterium-tritium" reactions are:

$$\begin{aligned} {}^2\mathrm{H} + {}^2\mathrm{H} &\rightarrow {}^3\mathrm{He} + n \\ {}^2\mathrm{H} + {}^2\mathrm{H} &\rightarrow {}^3\mathrm{H} + p \\ {}^2\mathrm{H} + {}^3\mathrm{H} &\rightarrow {}^4\mathrm{He} + n \; . \end{aligned}$$

These sum to

$$5({}^2\mathrm{H}) \rightarrow {}^3\mathrm{He} + {}^4\mathrm{He} + p + 2n.$$

Using the following masses in AMU,

(proton) p	1.007825
(neutron) n	1.008665
(deuteron) ^{2}H	2.014102
(helium 3) ^{3}He	3.016030
(triton) ^{3}H	3.016050
(alpha particle) ^{4}He	4.002603 ,

calculate (to 5%) the kinetic energy released when one liter of heavy water $({}^2\mathrm{H})_2\mathrm{O}$ undergoes deuterium-tritium fusion in an H-bomb. Express your answer in terms of tons of TNT (1 ton of TNT = 4.2×10^9 J of explosive energy).

4. The universe is filled with old cold photons that are remnants of the big bang. Typically their energy is $\approx 6.6 \times 10^{-4}$ eV.

A cosmonaut who is accelerated at 1 g for 10 years in her own rest frame attains a boost (= $\operatorname{arctanh}\beta$) of 10.34. As seen by her, what is the typical energy of these photons?

5. Prove that an isolated photon (zero mass) cannot split into two photons which do not both continue in the original direction.

6. The now retired Bevatron at Berkeley Lab is famous for having produced the first observed antiprotons (you may have glimpsed white-maned Nobelist Owen Chamberlain, one of the first observers, being helped to his seat at Physics Department colloquia). An economical reaction for producing antiprotons is

$$p + p \rightarrow p + p + p + \bar{p} \; ,$$

where the first proton is part of a beam, the second is at rest in a target, and $\bar{p}$ is an antiproton. Because of the CPT theorem, both p and $\bar{p}$ must have the same mass (= 0.94×10^9 eV).

At threshold, all four final state particles have essentially zero velocity *with respect to each other*. What is the beam energy in that case? (The actual Bevatron beam energy was 6×10^9 eV).

7. Using Eqs. 1.33 in the lecture notes, prove that $E^2 - B^2$, where E (B) is the magnitude of the electric (magnetic) field, is a Lorentz invariant.

8. You shine a one-watt beam of photons on a crow, who absorbs them. Calculate the force (in N) on the crow.