| Kinematic Correction Factors | | | | |
|---|---|---|---|---|
| Data Type | Variable | 1989 | 1990 | 1991 |
| Momentum | | | | |
| Data | $a_P$ | 1.022 | 1.018 | 1.0097 |
| Monte Carlo | $P_{smr}$ | 1.95 | 1.35 | 1.45 |
| | $a^u_P$ | 1.0051 | | |
| Range | | | | |
| Data | $b_R$ | 0 | 0 | 0.10 |
| Monte Carlo | $R_{smr}$ | 0.40 | 0 | 0.40 |
| | $a^u_R$ | 1.0309 | | |
| | $b^u_R$ | -1.1402 | | |
| Energy | | | | |
| Data | $a_E$ | 0.9270 | 0.9389 | 0.9125 |
| | $b_E$ | 10.845 | 9.5549 | 10.9308 |
| Monte Carlo | $a_s$ | $1.00 \times 10^{-3}$ | $0.93 \times 10^{-3}$ | $1.96 \times 10^{-3}$ |
| | $b_s$ | -0.047 | 0.025 | -0.096 |
| | $a^u_E$ | 1.0092 | | |

Table 4.4: Correction factors applied to raw kinematic quantities.

### 4.2.2 Energy

The total energy[4] was calculated in a four step procedure. The raw range stack energy $E^{raw}_{RS}$ was first rescaled by a factor of 1.95/1.905 to correct for a discrepancy between the actual RS layer thickness and the thickness used in the Monte Carlo simulation employed in the ADC calibration [49]. For stopping pions, the ADC gate included the 4.12 MeV muon energy from the $\pi^+ \to \mu^+\nu_\mu$ decay. A cut on the maximum allowed pion lifetime constrained the decay to occur within the ADC gate. This excess muon energy was corrected for by subtracting the saturated energy equivalent, 3.05 MeV. Saturation effects occur when a particle $dE/dx$ is large (equivalently, the ionization density); the scintillation mechanism is less efficient because of light quenching by damaged molecules. The saturation model used was,

$$\frac{dL}{dx} = \frac{dE/dx}{1 + K_B(dE/dx)},$$

where $L$ was the visible energy deposited with $K_B$, Birks' constant, equal to 0.01 [50]. The $dE/dx$ was calculated using the Bethe-Bloch equation. The sum of the range stack, target and I-counter energies were then unsaturated using the same model. The total track energy

[4]Unless otherwise stated, energy refers to kinetic energy.

was calculated as,

$$E_{tot} = F_{\rm unsat}(\frac{1.95}{1.905} \times E_{RS}^{raw} - 3.05 + E_{tgt} + E_{IC}) + E_{dead},$$

where $F_{\rm unsat}$ represented the unsaturation correction function and $E_{dead}$ was a correction made for known dead material in the DC and RS. For the RS, the path length in the RSPC chambers and scintillator log wrappings were converted into an equivalent energy using a $dE/dx$ determined from the track energy integrated inwards from the stopping counter. The typical correction was 1.8 MeV and 3.2 MeV for $K_{\pi 2}$ and $K_{\mu 2}$ events, respectively. The DC dead material correction was for the inner and outer carbon fiber support tube walls. The correction for each wall was typically 0.2 MeV. For the kinematic particle identification cuts, the range stack energy was defined as,

$$E_{RS} = F_{\rm unsat}(\frac{1.95}{1.905} \times E_{RS}^{raw} - 3.05) + E'_{dead},$$

where $E'_{dead}$ represented the RS and DC outer wall dead material corrections. As in the case of the total momentum, there remained systematic differences between the observed and accepted values for the $K_{\pi 2}$ and $K_{\mu 2}$ energies. Monte Carlo generated data also required additional smearing to properly simulate real data. [5] The scaling correction applied to real data was,

$$E'_{tot} = a_E \times E_{tot} + b_E,$$

where $a_E$ and $b_E$ were determined by constraining $E'_{tot}$ to the accepted $K_{\pi 2}$ and $K_{\mu 2}$ values. For Monte Carlo data, the total energy was rescaled using

$$E'_{tot} = a_E^u \times (E_{tot} + G \times F(E; a_s, b_s)),$$

where $a_E^u$ was a scaling factor, $G$ was a Gaussian random number and $F(E; a_s, b_s)$ was an energy dependent smearing factor of the form $(a_s E + b_s)\sqrt{E}$. For Monte Carlo data, a constant value for $a_E^u$ was sufficient for all three data sets. A summary of the final kinematic corrections applied to the data set is given in table 4.4. The year-to-year differences are possibly due to rate effects and aging of the RS scintillator. The peak positions and resolutions obtained after application of the above corrections is shown in table 4.5.

### 4.2.3 Range

The total track range was determined using the relation,

$$R_{tot} = R_{RS} + R_{tgt} + R_{IC} + R_{walls} + R_{gas},$$

[5] This was necessary even though the energies in individual counters were smeared using realistic photoelectron statistics for the range stack and target.