ORNL/TM-2001/164

Prediction of External Corrosion for Steel Cylinders—2001 Report

Rick Schmoyer
and
B. F. Lyon

This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from the Office of Scientific and Technical Information, P.O. Box 62, Oak Ridge, TN 37831; prices available from 615-576-8401, FTS 626-8401.

Available to the public from the National Technical Information Service, U.S. Department of Commerce, 5285 Port Royal Rd., Springfield, VA 22161.

ORNL/TM-2001/164

PREDICTION OF EXTERNAL CORROSION FOR STEEL STORAGE CYLINDERS—2001 REPORT

Rick Schmoyer
and
B. F. Lyon

Date Published: September 2001

Prepared for
U.S. Department of Energy
Office of Environmental Restoration and Waste Management
under budget and reporting code EW 20

Prepared by
OAK RIDGE NATIONAL LABORATORY
P.O. Box 2008
Oak Ridge, Tennessee 37831-6285
managed by
UT-Battelle, LLC
for the
U.S. DEPARTMENT OF ENERGY
under contract DE-AC05-00OR22725

CONTENTS

LIST OF FIGURES

LIST OF TABLES

EXECUTIVE SUMMARY

The United States Department of Energy (DOE) currently manages the UF_6 Cylinder Project. The project was formed to maintain and safely manage the depleted uranium hexafluoride (UF_6) stored in approximately 50,000 carbon steel cylinders. The cylinders are located at three DOE sites: the ETTP site (K-25) at Oak Ridge, Tennessee; the Paducah Gaseous Diffusion Plant (PGDP) in Paducah, Kentucky, and the Portsmouth Gaseous Diffusion Plant (PORTS) in Portsmouth, Ohio.

The System Requirements Document (SRD) (LMES 1997a) delineates the requirements of the project. The appropriate actions needed to fulfill these requirements are then specified within the System Engineering Management Plan (SEMP) (LMES 1997b). This report documents activities that in whole or in part satisfy specific requirements and actions stated in the UF_6 Cylinder Project SRD and SEMP with respect to forecasting cylinder conditions. The results presented here supercede those presented previously (Lyon 1995, 1996, 1997, 1998, 2000). Many of the wall thickness projections made in this report are conservative, because they are based on the assumption that corrosion trends will continue, despite activities such as improved monitoring, relocations to better storage, and painting.

For thin-walled cylinders (design nominal initial wall thickness 312.5 mils), the minimum wall thicknesses of interest used in this report are 0 (breach), 62.5 mils, and 250 mils (1 mil = 0.001 in.). For thick-walled cylinders (design nominal initial wall thickness 625 mils), the minimum wall thicknesses of interest used in this report are 0, 62.5 mils, and 500 mils. These thicknesses are preliminary boundaries identified within the project that indicate loss of material (UF6), safe handling and stacking operations, and standards for off-site transport and contents transfer criteria, respectively. In general, these criteria are based on area of wall thinning. However, the minimum thickness predicted in this report is essentially for a point (an area of about 0.01 sq. in), because this is the type of data used. For thicknesses criteria greater than zero, conclusions based on minimum point thicknesses are conservative. Because of the interaction of UF6, atmospheric moisture, and steel, a point breach would rapidly deteriorate to a larger hole, and so small-area approximations should be close for the breach criteria.

The most recently collected data, which were not available for the previous report (Lyon 2000), consisted of evaluations of wall loss of 48" cylinders: 100 at Paducah, 58 at ETTP and 155 at Portsmouth. Results for 144 scans made at Portsmouth, but not keypunched until FY01, were also incorporated. However, because of missing values, repeated measures on the same cylinders, outliers, and other data problems, not all of this data was considered usable. Some of the difficulty with the data is also due to the current mathematical approach to cylinder corrosion modeling, which focuses on maximum pit depths rather than minimum wall thicknesses. Because wall thicknesses are what is actually measured, fitting models to observed data entails inferring pit depths and initial thicknesses. This can be troublesome. For example if corrosion is uniform, initial thicknesses cannot be inferred from uncorroded cylinder areas. Furthermore, people doing the scanning inspections tend to restrict attention to cylinder areas of minimum thickness, which can introduce biases into the data. Therefore, corrosion models of minimum thickness rather than maximum pit depths are currently being investigated.

Bias in the data can also be caused by purposive or otherwise nonrandom sampling. Data bias is one of three main uncertainties in the corrosion modeling. The other two are model bias—uncertainty about the model as a mathematical approximation to the underlying corrosion physics, and statistical error—uncertainty in the estimates and projections computed from the model, due to sampling, which occurs even in the absence of data or model bias. The corrosion data appear to be widely variable, and statistical error is an important component of overall uncertainty.

A summary of projections of numbers of cylinders likely to fail various minimum thickness specifications is in Table 10 of this report. Some of these projections depend on initial thickness inferences that were affected by the data that is new for this 2001 report, and may be revised, particularly if an approach is developed in which minimum wall thickness is modeled directly. Most of the thin-walled cylinders predicted to have minimum point thickness below the 62.5 mil spec in 2001 are in K-1066-K yard at ETTP or various yards at PGDP. Based on data for the cylinder body, few of the approximately 2000 thick-walled cylinders are predicted to have a minimum point thickness on the cylinder body below any of the thickness criteria by 2020. In particular, only eighteen are predicted to have a minimum wall thickness below 500 mils by 2020, and none are predicted to have a minimum point thickness less than 62.5 mils by 2020.

Upper confidence limits for each projection in Table 10 provide alternative projections that account for statistical error in a way that is pessimistic but not unrealistic. In this report, the method for computing upper confidence limits in the FY2000 report has been improved, so that upper confidence limits are now closer to their corresponding point estimates. The new method, which is based on exactly the same model and assumptions as the method used for the FY2000 report, nevertheless makes more efficient use of several inequalities that are used as approximations, and therefore leads to tighter confidence limits. That is, the variability of the corrosion-model estimates is less than had been thought (though the estimates themselves are the same), and so the cost of statistical sampling error, in understanding and forecasting the corrosion process, is smaller than had been thought. This is discussed in Appendix B.

The painting program has reduced the predicted number of cylinders that do not meet the specified wall thickness criteria. For the C-745-G yard bottom row cylinders, it is predicted that the painting already completed will prevent almost 300 cylinders from failing the ANSI 14.1 thickness criterion for shipment by 2010 (assuming that painting halts corrosion for approximately 10 years). For K-1066-K yard, painting is predicted to prevent 70 cylinders from failing this criterion.

1. INTRODUCTION

The United States Department of Energy (DOE) currently manages the UF_6 Cylinder Project. The project was formed to maintain and safely manage depleted uranium hexafluoride (UF_6) stored in approximately 50,000 carbon steel cylinders. The cylinders located at three DOE sites: the ETTP site at Oak Ridge, Tennessee (K-25); the Paducah Gaseous Diffusion Plant in Paducah, Kentucky (PGDP), and the Portsmouth Gaseous Diffusion Plant (PORTS) in Portsmouth, Ohio.

The System Requirements Document (SRD) (LMES 1997a) delineates the requirements of the project. The appropriate actions needed to fulfill these requirements are then specified within the System Engineering Management Plan (SEMP) (LMES 1997b). The report presented herein documents activities that in whole or in part satisfy specific requirements and actions stated in the UF_6 Cylinder Project SRD and SEMP with respect to forecasting cylinder conditions. The wall thickness projections made in this report are based on the assumption that the corrosion trends noted will continue. Some activities planned may substantially reduce the rate of corrosion, in which case the results presented here are conservative. The results presented here are intended to supercede and enlarge the scope of those presented previously (Lyon 1995,1996, 1997, 1998).

System Requirement 1.2.2 states that performance shall be monitored and evaluated to identify potential risks within the project. The related **SEMP Action 2.1.2** is to model corrosion to project cylinder integrity. This report establishes the technique for modeling corrosion rates used in the project to forecast cylinder wall thickness conditions in the future.

System Requirement 4.1.2 calls for cylinder conditions to be monitored. The related **SEMP Action 3.1.2** is to statistically determine the baseline condition of cylinder populations by obtaining quantitative data. This report contains the statistical method used in the project to apply the available quantitative data to cylinder populations. Populations have been established based on historical storage locations (yard and position) and similarity of quantitative data. Wall thickness and corrosion pit depth data have been collected for several subpopulations of cylinders.

System Requirement 4.2.2 further states that cylinder conditions shall be forecast to direct surveillance and maintenance resources. **Technical Requirement 4.2.2a** is that specific information, as determined by the project, shall be tracked to project the current and future conditions of the system. In addition, **Technical Requirement 4.2.2.b** entails the development of mechanisms to consolidate information for summary level decision-making determinations. **SEMP Action 2.2.1** is to integrate cylinder condition elements to be forecast with cylinder categorization. **SEMP Action 3.1** is to forecast cylinder conditions using parameters identified. Wall thickness, the subject of this report, is one parameter identified in the project to forecast cylinder conditions. The available wall thickness data are used to forecast out year conditions.

SEMP Action 3.1.1 is to project the number of non-compliant cylinders. The disposition of any particular cylinder for storage, handling, and transfer is based on the condition of the cylinder, where "condition" is ultimately reflected by the minimum wall thickness of a cylinder. The wall thickness parameters (0, 62.5 mils, and 250 mils for thin-walled; 0, 62.5, 500 mills for thick-walled cylinders) used in this report are preliminary boundaries identified within the project that indicate loss of material, safe handling and stacking operations, and standard off-site transport and contents transfer criteria, respectively. In general, these criteria are based on area of wall thinning, rather than minimum thickness at what is essentially a point (an area of about 0.01 sq. in.), as used in this report. For thicknesses criteria greater than zero, conclusions based on minimum point thicknesses are conservative. Because of the interaction of UF6, atmospheric moisture, and steel, a point breach would rapidly deteriorate to a larger hole, and so point approximations should be close for the breach criteria.

2. BACKGROUND

The basic problem addressed here is to estimate how many cylinders will have a minimum thickness below some value z by time t. The current analyzed data only allow estimating the minimum thickness at a small point, and not the thickness over a large area. Additional analysis of the available data could provide estimates of the thickness over a larger area of structural significance.

Let $C_0(x)$ denote the initial wall thickness (mils) at a location x on the cylinder, and let $P(t,x)$ denote the amount of corrosion that has occurred (mils) at location x by time t. The minimum wall thickness at time t for a given cylinder, denoted here by $M(t)$, is given by

$$M(t) = \min_x \left\{ C_0(x) - P(t,x) \right\}$$

where the minimum is over all points x on the cylinder.

If the only concern is about the minimum thickness for a given cylinder at a given time, then knowledge of both $C_0(x)$ and $P(t,x)$ is not necessary. One can simply estimate the minimum wall thickness directly, although there will be measurement error that depends on the type of equipment used, as well as uncertainty as to the exact location of the minimum wall thickness. However, in order to make predictions for unsampled cylinders, or to make predictions for future time periods, assumptions must be made about the nature of the quantities $C_0(x)$ and $P(t,x)$.

Because the thicknesses of the cylinder walls were not recorded when they were first delivered, it is not possible to determine $C_0(x)$ as a function of x. For this reason, the joint distribution (in x) of $C_0(x)$ and $P(t,x)$ cannot be known. Assumptions must then be made about $C_0(x)$. One such assumption is to treat it as an independent (from $P(t,x)$) random variable, in which case the minimum thickness $M(t)$ for a given cylinder is then also a random variable, defined by

$$\begin{aligned} M(t) &= C_0 - \max_x \left\{ P(t,x) \right\} \\ &= C_0 - P(t) \end{aligned}$$

where $P(t)$ is defined as the maximum penetration depth for a given cylinder of age t. The corrosion rate is then dP/dt. Thus, even given the knowledge of the value of $P(t)$, there would be uncertainty in $M(t)$ due to uncertainty in the initial thickness where the maximum pit depth occurred, C_0, for the given cylinder. The design range for the initial thickness of thin-walled cylinders is from 302.5 to 345.5 mils (615 to 655 or 665 mils for thick-walled cylinders), and so it could be argued that, without sufficient supporting information, the lower end of the design range must be used to confidently bound the minimum thickness. However, if it is acceptable that where the maximum penetration depth occurs the initial thickness is actually larger than the minimum of the design range, then less conservative estimates may be possible.

The preceding discussion pertains to estimating the minimum thickness for a given cylinder. When estimating the thickness for a population of cylinders, there are two additional sources of variability: (1) variability across cylinders of the maximum penetration depth $P(t)$, and (2) variability across cylinders of the distribution of initial thickness C_0. Variability in $P(t)$ can be due to random variations in the corrosion process, the steel substrate, and differences in the storage history of the cylinders (and paint for painted cylinders). Variability in the distribution of initial thickness can be due to differences in the methods used by manufacturers.

Sampling of the cylinders provides estimates of the maximum penetration depth $P(t)$, and from these data predictions must be made for the maximum penetration depth for both the unsampled cylinders and for all cylinders as a function of age. Even for a fixed age, there will be variability in the maximum penetration depth. (And the data demonstrate that variability to be substantial. See Figures 1-15, Appendix A). Given the uncertainty in the storage histories for the cylinders, conservative estimation of the variability in $P(t)$ can be desirable.

In this report, approximations to $P(t)$ are of the form $P(t)=F(\alpha(t),\beta(t)),$ where $F(\alpha,\beta)$ is a two-parameter random variable (e.g., the parameters α and β could be the mean and standard deviation). The goal then is to accurately approximate the functions F, α, and β. The minimum thickness of a cylinder is given by

$$M(t) = C_0 - F(\alpha(t),\beta(t)),$$

and, for a given population of cylinders, the probability that a particular cylinder of age t will have a minimum thickness below a given thickness z is

$$Prob\{M(t)<z\} = Prob\{C_0 - F(\alpha(t),\beta(t)) < z\}.$$

With the exception of cylinders that are being purchased now, there is no way to know the distributions C_0. The data collected suggest that the wall thickness on relatively uncorroded areas of a cylinder is usually larger than the nominal design thickness. Ultimately, for a given population of cylinders, the total number of cylinders with a minimum thickness below a given value z at time T is estimated by

$$\text{\# Cylinders with minimum thickness below } z \text{ at time } T =$$

$$\sum_t \left(\text{\# cylinders of age } t \text{ at time } T\right) \times Prob\{M(t)<z\}.$$

3. MODELING METHODS

3.1. Modeling Maximum Pit Depth

An expected feature of the corrosion rate is that, in general, it should decrease with time. The problem that must be addressed is defensibly quantifying just what the decrease in corrosion rate is. It may be that in many cases (i.e., many subpopulations of cylinders) the corrosion process has reached a condition in which, whatever the past corrosion history for each cylinder may have been, each cylinder is corroding at some relatively constant (over the year) rate. If this is true, then the modeling is fairly straightforward: one determines the current condition for each cylinder, estimates what the current "constant rate" is, and then projections can be made for future times. To do this successfully, however, requires a reliable picture of the current conditions, and a determination of the current "constant" corrosion rates for all relevant populations. Both of these factors rely on the quantity and quality of the data available.

Prediction of the distribution of penetration depths $P(t)$ across cylinders is critical to the whole process of estimating the number of cylinders that have a minimum thickness below a given level. The simplest manner in which to do this is to assume that the general shape of the distribution of penetration depths is the same for all ages, but the mean or median (measures of central tendency) and standard deviation (i.e., the amount of "spread" in the distribution) is changing in a specified fashion. Focus is then directed to determining exactly how these factors depend on age.

The main approach utilized here allows modeling of the "leveling off" commonly observed, and is of the form $P(t)=A\ t^n$ where A and n are constants. This is often referred to as the "linear bilogarithmic law," and there are many applications of this model in long-term corrosion prediction (Felieu et al. 1993a; Felieu et al. 1993b; Legault and Preban 1975; Pourbaix 1982; Mughabghab and Sullivan 1989; Romanoff 1957). Determination of A and n are performed by doing a linear regression of the form $lnP=lnA + n\ lnt$. According to Pourbaix (1982), Passano (1934) was the first to use such a relationship in corrosion prediction. This law is considered to be valid for different types of atmospheres (rural, marine, industrial) and a number of materials. The parameter A can be interpreted as the corrosion in the first year, and the parameter n represents the attenuation of the corrosion because of the passivation of the material in the atmosphere (Pourbaix, p.115).

It is also possible to discuss this model in terms of the mean (or age-averaged) corrosion rate, since the mean corrosion rate is given by $P/t = A\ t^{n-1}$. If $n=1$ then this implies that the age-averaged corrosion rate is constant, while if $n<1$ (which is usually the case) then the corrosion rate decreases with time. Mechanistic interpretations of n have also been made (Horton 1964). If $n=0.5$, then the relationship is said to be parabolic, with the corrosion rate controlled by diffusion through the rust layer. If $n<0.5$, then this implies that the rust layer is showing protective properties, while if $n>0.5$, then the rust layer is not protective because of factors that may be preventing the homogeneous thickening of the rust layer. This approach is used in several Department of Energy models to predict time to breach due to external corrosion for carbon steel containers in soil. This approach should be used with caution, however, because estimates of the "leveling off" pattern usually expected for the penetration depth can be sensitive to narrow data ranges, outliers, and other data anomalies. Indeed, leveling off is not observed for several cylinder populations considered in this report, and a simpler method is then applied, which is the same except n is constrained to be 1 (Lyon 1995, 1996)

In order to address the variability inherent in the corrosion process, it is assumed that the penetration depths are lognormally distributed at each time. This can also be expressed as $lnP(t){\sim}N(lnA + n\ ln\ t, \sigma_L)$, where $N(\mu,\sigma)$ is the normal distribution with mean μ and standard deviation σ. For this model, the median is equal to At^n, the arithmetic mean μ is $At^n \, exp[0.5\sigma^2]$, and the arithmetic standard deviation is $At^n \, exp[0.5\sigma^2] \, [exp(\sigma_L^2)-1]^{1/2}$. The coefficient of variation (ratio of the standard deviation to the mean) is constant with time, and is equal to $[exp(\sigma_L^2)-1]^{1/2}$.

The lognormal assumption has been checked by goodness of fit tests discussed in previous cylinder reports (Lyon 2000) as well as in Section 5 and Appendix B of this report. Given that the data consist of what are considered to be maximum pit depths, it would also be natural to apply extreme-value statistics to this problem. Application of the extreme value distribution (without confidence limits) is discussed in several papers and has also been suggested for use within this project in Rosen and Glaser (1996). The basic premise underlying this theory is that the distribution of extreme values, under rather general assumptions, should have a specific (parametric) form. Extreme value models are being investigated for the cylinder corrosion. However, for the present analyses, lognormal-based methods are used because (1) lognormal-based confidence limits are more straightforward, (2) due to the substantial variability in the data, confidence limits are crucial in the analysis, and (3) the lognormal distribution has many of the same qualitative properties as the extreme value distribution.

3.2. Modeling the Initial Thickness

The initial thickness assumed is an important feature of the analysis. In Lyon (1995 and 1996) this was dealt with in a simple fashion. In this report, the initial thickness is modeled using a distribution that accounts for the variability in the initial thickness. In this analysis, the initial thickness is approximated using a truncated normal distribution, which is a normal distribution that is defined on a finite range. The parameters for the distribution are estimated based on the data available. With the exception of the data at the head/skirt interface (see below), these data consist of wall thickness measurements made on the cylinders evaluated in relatively uncorroded regions of the cylinder. These measurements were made using either an automated scanner or a hand-held probe, depending on the particular dataset. This initial thickness estimate does not include any general corrosion that may have occurred across the entire cylinder surface. This is motivated in part by concerns within the project that the variability in initial thickness could be a critical factor (e.g., Rosen and Glaser 1995).

It has been found that the wall thickness is typically much larger in the head/skirt interface than the design specifications would indicate. When the data were collected, five manual measurements were made on the cylinder head at a distance of about one inch from the cylinder head/skirt weld (Lykins and Pawel 1997). The initial thickness was set to the measured thickness at the center of the cylinder head plus 10 mils. The extra factor was used because it was found that on several 48G-type cylinders (these are thin-walled cylinders), the wall thickness was usually 10-20 mils less than that found beneath the plug. This difference was attributed to the forging process to form the contour of the head. That method is not used here because it was found that this does not guarantee that the initial thickness is larger than the measured wall thickness in the head/skirt area. Instead, the maximum of the five measurements plus 10 mils was used as an approximation to the initial thickness.

The lower bound of the range used to model the initial thickness is set to the lower bound of the design specifications: 302.5 mils for thin-walled cylinders, 615 mils for thick-walled cylinders. The upper bound is not taken directly from the design specifications, but is instead set to the largest observed value for initial wall thickness.

3.3. Calculation of Confidence Limits

The method used to calculate confidence limits is discussed in Appendix B. Note that the validity of the confidence limits depends on the probability model and on whether the sampling is random or emulates random sampling. To the extent these assumptions are violated, the confidence limits are approximations. As noted in Tables 1 and 2, the assumption of random sampling was sometimes violated in the cylinder measurement process. Of course validity of the corrosion model can also be adversely affected by bias in the physical corrosion model and by biases inherent in the data measurements (e.g., failure to properly capture initial thickness). Nevertheless, for the estimates and projection presented in this report, under the modeling, measurement, and sampling assumptions, the confidence limits provide best pessimistic but reasonable alternatives that account for sampling error. The confidence limits reflect the statistical error, and as the variability of the data about the fitted curves in Figures 1-15 (Appendix A) illustrates, the statistical error is substantial.

The method for computing confidence limits computed for this report differs from the method for the FY2000 report. Both methods are discussed in Appendix B. Both methods are premised on exactly the same model and assumptions. However, the confidence limits computed for this report tend to be closer to their corresponding point estimates (see the section "A Departure from the FY2000 Report" in Appendix B). This is important because it demonstrates that the cost of sampling error, though substantial, is not as great as previously had been thought.

4. DATA UTILIZED

In this section a summary of the data sets utilized is provided. The previous report (Lyon 2000) utilized wall thickness data that had been collected through August 1999. This report includes additional data that were collected since then.

Two main types of data are used:

(1) data for predicting overall minimum wall thickness at a point, not including the head/skirt interface

(2) data for predicting minimum wall thickness at the head/skirt interface

Some of the data available can be used to address more general issues, such as the average wall thickness over a given region, but such analyses were not done for this report.

Tables 1, 2 and 3 summarize the data collected by fiscal year. In each case, it is noted whether or not the data collected constitute a random sample. Random sampling is important because compromising it introduces biases into inferences about sampled populations. An initial sampling plan (Lyon and Lykins 1996) was prepared that included random sampling, and recommended that it be updated to more efficiently fit within the current budgetary and logistical constraints. Table 3 summarizes the data collected at the head/skirt interface, and Tables 4, 5, and 6 show more detail about the age ranges and yard locations of the evaluated cylinders. Plots of the data are shown in Figures 1-15 in Appendix A.

4.1. Summary of Measurement Methods

Several of the data collection efforts have used an automated scanner called a P-Scan system (see Schmidt et al. 1996 for a description of the equipment). The first effort was performed during 1994 at K-1066-K yard at ETTP. The second was performed during the fall of 1995 at the PGDP, the third was conducted between March and September 1996 at both the PORTS and PGDP sites as part of the cylinder relocation efforts. The most recent effort was conducted during FY97, primarily at Portsmouth. The pit depth data consist of measurements made with the automated scanner for a square region of width and height of about 2.54 mm (0.1 in). The wall thickness data used for the initial thickness consisted of either data collected with the automated scanner near where the maximum pit occurred (with a width and height of approximately the same size as the pit data), or was collected using a hand-held probe for a circular region with a radius of about 2 mm (0.08 in).

Hand-held ultrasonic thickness (UT) methods (Lykins and Pawel 1997) were used to collect all other wall thickness data.

Table 1. Chronological summary of data collection efforts, FY92-97 for estimating minimum wall thicknesses at a point, not including the head/skirt interface.

	FY92	FY94	FY95	FY96		FY97	
Thin-Walled Cylinders (nominal initial wall thickness of 312.5 mils)							
Yard(s)	K-1066-K	K-1066-K	C-745-B/F/K/L	C-745-F/G/K	X-745-C	C-745-G/L	X-745-C
Type of Data	Visual	PSCAN	PSCAN	PSCAN	PSCAN	PSCAN	PSCAN
Number cylinders	2	138	94	261	473	3	85
Random sampling?	No	No	No	No	Yes	No	Yes
Comment on Sampling	Breached cylinders	Intent was that cylinders be selected randomly, but limitations were imposed by scanner	Cylinders selected based on judgement of personnel (Blue, 1995a).	Intent was that 10% of cylinders moved would be evaluated. Space restrictions for selected cylinders prevented this from occurring.	10% of cylinders moved during FY96 were randomly selected and evaluated		

Table 1 (cont'd). Chronological summary of data collection efforts, FY92-97 for estimating minimum wall thicknesses at a point, not including the head/skirt interface.

	FY92	FY94	FY95	FY96		FY97	
Thick-walled Cylinders (nominal initial wall thickness of 625 mils)							
Yard(s)				X-745-C			
Type of Data				PSCAN			
Number cylinders				137			
Random sampling?				Yes			
Comment				10% of cylinders moved during FY96 were randomly selected and evaluated			

(No Model 30A Cylinders were sampled during FY92-97.)

Table 2. Chronological summary of data collection efforts, FY98-2000 for estimating minimum wall thicknesses at a point, not including the head/skirt interface.

	FY98			**FY99**			**FY2000**		
Thin-Walled Cylinders (nominal initial wall thickness of 312.5 mils)									
Yard(s)	K-1066-K	PAD	PORTS	K-1066-K	PORTS	PAD	K-1066-K	PAD	PORTS
Type of Data	Manual UT	Manual UT	Manual UT	Manual UT	Manual UT	Manual UT	Manual UT	Manual UT	Manual UT
Number cylinders	40	200	142	30	141	200	58	100	130
Random sampling?	Yes	Yes	Yes, but some cylinders were scanned previously	Yes for 29	Yes	Yes	Yes	Yes	Yes, but some cylinders were scanned previously
Comment		Only "good" locations on cylinders were evaluated, contrary to implement ation plan.	Delivery year is unknown for nine cylinders.	One cylinder was chosen by field personnel due to its known ground contact (#100581)	Methodology used to pick cylinders was developed by PORTS personnel		Top/bottom status is not ascertained (but it not used in the current modeling).		

Table 2 (cont'd). Chronological summary of data collection efforts, FY98-2000 for estimating minimum wall thicknesses at a point, not including the head/skirt interface.

	FY98			FY99			FY2000		
Thick-walled Cylinders (nominal initial wall thickness of 625 mils)									
Yard(s)				X-745-C					X-745-E
Type of Data				Manual UT					Manual UT
Number cylinders				11					25
Random sampling?				Yes, but some scanned previously					Yes, but some scanned previously
Comment									
Model 30A Cylinders									
Yard(s)				PAD					
Type of Data				Manual UT					
Number cylinders				100					
Random sampling?				No					
Comment				Preliminary list of cylinders to be evaluated was used in place of corrected list.					

Table 3. Counts of thin-walled cylinders used, not including data at the head/skirt interface (numbers in parentheses are the ages, or range of ages, of the cylinders when evaluated).

Site	Yard	Row	Fiscal Year Measured							
			1992	1994	1995	1996	1997	1998	1999	2000
ETTP	K-1066-K	Top	1 (29)[A]	60 (31-36)				21 (40-42)	13 (36-42)	58 (37-42)
		Bottom	1 (34)[A]	55 (31-38)				19 (40-42)	17 (36-42)	
PGDP	C-745-B	Top			4 (39)					
		Bottom			2 (39)					
	C-745-C and D	Top							2 (24-30)	1 (12-12)
		Bottom							7 (18-25)	1 (18-18)
	C-745-F	Top			13 (31-36)				16 (19-40)	5 (9-41)
		Bottom			13 (32-36)	6 (36-37)			20 (8-40)	15 (9-41)
	C-745-G	Top			9 (33-36)	137 (18-37)			15 (22-40)	12 (23-41)
		Bottom			17 (33-36)	98 (5-37)	2 (37)		44 (10-40)	39 (22-41)
	C-745-K and L	Top			17 (13-18)				14 (18-22)	8 (19-24)
		Bottom			25 (14-19)	6 (16-37)	1 (38)		16 (18-40)	11 ((7-41)
	C-745-M through T	Top							14 (8-36)	4 (11-14)
		Bottom							48 (10-40)	4 (10-11)
PORTS	X-745-C	Top				221 (6-40)	56 (20-36)	57 (8-36)[B]	56 (12-43)[B]	15 (10-38)[B]
		Bottom				252 (6-40)	29 (8-36)	63 (9-39)[B]	85 (12-43)[B]	10 (11-40)[B]
PORTS	X-745-E	Top						4 (9-11)[B]		60 (11-44)[B]
		Bottom						18 (8-9)[B]		45 (23-44)[B]

[A]These are the two cylinders that were determined to breach from external corrosion (Barber et al. 1994).
[B]Top/bottom status estimated from information available; historical information on top/bottom status not available for PORTS cylinders.

Table 4. Summary of data utilized for estimating wall thickness at head/skirt interface.

	FY97	FY2000
Thin-Walled Cylinders (312.5 mils)*		
Yard(s)	X-745-C	X-745-E
Type of Data	Manual UT	Manual UT
Number cylinders	233	87
Age Range (yr)	38-40	42-44
Random sampling?	Yes, but some scanned previously	Yes, but some scanned previously
Thick-walled Cylinders (625 mils)*		
Yard(s)	PORTS	PORTS
Type of Data	Manual UT	Manual UT
Number cylinders	115	23
Age Range (yr)	36-45	46-48
Random sampling?	Yes, but some scanned previously	Yes, but some scanned previously

*No nominal initial wall thickness

Table 5. Summary of thick-walled cylinder data utilized, not including data at the head/skirt interface (numbers in parentheses are the ages, or range of ages, of the cylinders when evaluated).

Site	Yard	Row	Fiscal Year Measured				
			1996	1997	1998	1999	2000
PGDP	C-745-C	Top	1 (42)				
		Bottom	1 (44)				
PORTS	X-745-C	Top	50 (42-45)	35 (42-45)		11 (45-48)	
		Bottom	65 (42-45)	35 (36-45			
	X-745-E	Top			1 (46-46)		16 (46-48)
		Bottom			1 (44-44)		9 (46-48)

Table 6. Summary of Model 30A cylinder data utilized (numbers in parentheses are the ages, or range of ages, of the cylinders when evaluated).

Site	Yard	Row	Number of Cylinders
			1999
PGDP		Top	50 (45)
		Bottom	50 (45)

4.2. Data Collection by Fiscal Year

In this section the data collection efforts are summarized in order by fiscal year in which they were obtained. Summaries by yard and/or subpopulation are discussed below.

FY92

These data consist of two breached cylinders discovered in 1992 in K-1066-K yard, for which it was deemed that external corrosion was the cause of the breach. There have been five other breaches discovered (two at ETTP in 1992, two at PORTS in 1990, and one at PGDP in 1992), but it was concluded that the breaches were induced by mechanical damage at the time of stacking rather than to external corrosion. In particular, the breaches were caused by a lifting lug of an adjacent cylinder that induced a small crack near a stiffening ring. While it is challenging to determine the best manner in which to include these data, it is felt that it is critical that these data be included. This is because the existence of these data show that extremely accelerated corrosion is possible (but rare), and hence any model utilized must incorporate this feature.

FY94

During the six month period 12/93 to 5/94, pit depth and wall thickness measurements were made for 136 cylinders in K-1066-K yard (Philpot 1995) using an automated scanner. It was intended that the cylinders selected for measurement be chosen at random, although a random number generator was not used to select the cylinders, and there were limitations imposed by the automated scanner (e.g., length of power cord, clearance between adjacent cylinders). For these reasons, it is not possible to objectively conclude that the cylinders selected are a representative sample from the population, although this actually may be the case. For the first 21 cylinders evaluated, only minimum wall thickness data were recorded, while pit depth data were also recorded for the rest of the cylinders. There is concern about the accuracy of the wall thickness data for the first group of cylinders. Further, since no pit depth data were recorded for these first 21 cylinders that would allow estimating how much corrosion had occurred, these cylinders are not included in this analysis.

FY95

During FY95, data were collected for 100 thin-walled cylinders at PGDP using the automated scanner (Blue 1995a). The primary purpose of this effort was to assess "the condition of the more vulnerable portion" of the cylinder population at PGDP (Blue 1995a). The cylinders were selected from the C-745-B/F/G/K/L yards based on the judgement of the personnel involved, and do not defensibly constitute a random sample from any of these yards.

FY96

During FY96, almost 900 cylinders were evaluated with the automated scanner at the PORTS and PGDP sites. Both thin-walled (nominal initial wall thickness 312.5 mils) and thick-walled cylinders were evaluated (nominal initial wall thickness 625 mils).

At PORTS, 10% of the cylinders that were relocated were selected using a random number generator to evaluate the wall thickness using manual UT measurements. The 10% evaluation criterion was required based on a Consent Decree with the Ohio Environmental Protection Agency.

Most of the cylinders evaluated at PGDP were from C-745-G yard, and had been set aside as part of the relocation efforts performed during FY95 and FY96. These cylinders were a subset of the approximately 390 cylinders set aside from the first 3900 cylinders moved out of the C-745-G yard. Because of the manner in which these cylinders were selected, these cylinders are a systematic sample only from the first 3900 cylinders moved out of G yard. An additional 6 cylinders from both C-745-F and C-745-K yard were also evaluated. For C-745-F yard, single stacked cylinders from the north end were selected, while the C-745-K yard cylinders were selected based on the ease of accessibility with the equipment. In both cases, these samples are not considered to be a random sample.

FY97

During FY97, data were collected for cylinders both at the head/skirt interface and for overall minimum wall thickness. Cylinders were evaluated at PORTS and PGDP. The head/skirt data were collected from cylinders that had been evaluated with the automated scanner in FY96 at PORTS, and from two cylinders at PGDP (Lykins and Pawel 1997). The cylinders at PORTS had originally been systematically set aside as part of the 10% criterion for evaluation. No specific criteria were used for the selection of cylinders from this subset, thereby weakening the defensibility of assuming that the sample is random; however, more than 75% of those cylinders originally set aside were evaluated.

The cylinders evaluated at PORTS with the automated scanner were randomly selected from those cylinders moved during the year. Originally, it was suggested that approximately 250 cylinders be evaluated (Lykins 1996). However, budget constraints allowed only 85 evaluations with the PSCAN.

Two cylinders that had been in the bottom row of C-745-G yard, and one cylinder from C-745-L bottom row, were also evaluated with the PSCAN during FY97. These were located in the north end of C-745-F yard when evaluated.

FY98

There were three basic populations sampled from in FY98. The first consisted of 40 thin-walled cylinders randomly selected from K-1066-K yard. These cylinders were chosen from a population of 400 cylinders that were moved to K-1066-E yard during FY98. The second consisted of 200 thin-walled cylinders randomly selected from Paducah yards. The third consisted of 142 thin-wall and 2 thick-wall cylinders in Portsmouth X-745-C and E yards. Some of the Portsmouth cylinders were also measured in 1996. In all cases, the ultrasonic thickness measurements were done manually. It was confirmed that the Paducah data were representative of relatively uncorroded locations on each cylinder; hence, these data cannot be used alone for determining either minimum wall thickness or wall loss. The Paducah cylinders were re-evaluated in FY99 with the purpose of determining estimates of the thinnest locations on each cylinder.

FY99

There were four separate sampling efforts in FY99. One consisted of 30 thin-walled cylinders randomly selected from K-1066-K yard. The cylinders were from a subpopulation of 155 cylinders that could be evaluated without requiring any cylinder movement. A second effort was directed at re-evaluating 200 thin-walled cylinders at Paducah that had been evaluated in FY98, with the purpose being to estimate the thinnest location on each cylinder. The third effort was conducted at PORTS, and included evaluations of both thin-walled and thick-walled cylinders. In both cases, the ultrasonic thickness measurements were done manually. The fourth effort consisted of the evaluation of 100 model 30A cylinders from a population of 1825 cylinders at Paducah

FY2000

Additional data for FY2000 include UT scan data for 58 48" cylinders from K-1066-K yard in Oak Ridge, 100 48" cylinders at Paducah, and 155 48" cylinders at Portsmouth.

4.3. Summary of Data by Subpopulation

In this section the data utilized are discussed relative to the subpopulation to which they are assumed to belong. Some of the information is redundant with that in the previous section, but is included for clarity.

K-Yard at Oak Ridge, Thin-Walled Cylinders

K-1066-K yard, located at the ETTP plant in Oak Ridge contains 2942 thin-walled cylinders, ranging in age (in 2001) from about 37 years to 44 years. These cylinders were initially stored at K-1066-G yard at Oak Ridge starting at about 1966, and relocated in 1983 (Barber et al. 1994). During the six month period 12/93 to 5/94, pit depth and wall thickness measurements were made for 136 cylinders (Philpot 1995) using an automated scanner. It was intended that the cylinders selected for measurement were chosen at random, although a random number generator was not used to select the cylinders, and there were limitations imposed by the automated scanner (e.g., length of power cord, clearance between adjacent cylinders). For these reasons, it is not possible to objectively conclude that the cylinders selected are a representative sample from the population, although this actually may be the case. For the first 21 cylinders evaluated, only minimum wall thickness data were recorded, while pit depth data were also recorded for the rest of the cylinders. There is concern about the accuracy of the wall thickness data for the first group of cylinders. Further, since no pit depth data were recorded for these cylinders that would allow estimating how much corrosion had occurred, these cylinders are not included in this analysis.

It is noted that the accuracy of the equipment used when these data were collected was such that only increments of 5 mils were recorded for pit depth. As a result, there may be several cylinders with the same pit depth measurement, which, due do data overlaying, cannot be seen in the plot of this data (Appendix A, Figure 1).

Also included in the dataset are two breached cylinders discovered on K-1066-K yard in 1992 (Barber et al. 1994).

In FY98, 40 cylinders were evaluated out of a subpopulation of 400 cylinders that were being moved to K-1066-E yard. These cylinders were randomly selected for evaluation, and evaluated with manual ultrasonic thickness methods. In FY99, 30 more cylinders were evaluated from a subpopulation of 155 cylinders which could be evaluated without requiring any cylinder movements. All but one of these cylinders were chosen randomly, with the additional one selected by field personnel based on its history of ground/water contact. Fifty-eight more cylinders were measured in FY2000.

Paducah Thin-Walled Cylinders, Non-C745-G Yard

For the purpose of modeling, all thin-walled cylinders, excluding those cylinders that were in C-745-G yard, are treated as a single population. This decision was based on judgment about conditions of the cylinder yards and data availability. For the top row, the C-745-G yard cylinders are included as well. This makes yard designations of other than “G” or “other” not very critical for the Paducah cylinder groups. Two separate efforts performed in FY98 and FY99 collected data from this population; however, concerns about the quality of these data preclude inclusion in the current report. For FY2000, 152 cylinder measurements were added to the samples for this population.

C-745-B Yard, Thin-Walled Cylinders. This yard contains about 1500 thin-walled cylinders manufactured between 1954 and 1988. In 1995, six cylinders were inspected, all type 48T and manufactured in the period 1956-57. These cylinders had been stored on 10" high concrete piers above the yard surface since April 1967 (Blue 1995a). Four top row and two bottom row cylinders were evaluated, this particular choice of cylinders being a matter of convenience for the material handlers (Blue 1995a).

C-745-C and D Yards, Thin-Walled Cylinders. Manual UT scans for two top and seven bottom row C-yard cylinders were added to the database in FY99. Scans for one top and one bottom row D-yard cylinders were added in FY2000.

C-745-F Yard, Thin-Walled Cylinders. C-745-F yard contains approximately 4500 cylinders. The top and bottom rows of this yard were interchanged in 1992 when all bottom row cylinders were put on concrete chocks, rather than wood as had been previously used (each row was also relocated south one row). It is likely that some fraction of the bottom row cylinders were in water contact for extended periods of time, although none are now.

In 1995 26 cylinders were evaluated, with 13 from both the bottom and top rows. Both the pit depths and wall thickness were recorded for these cylinders. It is noted that hand-held measurements using a 2 mm probe were made in 1994 to estimate the minimum wall thickness for 21 cylinders in C-745-F yard, but the pit depths were not recorded (Blue 1994). These data are not included in this analysis because it is not possible to reliably estimate the pit depths. These data were used in the analysis discussed in Lyon (1995) because these were the only data available for this yard at that time.

Six cylinders were also evaluated in FY96 from C-745-F yard. Sixteen top and twenty bottom row cylinder measurements were added to the database in FY99. Five top and fifteen bottom row measurements were added in FY2000.

C-745-K and L Yards, Thin-Walled Cylinders. The C-745-K and C-745-L yards contain a total of about 9000 Type OM and G cylinders manufactured in the period 1958-1992. These cylinders have been stored on five-inch concrete saddles in gravel yards constructed with an underground drainage system. Data were collected from these yards in 1995, 1996 and 1997. The sampling in 1995 was limited to those cylinders that were manufactured during the period 1976-1982 that had lost large portions of their protective coating (Blue 1995a). A total of 42 cylinders were inspected (39 from K yard, 3 from L yard). Twenty-five cylinders were from the bottom row, and 17 were from the top row. In 1996, 6 cylinders from the bottom row of C-745-K yard were evaluated. In 1997, one cylinder that had been in the bottom row of C-745-L yard was evaluated; it was located in the north end of C-745-F yard when evaluated. Fourteen top and sixteen bottom C-745-K yard cylinder measurements were added for FY99. Eight top and eleven bottom cylinders were added in FY2000.

C-745-M through T Yards, Thin-Walled Cylinders. A few cylinders came from yards with designations higher than L (three in FY95 and one in FY97). Twelve top and 48 bottom cylinders in T-yard were measured in FY99. Five top and three bottom row cylinders in M and N yards were measured in FY2000.

Paducah Thin-Walled Cylinders, C-745-G Yard

C-745-G Yard, Thin-Walled Cylinders. The population modeled as C-745-G yard actually consists of those cylinders that were originally in C-745-G yard prior to construction of the new yard, and have not been painted. A painting program was initiated for the cylinders moved from C-745-G to C-745-S yard in 1996. All 2168 cylinders in C-745-S were painted during FY96-1997.

There are five datasets available for C-745-G yard that were utilized, three of which were collected using the automated scanner. (1) The first consists of data for 26 cylinders that were evaluated in 1995 (Blue 1995a, 1995b). (2) The second dataset consists of measurements made between March and September 1996 on cylinders set aside as part of the relocation efforts performed during 1995 and 1996. A total of 235 cylinders were evaluated (137 from the top row, and 98 from the bottom row). These cylinders are a subset of the approximately 390 cylinders set aside from the first 3900 cylinders moved out of the C-745-G yard. Because of the manner in which these cylinders were selected, these cylinders are a systematic sample only from the first 3900 cylinders moved out of G yard. This weakens the statistical defensibility of statements made for the whole C-745-G yard population based on the trends observed for these data. There was concern that the condition of the bottom row cylinders in C-745-G yard affected the accuracy of the equipment for the cylinders evaluated in FY95, as material at the bottom of the pits can result in the equipment underestimating the actual pit depth (Blue 1995c). Checks with hand-held instruments indicated that the pit depths may be underestimated by about 15 mils (Blue 1995c), and for this reason a factor of 15 mils was added to the measured maximum pit depth for these cylinders.

(3) The third data set consists of two bottom row cylinders that were evaluated in FY97; these were located in the north end of C-745-F when evaluated. It is noted that hand-held measurements using a 2-mm probe were made in 1994 to estimate the minimum wall thickness for eight cylinders in C-745-G yard, but the pit depths were not recorded (Blue 1994). These data are not included in this analysis because it is not possible to reliably estimate the pit depths. These data were used in the analysis discussed in Lyon (1995) because these were the only data available for this yard at that time.

Data set was collected at C-745-G yard as a result of two separate efforts performed in FY98 and FY99. Concerns about the quality of these data preclude inclusion in the current report. (4) UT measurements for fifteen top and 44 bottom cylinders in C-745-G yard were evaluated in September FY99 and were used in this report. Finally, (5) in FY2000, 12 top and 39 bottom G-Yard cylinders were evaluated. The FY2000 data is also used in this report.

Portsmouth, Thin-Walled Cylinders

There are approximately 14,000 thin-walled cylinders ranging in age from a few years to over 40 years located at the PORTS site. Prior to FY96, there were four cylinder storage yards at PORTS. These yards were designated X-745-A, X-745-C, X-745-E, and X-745-F. The X-745-A and X-745-C yards were essentially the same yard, but were separated into different sections. The X-745-C yard had six sections, while the X-745-A yard had three sections. The X-745-A and X-745-C yards had a two tier stacking configuration. The cylinders from the X-745-F yard were single stacked cylinders. The X-745-E yard was a compacted gravel storage area, but was reconstructed during FY95-96 to a reinforced concrete storage yard. In FY96, a total of 5708 cylinders were relocated at PORTS to meet the new storage requirements.

Cylinders at PORTS were moved from single row storage to a two tiered arrangement around 1976. Prior to this, there were no top row cylinders at PORTS. The cylinders had been in their current location until movement activities in FY96. Thus, the "top" row cylinders at PORTS discussed here have only been in the top row for about 20 years.

In FY96, 10% of the cylinders that were relocated were selected using a random number generator to evaluate the wall thickness using ultrasonic thickness (UT) measurements. The 10% evaluation criterion was required based on the Consent Decree with the Ohio Environmental Protection Agency. These cylinders, as well as other cylinders with handling or storage damage, were evaluated using the automated scanner P-Scan system and hand-held measurements. A total of 609 cylinders were evaluated at PORTS in FY96.

For the purpose of modeling, all of the thin-walled cylinders at PORTS are considered as one yard, and the top and bottom rows are treated separately. During FY96, 473 thin-walled (i.e., nominal thickness 312.5 mils) were evaluated (221 from the top row, 252 from the bottom row), with an age range of 6-40 years. In FY98, 142 cylinders were evaluated. During FY99, 141 more cylinders were evaluated. And in FY2000, 130 more were evaluated. Many of the cylinders evaluated in FY97-FY2000 were also measured in FY96. The more recent data indicate a much "thicker" population than the previous data had suggested, but it is not possible at this time to defensibly choose one of these data sets over the other. Due to differences in these data with the previous data, a separate analysis is performed using the newer data to derive the corrosion model. Many of the newer data points are duplicates—measurements made on a single cylinder during different FY's. In these cases, only the most recent measurements were used in the modeling, which, because of statistical independence requirements, assumes that all measurements on any given cylinder were made at (essentially) the same time.

In Lyon (1995), a different dataset was used for the PORTS site. These data consisted of hand-held ultrasonic thickness measurements made in 1994 on 125 cylinders. These data are not used in the present analysis because the measurements were not taken at areas known to have accelerated corrosion, such as the saddle interface. Further, the evaluation techniques currently used are more stringent and provide more accurate data than that obtained previously.

Thick-Walled Cylinders

There are approximately 1800 thick-walled cylinders (nominal wall thickness 625 mils) located at the three sites. During FY96, 137 thick-walled cylinders were evaluated with the PSCAN as part of the relocation efforts (135 at PORTS, 2 at PGDP). At this time, row location when evaluated (top vs. bottom) is uncertain for 20 of the cylinders evaluated at PORTS, and these data are not included in the present analysis. In FY98 two more Portsmouth thick-wass cylinders were evaluated, 11 cylinders were evaluated in FY99, and 25 were measured in FY2000.

Skirted Cylinders

There are about 1500 thin- and thick-walled 48" cylinders at the three sites that have skirted ends. Most of these cylinders were manufactured before 1970. There is a concern that accelerated corrosion in the head/skirt interface crevice is possible because of a combination of extended time of wetness and differential aeration (Lykins and Pawel 1997). In order to comply with the Director's Findings and Orders with the Ohio EPA at PORTS for cylinder movements performed in FY96, wall thickness data were taken during FY97 at the head/skirt interface for both thin-walled (233) and thick-walled (115) cylinders. In FY2000, 48 top and 38 bottom-row thin-wall cylinders (and one with indeterminate top/bottom status) and 15 top and 8 bottom thick-wall cylinders were also evaluated at the head skirt interface. These data are used to project the conditions at the head/skirt interface for the entire population of skirted cylinders. Two thick-walled cylinders were also evaluated at PGDP. These cylinders were selected because of ease of accessibility (Lykins and Pawel 1997).

Model 30A Cylinders at Paducah

There are 1825 model 30A cylinders located at Paducah. Precise historical information is not available on each cylinder, but it is known that all of these cylinders were manufactured around 1954. During FY99, 100 of these cylinders were evaluated via manual ultrasonic thickness methods. There were two lists of cylinders to be evaluated. The first was generated in June of 1999. Errors in the list required that it be replaced with an updated list that reflected random sampling. The second list was generated and disseminated in July 1999. The cylinders actually evaluated in August 1999 were apparently chosen based on the first list; with this list, the cylinders were chosen from just a few rows of the yard. These data were used, with caveats as discussed in Uncertainties and Limitation (Section 8).

5. DATA ANALYSIS

Table 7 shows the basic groupings made with the current sampled data. Scatterplots for the data for each grouping are shown in Figures 1-15 in Appendix A. The same groupings applied to the complete cylinder population (both sampled and unsampled) define target populations for each group. For each group, the sampled data should, in theory, represent its corresponding target population. (In practice, purposive rather than random sampling, and other obstacles, may impede that theoretical objective.) The data and model fitting for each of these cylinder populations are discussed below.

5.1. K-1066-K Yard

This population is treated separately from the other populations, because a large portion of these cylinders were in ground contact for extended periods while they were in a previous yard (K-1066-G yard). The narrow age range, coupled with a lack of data for the same cylinders at significantly different times, makes it difficult to determine and/or defend any dependence of corrosion on age. In fact, it may be that the age of the cylinder is no longer relevant for predicting future corrosion.

With these data there are several issues that require making assumptions: (1) how to incorporate the two cylinders discovered in 1992 that were deemed to breach due to external corrosion (Barber et al. 1994), (2) whether or not the top and bottom row populations should be modeled separately, and (3) how to incorporate the data collected in 1998-2000. In this report, two separate predictions are made for this yard: in the first, the breaches are included with the data collected in 1994 , and in the second the data collected in 1998 are used separately, not including the breaches. See Appendix D for a discussion of the data collected in 1994.

The data collected via manual ultrasonic thickness evaluation in 1998 and 1999 are statistically different (i.e., medians are different) from the data collected before that, with the PSCAN equipment: the manually collected data show in general both a lower amount of wall loss and larger minimum thickness. This is consistent with the results obtained in Schmidt et al. (1996), where it was found that the PSCAN measurements under-predicted minimum wall thickness. They found that, generally, the PSCAN method resulted in underestimates of minimum wall thickness by an average of 10-20 mils. However, rather than manipulate the available data (either the PSCAN data or the more recently collected data), in this report the more recent data are used to provide an additional projections about the cylinders.

Although there was no a priori intention of comparing the PSCAN and manual UT methods, such comparisons are nevertheless useful. (Also, evaluators would not likely be biased about instrumentation methods, since they were not focused on any such comparisons.) Since six of the cylinders selected at random for evaluation in 1998 and 1999 from K-1066-K yard were previously evaluated with the PSCAN equipment in 1994, additional comparisons were possible (see Table 8 below). In fact, these comparisons were required as part of the process of determining how or even if the different data sets could be combined. The results obtained here are consistent with those obtained in the Schmidt et al. analyses: manual evaluation results in a thicker estimate of the minimum wall thickness. The mean difference here is 50 mils (standard error = 8.8, significance level = .002). The problem then is how to best address this issue for the purpose of cylinder condition prediction. One possibility would be to use the data in Table 8 and other similar data to adjust the results for one or both methods, so that they would be closer to equivalent. At this time, however, these data are not combined; rather, the newer data are used to provide a separate estimate of the future condition of the cylinders.

Table 7. Summary of Populations and Modeling Assumptions

Cylinder Grouping	Population	Thick-ness	Model	Sample Size	Inter-cept	Slope	Std. Dev.	Initial Thick-ness Sample Size	Initial Mean	Initial Std	Initial Thick-ness Inter-val	Total in Popu-lation
Thin-Walled Cylinder Populations	K-1066-K, top and bottom, pre-FY98	Thin	Slope Set to 1	117	0.532	1.000	0.456	117	315.1	9.8	[302.5, 340]	2,542
	K-1066-K, evaluated FY98-2000	Thin	Slope Set to 1	128	-.325	1.000	0.760	128	331.6	8.4	[302.5, 354]	2,542
	C-745-G, bottom	Thin	Fitted Slope	198	2.840	0.200	1.030	198	333.6	10.1	[302.5, 363]	2,064
	PGDP bottom, except G-yard	Thin	Fitted Slope	174	1.737	0.330	0.856	174	328.6	13.1	[302.5, 362]	10,299
	PGDP top	Thin	Fitted Slope	267	0.466	0.816	0.858	267	332.3	9.0	[302.5, 356]	12,281
	PORTS, top, pre-FY99	Thin	Fitted Slope	337	2.734	0.306	0.298	337	335.7	16.6	[302.5, 399]	8,014
	PORTS bottom, pre-FY99	Thin	Fitted Slope	346	2.596	0.400	0.396	346	337.9	16.9	[302.5, 397]	8,014
	PORTS Thin, Top, FY99 and later	Thin	Fitted Slope	128	1.979	0.390	0.472	128	357.9	14.1	[302.5, 389]	8,014
	PORTS Thin, Bottom, FY99 and later	Thin	Fitted Slope	138	1.833	0.426	0.469	138	358.2	13.4	[302.5, 387]	8,014

Table 7 (cont'd). Summary of Populations and Modeling Assumptions

Cylinder Grouping	Population	Thick-ness	Model	Sample Size	Inter-cept	Slope	Std. Dev.	Initial Thick-ness Sample Size	Initial Mean	Initial Std	Initial Thick-ness Inter-val	Total in Popu-lation
Thick-Walled Cylinder Populations	PORTS Thin and PORTS and PGDP Thick, top	Thick	Fitted Slope	435	2.722	0.305	0.315	59	650.3	22.0	[615, 699]	931
	PORTS Thin and PORTS and PGDP Thick, bottom	Thick	Fitted Slope	437	2.876	0.289	0.410	67	644.1	15.7	[615, 697]	931
Skirted Cylinder Populations	PORTS Thin, skirted, top	Thin	Slope Set to 1	135	-.350	1.000	0.620	135	350.2	17.9	[302.5, 392]	3,485
	PORTS Thin, skirted, bottom	Thin	Slope Set to 1	145	-.191	1.000	0.713	145	346.9	14.8	[302.5, 387]	3,574
	PORTS Thick, skirted, top and bottom	Thick	Slope Set to 1	124	0.077	1.000	0.700	124	770.4	23.5	[615, 846]	1,861
Model 30A Cylinders	Paducah 30As, evaluated FY99	30As	Slope Set to 1	100	0.155	1.000	0.707	100	507.2	29.2	[490, 568]	1,825

Table 8. Comparison of Estimated Minimum Point Wall Thickness Using Different Measurement Methods for Cylinders at K-1066-K Yard.

Cylinder	Estimated Minimum Wall Thickness (mils) Using PSCAN Method (1994)	Estimated Minimum Wall Thickness (mils) Using Manual Methods (1998/1999)	Difference
5280	230	311	–81
6294	260	304	–44
6622	250	304	–54
7340	140	200	–60
7486	205	220	–15
14375	280	326	–46
		Mean (Std. Err.):	49.8 (8.8)

When a power law is fit to the maximum pit depths, the resulting model is unrealistic, whether or not the breaches are included. For example, the median maximum pit depth using the 1994 data would be given by $P(t)=0.5\ t^{1.4}$(with breaches included) or $P(t)=0.1\ t^{1.7}$(without breaches included) . This would imply that the corrosion rate is *increasing* for these cylinders at a very high rate, which seems unlikely (note that an example has been found in the literature where a value above 1 is reported[1], but the conditions do not seem relevant).

For the present analysis, the same method for pit depth discussed in Lyon (1995, 1996) is used for the K-1066-K yard cylinders for both analyses. With this method, the distribution of penetration depth is given by $P(t)=R\ t$, where R is the distribution of age-averaged corrosion rates. This method is most applicable for short-term prediction due to the uncertainty about the current corrosion rates and the narrow age range for the cylinders: any model that would fit the current data should agree for near-term predictions since they must agree with the data available.

5.2. Paducah Yards Except G, Bottom Rows

This population is used to represent cylinders at PGDP that were not in ground contact for extended periods. The dataset consist of two basic age groups: cylinders about 18-25 years old (from K and L yards), and cylinders about 33-41 years old (from B and F yards). If more data for older cylinders were available for K or L yard bottom row (or any of the other better-condition yards, such as M or N yard), then the F yard cylinders would not be included in this dataset. Use of the F yard bottom row cylinders is conservative because some may have been in water contact, and hence more corrosion may have occurred. C-745-G yard, which was the only other population of cylinders from PGDP for which data are available, was not considered appropriate due to the poorer conditions of the yard compared to the C-745-F yard.

The power law model that best fits (via least squares) the pit depth data is *Log(1.74 + 0.33 log t,*

[1] A value above 1 was reported for galvanized steel in an industrial environment in Chicago Pourbaix (1982).

0.86), which has a median predicted pit depth of $5.7\ t^{0.33}$. This is slightly different from the fitted model for this population in the previous (FY2000) report, where the pit depth model was *Log(1.79+ 0.60 log t, 0.37)*, which has a median predicted pit depth of $6.0\ t^{0.60}$. The difference is due to the additional data points for C, F, K, and T yards and the generally high variability of the data.

5.3. C-745-G Yard, Bottom Rows

The C-745-G yard cylinders represent the worst conditions at the PGDP site. Many of these cylinders were in ground contact for extended periods. Unlike K-1066-K yard at Oak Ridge, there is a wide range of ages for these cylinders.

A power law model that fits (via least squares) the pit depth data is *Log(2.8 + 0.20 log t, 1.0)*, which has a median predicted pit depth of $17.1\ t^{0.20}$. This is different from the fitted model in the previous report, where the penetration model used was *Log(1.85 + 0.71 log t, 0.37)*, which has a median predicted pit depth of $6.4\ t^{0.71}$. The difference is due to the additional data added in FY99 and FY2000 and the generally high variability of the data.

5.4. Paducah Yards, Top Rows

Few, if any, of the cylinders in the top rows of these yards were ever in extended ground contact, and this is assumed to be the case for all of the top row cylinders at PGDP (note that what is modeled here as the C-745-F top row cylinders are currently in the bottom row, and vice versa, due to the relocation that took place in 1992).

A power law model that fits (via least squares) the pit depth data is *Log(0.47 + 0.82 log t, 0.86)*, which has a median predicted pit depth of $1.6\ t^{0.82}$. The previous fit, for FY2000, which was *Log(2.04 + 0.50 log t, 0.25)*.

5.5. Portsmouth thin-walled cylinders, Top Rows

As with the top row cylinders at PGDP, few of the PORTS top row cylinders have ever been in extended ground contact. The data collected for the top row cylinders at PORTS are used to represent both the PORTS top row cylinders as well as the top row cylinders in the K-1066-B/E/J yards at the ETTP site at Oak Ridge until more data for the K-1066-B/E/J yards are collected. There are two datasets available, and each is used in a separate analysis to predict the future condition of the population. The first data set consists of evaluations for 277 cylinders, collected using the automated PSCAN equipment prior to 1998. The second consists of data collected for 85 cylinders with manual methods in 1999.

A power law model that fits (via least squares) the pit depth data collected prior to 1999 is *Log(2.73 + 0.31 log t, 0.30)*, which has a median predicted pit depth of $15.4\ t^{0.31}$. For the data collected in 1999 and later, the corresponding power law model is *Log(1.98 + 0.39 log t, 0.47)*, which has a median predicted pit depth of $7.2\ t^{0.39}$. The model fit with data collected before FY99 has larger predicted median penetration depths (for times less than about 13,000 years[2]).

---

[2]One can find the time at which the curves cross by solving the equation $7.2\ t^{0.39} = 15.4\ t^{0.31}$, which reduces to $t = (15.4/7.2)^{1/.08} \approx 13{,}000$ years.

5.6. Portsmouth thin-walled cylinders, Bottom Rows

Few of the PORTS bottom row cylinders have ever been in extended ground contact. The data collected for the bottom row cylinders at PORTS are used to represent both the PORTS bottom row cylinders as well as the bottom row cylinders in the K-1066-B/E/J yards at the ETTP site at Oak Ridge until more data for the K-1066-B/E/J yards are collected. There are two datasets available, and each is used in a separate analysis to predict the future condition of the population. The first data set consists of evaluations for 281 cylinders, collected using the automated PSCAN equipment prior to 1998. The second consists of data collected for 56 cylinders with manual methods in FY99.

A power law model that fits (via least squares) the pit depth data collected prior to FY98 is *Log(2.60 + 0.40 log t, 0.40)*, which has a median predicted pit depth of $13.4\ t^{0.40}$. For the data collected in FY99 and later, the corresponding power law model is *Log(1.93 + 0.39 log t, 0.45)*, which has a median predicted pit depth of $6.9\ t^{0.39}$. The model fit with the data collected before FY99 has higher predicted median penetration depths than the model for the FY99 and later data.

5.7. Thick-walled cylinders, Top Rows

There were 59 thick-walled cylinders from the top row evaluated in FY96-2000 (58 from PORTS, 1 from C-745-C), with an age range of 42-48 years. Due to a concern about using such a narrow age, and since it is expected that the penetration depth for the thin-walled cylinders will be similar to that for the thick-walled cylinders, the data for the thin-walled cylinders in the top row at PORTS (375 cylinders) were added to the dataset, and a model for penetration depth was derived. This model was applied for top row thick-walled cylinders at all yards. It is noted that there are approximately 420 thick-walled cylinders at K25 in the K-1066-B/E/J yards, 275 at PGDP in the C-745-B/C/D yards, and 1166 thick-walled cylinders at PORTS. Using the combined dataset, the resulting power law model that fits (via least squares) these pit depth data is *Log(2.72 + 0.31 log t, 0.32)*, which has a median predicted pit depth of $15.2\ t^{0.31}$.

5.8. Thick-walled cylinders, Bottom Rows

There were 69 thick-walled cylinders from the bottom row evaluated in FY96-2000 (68 from PORTS, 1 from C-745-C), with an age range of 42-48 years. As for the top row thick-walled cylinders, the data for the thin-walled cylinders in the bottom row at PORTS (370 cylinders) were added to the dataset, and a model for penetration depth was derived. This model was applied for bottom row thick-walled cylinders at all yards. Using the combined dataset, the resulting power law model that fits (via least squares) these pit depth data is *Log(2.88 + 0.29 log t, 0.41)*, which has a median predicted pit depth of $17.7\ t^{0.29}$.

5.9. Thin-walled Skirted Cylinders

The wall thickness in the head/skirt interface was evaluated for 279 thin-walled skirted cylinders at PORTS during FY96-2000. As for the thick-walled cylinders, an extremely narrow age range (38-44 yr) precluded fitting a time-dependent model for penetration depth. As there are similar issues with the thick-walled skirted cylinders, combining the thin- and thick-walled datasets (as is done for the thick-walled cylinders above) is not considered. Instead, the penetration depth is modeled as $P(t)=Rt$, where R is the distribution of age-averaged corrosion rates (this is the same method currently used for the K-1066-K yard cylinders). The fitted model for the top row cylinders is *Log(-0.35, 0.62) t*, which has a median predicted pit depth of *0.70 t*. The median wall loss for the bottom row cylinders is slight larger, although the variability is similar; the fitted model for the top row cylinders is *Log(-0.19, 0.71) t*, which has a median predicted pit depth of *0.83 t*. In both cases it could be accepted at the 0.05 level of significance that the distribution of age-averaged corrosion rates were lognormally distributed, using the method described in

Lilliefors (1967; also see Appendix B).

5.10. Thick-walled Skirted Cylinders

The wall thickness in the head/skirt interface was evaluated for 123 thick-walled skirted cylinders at PORTS during FY97 (Lykins and Pawel 1997); for 45 of these cylinders, the row (top/bottom status) was not available. The available data could not be fit with a power law, although the age range (42-48 plus one cylinder age 36 when measured) was larger than that for the thin-walled skirted cylinders. As for the thin-walled skirted cylinders, the penetration depth is modeled as $P(t)=Rt$, where R is the distribution of age-averaged corrosion rates (this is the same method currently used for the K-1066-K yard cylinders). For the FY97 and earlier data, the top and bottom row populations (35 in each) have a similar fit using this approach: $Log(0.32,0.63)\ t$ for top row, $Log(0.34,0.71)\ t$ for bottom row. Application of the F-test (see, e.g., Snedecor and Cochran 1978, pp.116-117) indicated that one can accept that the variances of the logarithms of the pit depths for the top and bottom row populations are equal with 5% significance. Similarly, application of the t-test with unequal variances (Casella and Berger 1990), one can conclude that the medians of the distributions are the same at the same level of significance. The goodness-of-fit tests indicated that the use of a lognormal distribution must be considered a conservative approximation, as the fitted lognormal distribution has a slightly higher probability of obtaining age-averaged corrosion rates above 2.25 mils/yr. These results suggest that it is not unreasonable to treat the top and bottom rows as a single population. Thus, in order to allow inclusion of the cylinders for which the row is unknown, the top and bottom rows are combined, and the other cylinders were added to the dataset. The combined fitted model is $Log(0.08,\ 0.70)\ t$, which has a median predicted pit depth of $1.08\ t$.

5.11. Model 30A Cylinders at Paducah

There are 1825 model 30A cylinders located at PGDP. In the summer of 1999, 100 of these cylinders were evaluated using manual ultrasonic thickness techniques. There were 50 cylinders evaluated from the top row and 50 from the bottom row As for the K-1066-K yard cylinders, there is a narrow age range for these cylinders; in fact, precise information on the manufacture date is not known, except that all were manufactured around 1954. For the purpose of the current analysis, it is assumed that all cylinders were manufactured in 1954, and the distribution of penetration depth is given by $P(t)=R\ t$, where R is the distribution of age-averaged corrosion rates. The Lilliefors test of normality indicated that this was an adequate fit at the 0.01 level of significance, but not at the 0.05 level of significance. There is a slight difference between the corrosion rates for the different rows, with the bottom row having a slightly higher corrosion rate. Table 9 below shows the sample statistics for the two populations.

Table 9. Comparison of Corrosion Rates for Model 30A Cylinders at Paducah, Evaluated 1999

	Average Corrosion Rate (mil/yr)	Standard Deviation of Corrosion Rate
Bottom Row (50 cylinders)	1.59	1.69
Top Row (50 cylinders)	1.53	1.37

Using the t-test with unequal variances (Casella and Berger 1990), one can conclude that the medians of the distributions are not different; i.e., there is not a statistically significant difference between the medians of the distributions for the top and bottom row cylinders. For this reason, these populations are combined for the purpose of the present analysis.

6. RESULTS

Using the assumptions discussed above, projections were made of the number of cylinders with a minimum wall thickness less than preliminary criteria values. For thin- and thick-walled cylinders, these minimum wall thickness criteria are:

1. A wall thickness that indicates a possible loss of contained material (a breach)

2. A wall thickness below which ordinary safe handling and stacking is impaired (62.5 mils)

3. A wall thickness representing applicable standards for off-site transport and contents transfer (based on ANSI 14.1 1995; 250 mils for thin-walled cylinders, 500 mils for thick-walled cylinders)

Several additional thickness criteria including 0 (breach) are also considered in the projections made here (Table 10). For the model 30A cylinders, there are no published criteria for minimum thicknesses of interest; based on personal communication with S. J. Pawel, two criteria are reported in the modeling results presented here: 100 mils (the minimum thickness for regular hot feeding), and breach.

It is important to note that, in general, these criteria are based on *area* of wall thinning. However, the minimum thickness predicted in this report is for an area of about 0.01 sq. in, essentially a point. For thickness criteria greater than zero (breach), using a point thickness may add conservatism to the results in this report. On the other hand, because of the interaction of UF6 with atmospheric moisture and the substrate steel, the approximation of a small-area breach with a point breach is probably close:

> A breach in a cylinder allows the external atmosphere to react slowly with the UF6. The solid reaction product tends to plug the breach; however, the HF formed releases slowly, attacks the metal cylinder, and enlarges the breach over time. The hole diameter is estimated to increase at a rate of approximately one inch per year. (DNFSB 1995).

Table 10 shows predicted number of cylinders with a minimum wall thickness below various thickness criteria. The thin- and thick-walled cylinder populations include skirted cylinders, but the results do not specifically apply to the thickness at the head/skirt interface because the data used do not address the thickness there. The results for the skirted cylinders are based on data specifically collected at the head/skirt interface. These results are based on grouping unsampled cylinder populations with similar populations that have been sampled (e.g., K-1066-B yard is added to the PORTS population).

Also included in Table 10 are additional alternative analyses for the cylinders with a nominal wall thickness of 312.5 mils. These analyses are based on the models derived from the 1998/1999 wall thickness data from K-1066-K and Portsmouth. Three populations are addressed in the additional analyses: K-1066-K yard, and the top and bottom rows from PORTS and K-1066-B/E/J.

The numbers in the columns labeled "Estimate" are computed from the least square estimates of the model parameters. The confidence limits are upper bounds on the upper 95% confidence limit. These confidence limits are calculated using a different and somewhat simpler method than the method used for the previous report. Both methods are described in Appendix B. The new approach leads to confidence limits that are closer to their corresponding point estimates. Yet both sets of confidence limits are premised on exactly the same assumptions and have the same nominal 95% confidence level. The tighter confidence bounds demonstrate that the penalty for statistical (sampling) uncertainty is smaller than had previously been thought. This is important, because, as the variability of the measured data points about the fitted curves in Figures 1-15 demonstrates, there is substantial variability in the cylinder data.

For the populations other than the K-1066-K yard and skirted cylinders, the differences between the upper confidence limits and the estimates depend on two factors: the total number of cylinders sampled, and the ages of the cylinders sampled. Basically, the larger the spread in ages of the cylinders sampled, the smaller the difference between the confidence limits and the estimated values. The confidence limits for the K-1066-K yard, the model 30A cylinders, and the skirted cylinders depend on the number of samples but not on the spread of the ages of the cylinders sampled.

6.1. Thin-Walled Cylinders

The results for predicted cylinders with minimum thickness below 250 mils show that the populations can be naturally divided into three groups: (1) K-1066-K yard and C-745-G yard bottom row, (2) C-745-B/F/K/L and PORTS bottom row, and (3) C-745-B/F/G/K/L and PORTS top row. The results for the number of cylinders with minimum thickness below 62.5 mils or breaches are similar, although there is a more pronounced difference between the K-1066-K and C-745-G yard bottom row cylinders.

There is a marked difference between the results obtained using the 1998/2000 data versus the previously available data. This is a reflection of the differences in the data itself, with the more recent data suggesting that less corrosion has occurred than previous data had implied. However, there are substantial differences between the measurements methods (PSCAN vs manual UT) used to collect these sets of data, and hence it cannot be concluded which of the datasets is closer to reality.

For two cylinder populations, K-1066-K at ETTP and the cylinders that were in the bottom row of C-745-G at PGDP, a large number of cylinders are predicted to have a minimum point thickness below 250 mils at present. Both of these populations have had a large fraction of cylinders that were in ground contact at one time, although they are no longer in ground contact. The change in conditions is not incorporated into projections for these cylinders. For several of the populations, including the bottom row of C-745-G at PGDP, quite a few breaches are predicted to have occurred by 2001, many more than would seem very likely. (Note that these estimates include any breaches already known to have occurred.) Again, much of this apparent overestimation can be attributed to changes in the conditions these cylinders are being stored under. Some of the overestimation is due simply to sampling error, which, as can be seen in Figures 1-15, is considerable. Some of the overestimation may be due to the lognormal distribution assumption.

For all of the cylinder populations, bottom row cylinders are generally predicted to have a greater number less than 250 mils than top row cylinders. Generally, the confidence limits increase with increasing time in a manner that depends on the number of samples and the spread of the ages of the cylinders evaluated. Increasing either the sample size or the spread of the ages will result in a decrease in the difference between the expected value and the upper confidence limit.

6.2. Thick-Walled Cylinders

The results for the thick-walled cylinders (nominal wall thickness 625 mils) are similar to those for the PORTS yard. Few are predicted to have a minimum point thickness below the ANSI N14.1 standard (ANSI 1995) of 500 mils, and none are predicted to have a thickness below 62.5 or have a breach, through 2020. The data available for the thick-walled cylinders are for a narrow age range that precludes finding a fit with a power law model (data are for thick-walled cylinders at PORTS). For this reason, the maximum pit data for the thin-walled cylinders at PORTS were included with the data for the thick-walled cylinders in order to derive the model for penetration depth.

Table 10. Summary Projections for Specified Target Years and Minimum Thickness Values

Cylinder Grouping	Population	Specified Value	Thick-ness	Model	Projected Number of Cylinders Below Minimum Thickness Value											
					2001		2005		2010		2015		2020		2025	
					Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB
Thin-Walled Cylinder Populations	K-1066-K, top and bottom, pre-FY98	250	Thin	Slope Set to 1	1,363	1,572	1,559	1,780	1,769	1,989	1,939	2,145	2,075	2,260	2,181	2,343
		197	Thin	Slope Set to 1	307	501	421	631	581	798	751	964	924	1,124	1,094	1,276
		62.5	Thin	Slope Set to 1	6	31	11	47	22	75	37	110	59	154	87	205
		0	Thin	Slope Set to 1	1	10	3	16	5	27	10	43	17	63	27	89
	K-1066-K, evaluated FY98-2000	250	Thin	Slope Set to 1	239	411	295	480	369	565	445	650	524	733	603	813
		197	Thin	Slope Set to 1	60	151	80	186	108	231	140	279	175	329	213	380
		62.5	Thin	Slope Set to 1	5	24	7	32	11	43	15	56	21	71	28	87
		0	Thin	Slope Set to 1	2	13	3	17	5	23	7	31	10	40	13	50
	C-745-G, bottom	250	Thin	Fitted Slope	418	648	430	670	444	698	457	727	468	759	479	794
		197	Thin	Fitted Slope	194	373	202	389	210	411	218	434	225	459	231	487
		62.5	Thin	Fitted Slope	49	136	51	144	54	155	57	167	59	181	61	196
		0	Thin	Fitted Slope	30	95	31	101	33	109	35	118	37	129	38	141
	PGDP bottom, except G-yard	250	Thin	Fitted Slope	340	917	391	962	451	1,033	507	1,129	561	1,250	612	1,385
		197	Thin	Fitted Slope	73	306	86	325	103	357	119	400	135	455	151	518

Table 10 (cont'd). Summary Projections for Specified Target Years and Minimum Thickness Values

					Projected Number of Cylinders Below Minimum Thickness Value											
					2001		2005		2010		2015		2020		2025	
Cylinder Grouping	**Population**	**Specified Value**	**Thickness**	**Model**	**Estimate**	**95% UCB**	**Estimate**	**95% UCB**	**Estimate**	**95% UCB**	**Estimate**	**95% UCB**	**Estimate**	**95% UCB**	**Estimate**	**95% UCB**
Thin-Walled Cylinder Populations	PGDP bottom, except G-yard	62.5	Thin	Fitted Slope	5	47	7	51	8	57	10	67	12	79	14	93
		0	Thin	Fitted Slope	2	24	3	27	3	30	4	36	5	43	6	51
	PGDP top	250	Thin	Fitted Slope	852	1,676	1,072	1,961	1,368	2,321	1,678	2,648	1,994	2,995	2,311	3,436
		197	Thin	Fitted Slope	261	674	342	821	459	1,017	591	1,200	734	1,399	888	1,671
		62.5	Thin	Fitted Slope	33	134	45	173	65	228	89	282	117	343	150	433
		0	Thin	Fitted Slope	16	75	22	99	32	134	45	168	60	206	79	265
	PORTS, top, pre-FY99	250	Thin	Fitted Slope	165	304	216	380	288	484	368	600	455	728	546	866
		197	Thin	Fitted Slope	1	5	1	6	2	10	3	14	5	19	7	26
		62.5	Thin	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
		0	Thin	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
	PORTS bottom, pre-FY99	250	Thin	Fitted Slope	690	1,044	866	1,252	1,097	1,521	1,333	1,800	1,568	2,087	1,799	2,373
		197	Thin	Fitted Slope	48	118	66	155	94	210	128	275	167	351	212	437
		62.5	Thin	Fitted Slope	0	1	0	1	0	2	1	3	1	5	1	7
		0	Thin	Fitted Slope	0	0	0	0	0	0	0	1	0	1	0	1

Table 10 (cont'd). Summary Projections for Specified Target Years and Minimum Thickness Values

					Projected Number of Cylinders Below Minimum Thickness Value											
					2001		2005		2010		2015		2020		2025	
Cylinder Grouping	Population	Specified Value	Thick-ness	Model	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB
Thin-Walled Cylinder Populations	PORTS Thin, Top, FY99 and later	250	Thin	Fitted Slope	16	99	22	120	31	149	41	183	53	222	67	270
		197	Thin	Fitted Slope	1	11	1	14	2	19	2	24	3	32	4	41
		62.5	Thin	Fitted Slope	0	0	0	0	0	0	0	0	0	1	0	1
		0	Thin	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
	PORTS Thin, Bottom, FY99 and later	250	Thin	Fitted Slope	13	85	18	105	26	133	35	165	47	203	60	250
		197	Thin	Fitted Slope	1	9	1	12	1	16	2	21	3	28	4	37
		62.5	Thin	Fitted Slope	0	0	0	0	0	0	0	0	0	1	0	1
		0	Thin	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
Thick-Walled Cylinder Populations	PORTS Thin and PORTS and PGDP Thick, top	500	Thick	Fitted Slope	0	1	1	2	1	3	1	3	1	4	2	5
		62.5	Thick	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
		0	Thick	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0

Table 10 (cont'd). Summary Projections for Specified Target Years and Minimum Thickness Values

Cylinder Grouping	Population	Specified Value	Thick-ness	Model	2001 Esti-mate	2001 95% UCB	2005 Esti-mate	2005 95% UCB	2010 Esti-mate	2010 95% UCB	2015 Esti-mate	2015 95% UCB	2020 Esti-mate	2020 95% UCB	2025 Esti-mate	2025 95% UCB
					Projected Number of Cylinders Below Minimum Thickness Value											
Thick-Walled Cylinder Populations	PORTS Thin and PORTS and PGDP Thick, bottom	500	Thick	Fitted Slope	9	20	11	23	13	28	15	32	17	37	19	41
		62.5	Thick	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
		0	Thick	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
Skirted Cylinder Populations	PORTS Thin, skirted, top	250	Thin	Slope Set to 1	111	236	148	295	203	376	267	462	337	553	415	647
		62.5	Thin	Slope Set to 1	1	5	1	7	2	11	3	16	4	22	6	30
		0	Thin	Slope Set to 1	0	2	0	3	0	5	1	7	1	10	2	14
	PORTS Thin, skirted, bottom	250	Thin	Slope Set to 1	289	489	359	577	453	691	553	806	658	920	765	1,033
		62.5	Thin	Slope Set to 1	6	28	9	38	14	52	20	68	28	87	37	109
		0	Thin	Slope Set to 1	2	14	4	19	6	27	9	37	12	48	17	61
	PORTS Thick, skirted, top and bottom	500	Thick	Slope Set to 1	18	57	24	70	33	89	44	110	56	132	70	156
		62.5	Thick	Slope Set to 1	0	2	0	3	0	4	1	6	1	7	1	10
		0	Thick	Slope Set to 1	0	1	0	2	0	3	0	4	1	5	1	7

Table 10 (cont'd). Summary Projections for Specified Target Years and Minimum Thickness Values

					Projected Number of Cylinders Below Minimum Thickness Value											
					2001		2005		2010		2015		2020		2025	
Cylinder Grouping	Population	Specified Value	Thick-ness	Model	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB
Model 30A Cylinders	Paducah 30As, evaluated FY99	100	30As	Slope Set to 1	4	22	5	29	8	37	11	48	15	59	20	72
		0	30As	Slope Set to 1	1	11	2	15	3	20	5	26	6	32	9	40

6.3. Results at the Head/Skirt Interface for Skirted Cylinders

The results for the predicted minimum wall thickness at the head/skirt interface, in Table 10, present some noteworthy differences, compared to the other results. In particular, for thick-walled cylinders, all of which are skirted, more cylinders are predicted to have minimum thickness below the various criteria thicknesses, when compared to the results using the data for the body of the cylinder. However, the narrow age range of cylinders evaluated precluded a power law fit for the penetration depth, and a model of the form $P(t)=Rt$ was used instead. This form of model will generally be more conservative than one using a power law, $P(t)=At^n$. For the wall thickness data from the body of the cylinder, a power law model was used for most of the thin-walled cylinder populations, and all of the thick-walled populations. It is also worth noting that unlike the cylinder ground contact problem, which has been eliminated over the years, the head/skirt crevice is a feature that has persisted (except for painting, which is not considered for these head/skirt interface cylinders).

6.4. Model 30A Cylinders

The penetration model used for the model 30A cylinders suggests that for a few cylinders significant corrosion may have occurred; in fact, it is predicted that several may have a minimum thickness less than 100 mils or even breach at this time. This is not surprising given that two of the cylinders evaluated in 1999 had an estimated minimum wall thickness less than 100 mils.

6.5. Impact of Cylinder Painting

The painting program initiated in FY97 has resulted in reducing the forecasted number of cylinders with wall thicknesses below the specified criteria levels, with the more substantial reduction occurring for the C-745-G yard bottom row population. The tables below summarize one aspect of the potential impact of the painting program, assuming that the painting will essentially halt corrosion for approximately 10 years. The impacts are more noticeable for C-745-G yard bottom row because a larger proportion of this population has been painted than for K-1066-K yard.

Table 11. Illustration of impact of current painting program on C-745-G yard bottom row cylinders

	Predicted number of cylinders with minimum point wall thickness below 250 mils			
	1998	2000	2004	2008
C-745-G bottom row, if no painting had occurred	1387	1528	1800	2053
C-745-G bottom row, accounting for painting completed in 1998*	1387	1490	1688	1872
C-745-G bottom row, accounting for painting completed by 2000*		1490	1642	1784
Predicted number of cylinders that pre-1998 painting prevents from reaching 250 mil minimum point thickness	0	38	112	181
Predicted number of cylinders that all painting prevents from reaching 250 mil minimum point thickness	0	38	158	269
* For 1998, this number is the total for the unpainted cylinders alone as well as the cylinders that have been painted by 1998. After 1998, it is the number of unpainted cylinders that fail the criterion during each time period In this illustration, it is assumed that after 1998 the corrosion on the painted cylinders will be halted for a ten year period. A similar calculation is carried out when accounting for the painting completed by 2000.				

Table 12. Illustration of impact of current painting program on K-1066-K yard cylinders.

	Predicted number of cylinders with minimum point wall thickness below 250 mils			
	1998	2000	2004	2008
K-1066-K yard, if no painting had occurred	1417	1543	1775	1977
K-1066-K yard, , accounting for painting completed in 1998	1417	1527	1730	1907
Predicted number of cylinders that painting prevents from reaching 250 mil minimum point thickness	0	16	45	70

7. MODEL EVALUATION

In this section, analyses are conducted to compare results predicted results using the modeling results in the FY2000 (Lyon 2000) report with the data actually collected during FY2000. Similar comparisons are made in the FY2000 report using predictions based on even earlier predictions. In its simplest form, the basic question is posed as follows:

Using the model from the previous report, how "well" did it do in characterizing the newer data collected?

There are several important factors complicating such an evaluation; the primary one here is the observed difference in the data for the different measurement techniques used. Further, the models used here provide a statistical estimate of the probability of exceeding a given thickness, while the evaluation is a discrete event. For the purpose of the evaluation here, a comparison is made of the cylinders which were observed to fail the various criteria, and the number predicted by the relevant model(s). More evaluations will be possible as more data are collected.

Predicted and observed results are compared in Table 13. For the various cylinder populations, the table shows numbers of cylinders falling below 80%, 60%, 20% and 0% of their nominal thicknesses. The "Portsmouth Thin and Thick..." "base population" includes thin as well as thick-wall cylinders, but the actual target population for these groups is Portsmouth thick-wall cylinders only.

With one exception, the comparisons show that the model predictions are fairly good, at least for these specified thickness. (Minor differences between predicted and observed results can be attributed to statistical variation in both the FY2000 counts and the data on which the predictions are based.) The one exception, is the PGDP G-yard bottom predictions. As discussed in the last section and in the model evaluation section of the FY2000 report, the G-yard bottom predicted numbers of cylinders falling below the various criteria are biased high because they do not reflect the improved storage conditions for those cylinders.

Table 13. Comparison of FY2000 Projected and Observed Counts for Sampled Cylinders

Base Population	Total in Target Population	Number Sampled	Thickness Spec.	Projected Rate	Projected Number Out of Spec.	Observed Number Out of Spec.
K-1066-K, evaluated 1998-2000	2,542	58	250	1.24E-01	7	5
			187.5	2.61E-02	2	0
			62.5	2.56E-03	0	0
			0	1.02E-03	0	0
C-745-G, bottom	2,064	39	250	4.18E-01	16	0
			187.5	5.60E-02	2	0
			62.5	6.84E-04	0	0
			0	8.81E-05	0	0
PGDP bottom, except G-yard	10,299	31	250	4.32E-02	1	0
			187.5	1.10E-03	0	0
			62.5	1.84E-06	0	0
			0	1.20E-07	0	0
PGDP top	12,281	30	250	7.82E-03	0	0
			187.5	3.31E-06	0	0
			62.5	2.03E-12	0	0
			0	4.44E-15	0	0
PORTS Thin and Thick, and C-745-C Thick, top, pre-1999	931*	12	500	5.41E-05	0	0
			375	6.75E-12	0	0
			125	1.16E-22	0	0
			0	5.87E-27	0	0

*Thick-wall cylinders only.

Table 13 (cont'd). Comparison of FY2000 Projected and Observed Counts for Sampled Cylinders

Population	Total in Population	Number Sampled	Thickness Spec.	Projected Rate	Projected Number Out of Spec.	Observed Number Out of Spec.
PORTS Thin and Thick, and C-745-C Thick, bottom, pre-1999	931*	7	500	2.51E-03	0	0
			375	1.11E-07	0	0
			125	1.10E-14	0	0
			0	1.61E-17	0	0
PORTS Thin, skirted, top	3,485	47	250	4.61E-02	2	0
			187.5	2.57E-03	0	0
			62.5	2.41E-05	0	0
			0	3.50E-06	0	0
PORTS Thin, skirted, bottom	3,574	39	250	8.90E-02	3	0
			187.5	7.06E-03	0	0
			62.5	1.01E-04	0	0
			0	1.70E-05	0	0
PORTS Thick, skirted, top and bottom	1,861	23	500	1.19E-02	0	0
			375	2.27E-03	0	0
			125	1.76E-04	0	0
			0	6.20E-05	0	0
PORTS Thin, Top, 1999	8,014	74	250	4.15E-06	0	0
			187.5	8.96E-10	0	0
			62.5	3.69E-15	0	0
			0	2.75E-17	0	0

*Thick-wall cylinders only.

Table 13 (cont'd). Comparison of FY2000 Projected and Observed Counts for Sampled Cylinders

Population	Total in Population	Number Sampled	Thickness Spec.	Projected Rate	Projected Number Out of Spec.	Observed Number Out of Spec.
PORTS Thin, Bottom, 1999	8,014	54	250	9.86E-04	0	0
			187.5	2.62E-05	0	0
			62.5	1.12E-07	0	0
			0	1.26E-08	0	0

8. UNCERTAINTIES AND LIMITATIONS

The purpose of this report is to estimate the extent of corrosion on populations of cylinders as a function of time, and this requires that assumptions be made regarding the dependence of the distribution of corrosion rates on time. The data currently available consist of wall thickness measurements made on different cylinders of different ages. There is little data from the same cylinder at substantially different times. Taking repeated measurements on the same cylinders at different times tends in fact to be contrary to the goal of inspecting as many cylinders as reasonably possible. Nevertheless, if repeated measurements were made over time, cylinders could serve as their own controls, and more accurate estimates of corrosion could be obtained. Instead, it is necessary to make conservative assumptions regarding the change of corrosion with time or to use measurements from different cylinders at different ages to estimate corrosion with time. Implicit in any trend derived from the current data is an assumption of age invariance—that older cylinders had corrosion similar to younger cylinders when they were younger; the distribution of pit depths for 10 year old cylinders in a given population is the same no matter when it is measured. This assumption is unavoidable without data for a fixed cylinder at different ages.

Significant differences between the data collected at different times for the same yards have been observed. In all cases, the more recent data suggest that the amount of corrosion is less than previous data indicated. However, uncertainty in the differences between measurement methods and techniques used prevents an assessment as optimistic as the more recent data suggest. Another source of uncertainty is changing storage conditions. This complication increase when cylinders have been moved numerous times throughout their history.

The results are also based on the assumption that the data are representative of the population from which they are taken, which is difficult to ascertain for the older data for some populations (e.g., data for C-745-B and L yard cylinders). The PORTS data are probably most representative because they were randomly sampled when the cylinders were relocated during FY96. For most of the data collected since 1998, the cylinders sampling plans called for random sampling, but practical difficulties in moving cylinders has sometimes forced compromises in the sampling plans.

Other important caveats and difficulties in cylinder modeling include

- Initial wall thicknesses are estimates from maximum wall thicknesses from monitoring mainly to identify minimum wall thicknesses
- Storage (e.g., ground contact) conditions have changed for many cylinders
- Some cylinders have been painted
- Literature data for atmospheric corrosion of steel does not apply to cylinder corrosion modeling, because of the thermal inertia of the cylinders[3]
- Environmental changes such as acid rain are not accounted for
- Corrosion appears to be only very weakly related to cylinder age

---

[3]Steve Pawel, personal communication.

9. CONCLUSIONS AND RECOMMENDATIONS

The United States Department of Energy (DOE) currently manages depleted uranium hexafluoride that is stored in approximately 50,000 carbon steel cylinders located at three DOE sites. Using either a hand-held ultrasonic transducer or an automated scanner, wall thickness and corrosion pit depth data have been collected for several subpopulations of cylinders. In this report, the most recently collected wall thickness data were used, along with previously collected data, to make projections about the condition of the cylinders located at ETTP, PGDP, and PORTS. The results presented here are intended to supercede and enlarge the scope of those presented previously (Lyon 1995,1996, 1997, 1998, 2000). In particular, projections are made for thin-walled cylinders (nominal initial thickness 312.5 mils), thick-walled cylinders (nominal initial thickness 625 mils), and model30A cylinders (nominal initial thickness 500 mils). In addition, an analysis is conducted for the minimum thickness at the head/skirt interface for skirted cylinders.

The most recently collected data, which were not available for the previous report (Lyon 2000), consisted of evaluations of the wall loss for additional cylinders. In some cases the more recent data generally suggest a more "optimistic" view of the current minimum point wall thickness for the cylinder populations, although in some cases there is a wider spread (i.e., variance) in the newer data that can impact the predictions for rare events such as breaches. In some cases (e.g., Paducah G-yard bottom cylinders), this is due to relatively recent changes in cylinder storage conditions. However, some of the differences are also due to differences between the manual UT and PSCAN measurement methods, differences for which further investigation is probably warranted.

On a percentage basis, most of the thin-walled cylinders predicted to have a minimum point thickness less than 250 mils in 2001 are located in the bottom rows of C-745-G yard at PGDP and K-1066-K yard at ETTP. Some of these cylinders are also expected to fall below 62.5 mils and some all the way to 0 mils (breach). Cylinders in K-1066-K yards population, all PGDP populations (including 30As), and Portsmouth thin skirted bottom populations are all predicted to have one or more cylinders in the breach range. These predictions almost surely reflect the conservative nature of the models on which they are based. Note that both the variance and location estimates in the fitted models affect the predictions for the thickness extremes.

The painting program has reduced the predicted number of cylinders that do not meet the specified wall thickness criteria. For the C-745-G yard bottom row cylinders, it is predicted that the painting already completed will prevent almost 300 cylinders from failing the ANSI 14.1 thickness criterion by 2008 (assuming that painting halts corrosion for approximately 10 years). For K-1066-K yard, painting is predicted to prevent 70 cylinders from failing this criterion.

Other than at the head/skirt interface, few of the approximately 1862 thick-walled cylinders are predicted to have a minimum point thickness below any of the thickness criteria by 2020. In particular, less than 50 are predicted to have a minimum wall thickness below 500 mils by 2020, and none are predicted to have a breach or minimum point thickness less than 62.5 mils by 2020. Some of the thick-wall cylinders are expected to be below these specs at the head/skirt interface however. An analysis of data specifically collected at the head/skirt interface during FY97 confirmed the accelerated nature of corrosion in the skirt crevice compared to the general body. In particular, some thick-walled cylinders, all of which are skirted, were predicted to have a minimum thickness below 62.5 mils, or even a breach, at the interface by 2020.

Recommendations:

1. A re-evaluation of the measurement techniques (PSCAN and manual UT) should be performed, with the goal being to ensure a uniform implementation of the documented measurement methods across sites and personnel. In several cases, the recent data collected indicate a significantly different picture than the previous data had suggested; however, it is not clear if these differences are due to various measurement techniques/personnel, or reflect a more accurate assessment of the cylinder populations.

2. More wall thickness evaluations are needed in the head/skirt interface for skirted cylinders. In particular, a wider age range of skirted cylinders should be evaluated.

3. The values in this report are based on the assumption that the historical trends will continue, and thus represent all baseline projections. Many of the yards are being improved, in which case the corrosion rates will probably be reduced. Future analyses should incorporate these changes, if they can be reasonably quantified and accounted for. This will require additional discussion with the site technical personnel.

4. In this report minimum thickness is modeled through models of both initial thickness and maximum pit depth. Minimum wall thickness can be modeled directly, however, and, for several reasons, modeling minimum wall thickness would probably be easier than modeling maximum pit depths. One reason is that the initial thickness distribution would then not have to be modeled separately, and the assumption that initial thickness and pit depth are statistically independent could be avoided. That assumption may fail, for example, if the quality of the steel and the initial thickness are correlated. Another reason that modeling minimum wall thickness would be easier than modeling maximum pit depth is that wall thicknesses, not pit depths, are what is actually measured. Because thickness-when-new measurements were not made, measuring maximum pit depth entails measuring thickness in relatively uncorroded areas of cylinder surfaces, which are assumed to be "as new." Working and safety specifications, however, are expressed in terms of minimum wall thickness, not maximum pit depth, and field personnel tend naturally to focus on areas of minimum wall thickness. This has caused a rift between model and practice, which has resulted in deficiencies in data intended for modeling pit depth. Note that by comparing results over years, corrosion rates could be estimated with direct minimal thickness models.

5. The extreme value distribution should be investigated as an alternative to the lognormal model. The extreme value distribution has a physical basis for models of minima or maxima, and so, in the absence of evidence to the contrary, it should be considered in addition to alternatives such as the lognormal distribution in models of either minimum wall thickness or maximum pit depths (which are extremes). In this report, the fitted lognormal model fails to conform, in many cases (see Table 7), with the expectation that corrosion rates attenuate over time (i.e., that the slope of the fitted line relating the log of maximum pit depth to the log of cylinder age is less than 1). This failure could be due to improperly weighting the data in the model fitting. The weighting is a reflection of the underlying statistical distribution (e.g., lognormal) that is assumed.

10. REFERENCES

American National Standards for Nuclear Materials-Uranium Hexafluoride-Packaging for Transport, ANSI N14.1, American National Standards Institute, New York, 1995 edition.

Barber, E.J., J.H. DeVan, J.M. Googin, and M.S. Taylor (1994). *Investigation of Breached Depleted UF_6 Cylinders at the K-25 Site*. ORNL/TM-12840, Oak Ridge, Tennessee, October 1994.

Blue, S.C. (1994). Facsimile from S.C. Blue to M. Taylor, October 27, 1994.

Blue, S.C. (1995a). Memorandum from S.C. Blue to A.K. Balding, September 25, 1995.

Blue, S.C. (1995b). Facsimile from S.C. Blue to B.F. Lyon, October 30, 1995.

Casella, G. And R.L. Berger (1990). *Statistical Inference*. Wadsworth and Brooks/Cole Advanced Books and Software, Pacific Grove, California.

DNFSB (1995) "Integrity of Uranium Hexafluoride Cylinders," Defense Nuclear Facilities Safety Board, DNFSB/TECH-4, May 5, 1995 (http://www.dnfsb.gov/techrpts/tech-4.html).

Feliu, S., M. Morcillo and S. Feliu, Jr. (1993a). The Prediction of Atmospheric Corrosion from Meteorological and Pollution Parameters--Ii. Annual Corrosion in *Corrosion Science*, Vol. 34, No.3, pp.403-414.

Feliu, S., M. Morcillo and S. Feliu, Jr. (1993b). The Prediction of Atmospheric Corrosion from Meteorological and Pollution Parameters--II. Long-term Forecasts in *Corrosion Science*, Vol. 34, No.3, pp.415-422.

Horton, J.B. (1964). The Composition, Structure and Growth of the Atmospheric Rust on Various Steels. Thesis Lehigh University 1964, Bethlehem, PA.

Lawless, J.F. (1982). *Statistical Models and Methods for Lifetime Data*. John Wiley and Sons, New York.

Legault, R.A. and G. Preban (1975). *Corrosion-NACE*, Vol. 31, p.117.

Lockheed Martin Energy Systems, Inc. (LMES) (1997a). *UF6 Cylinder Project System Requirements Document (SRD)*. K/TSO-001, Rev.3. May 1997. U.S. Department of Energy.

Lockheed Martin Energy Systems, Inc. (LMES) (1997b). *UF6 Cylinder Project System Engineering Management Plan (SEMP)*. K/TSO-017. July 1997. U.S. Department of Energy.

Lykins, M.L. and S.J. Pawel (1997). *Evaluation of Wall Thickness in the Cylinder Head/Skirt Interface on Skirted Cylinders*. Memorandum from M.L Lykins and S.J. Pawel to M.S. Taylor, ORNL/CST-SP-102297-1, October 22, 1997.

Lyon, B.F. (1995). *Prediction of External Corrosion for UF_6 Cylinders: Results of an Empirical Method*, ORNL/TM-13012. June 1995. Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Lyon, B.F. (1996). *Prediction of External Corrosion for Steel Cylinders at the Paducah Gaseous*

Diffusion Plant: Application of an Empirical Method, ORNL/TM-13192. February 1996. Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Lyon, B.F. and M.L. Lykins (1996). *Ultrasonic Thickness Sampling Plan for the Depleted Uranium Hexafluoride Program*, ORNL/TM-13280. July 1996. Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Lyon, B.F. (1997). *Prediction of External Corrosion for Steel Cylinders*, ORNL/TM-13359. February 1997. Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Lyon, B.F. (1298). *Prediction of External Corrosion for Steel Cylinders - 1998 Report*, ORNL/TM-13568. February 1998. Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Lyon, B.F. (2000). *Prediction of External Corrosion for Steel Cylinders - 2000 Report*, ORNL/TM-2000/96. June 2000. Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Mughabghab, S.F. and T.M. Sullivan (1989). Evaluation of the Pitting Corrosion of Carbon Steels and Other Ferrous Metals in Soil Systems. *Waste Management*, Vol. 9, pp. 239-251.

Philpot, H.E. (1995). Memorandum from Halen Philpot to Valerie Newman, February 17, 1995.

Pourbaix, M. (1982). The Linear Bilogarithmic Law for Atmospheric Corrosion, in *Atmospheric Corrosion* (W.H. Ailor, ed.). John Wiley and Sons, New York.

Romanoff, M. (1957). Underground Corrosion. *NBS Circular 579*, National Bureau of Standards, Washington, D.C.

Rosen, R.S. and R.E. Glaser (1995). Letter to C.E. Bradley from R.S. Rosen and R.E. Glaser, November 8, 1995.

Rosen, R.S. and R.E. Glaser (1996). Letter to B.F. Lyon from R.S. Rosen and R.E. Glaser, April, 10, 1996.

Schmidt, M.A., J.K. Harper, and J.A. Broders (1996). *A Comparison of Wall Thickness Measurements on UF_6 Cylinders using Scanning vs Hand-held Ultrasonic Probes*. K/TSO-019, November 1996. Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Snedecor, G.W. and W.G. Cochran. (1978). *Statistical Methods*, Sixth Edition, Iowa State University Press, Ames, Iowa.

APPENDIX A: FIGURES

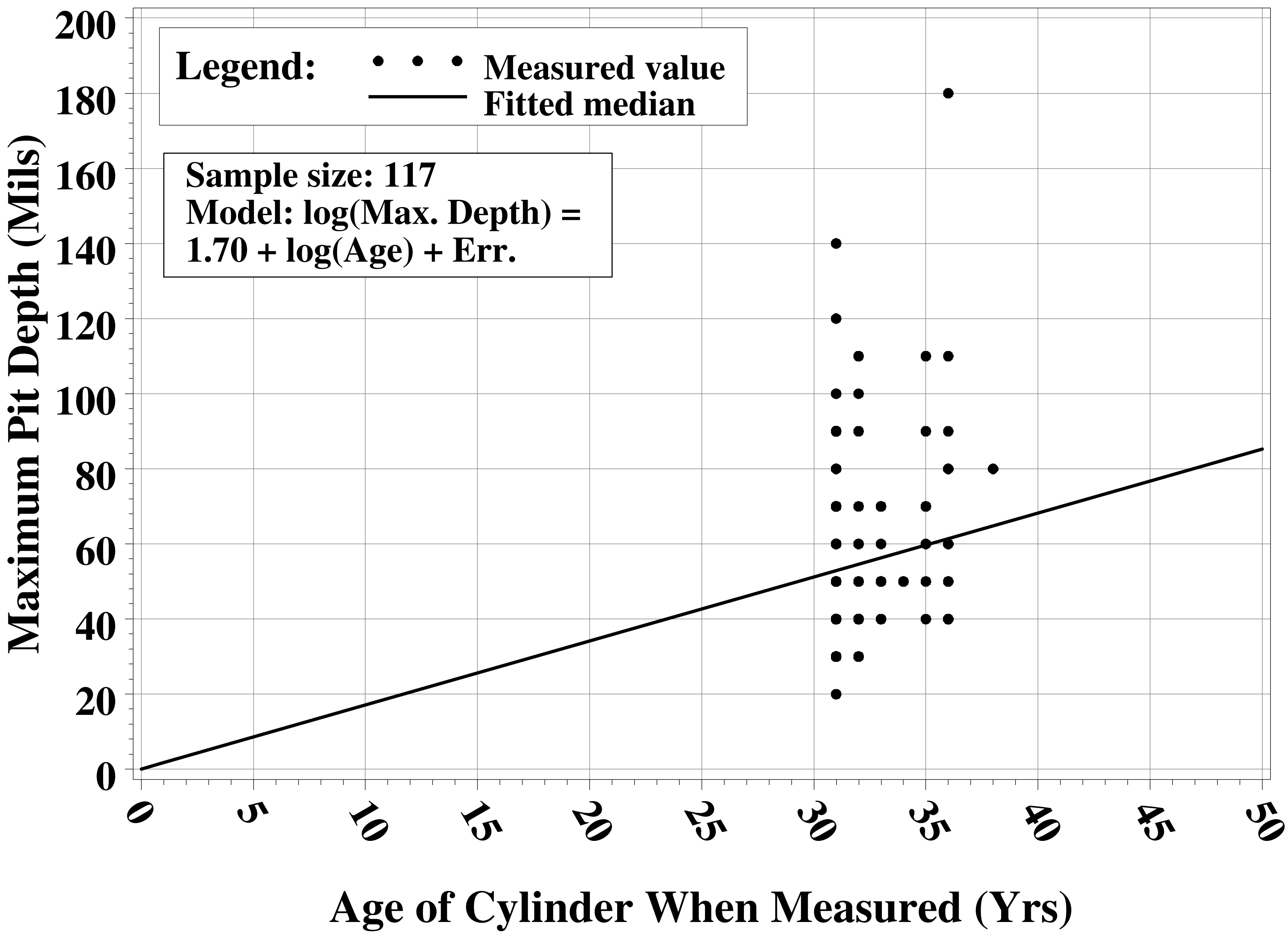

Figure 1. Pit Depths for K-1066-K, top and bottom, pre-FY98.

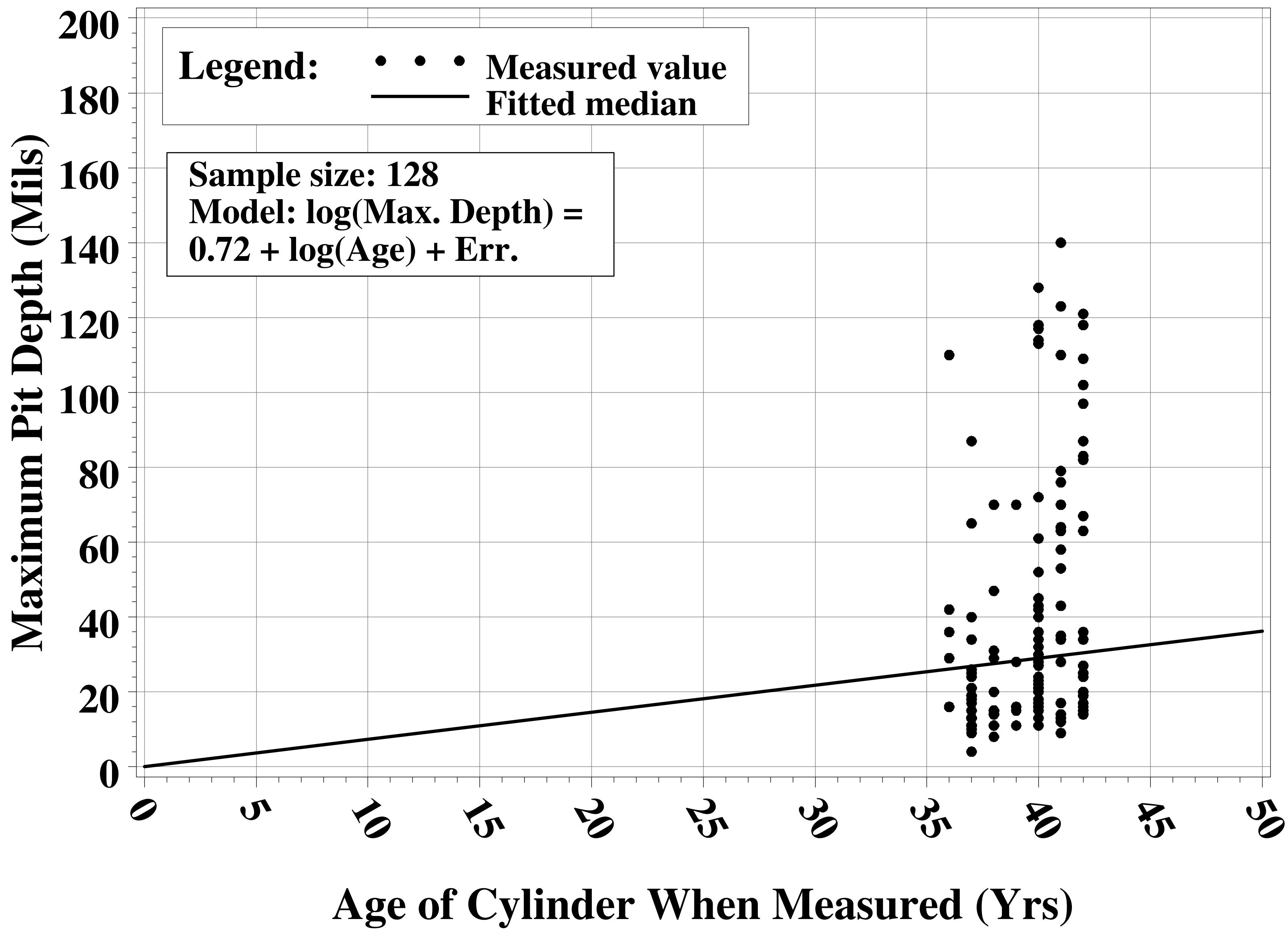

Figure 2. Pit Depths for K-1066-K, evaluated FY98-2000.

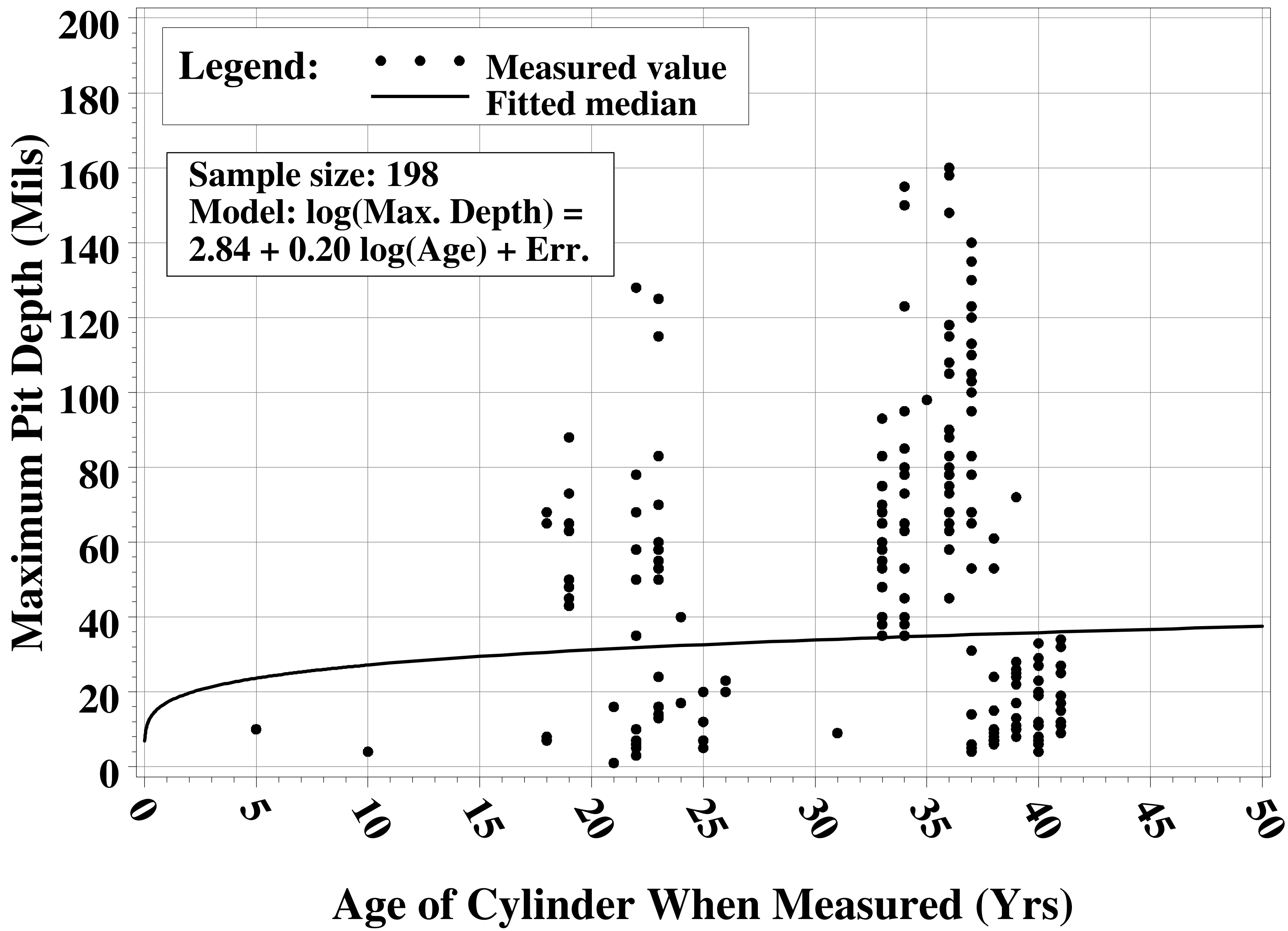

Figure 3. Pit Depths for C-745-G, bottom.

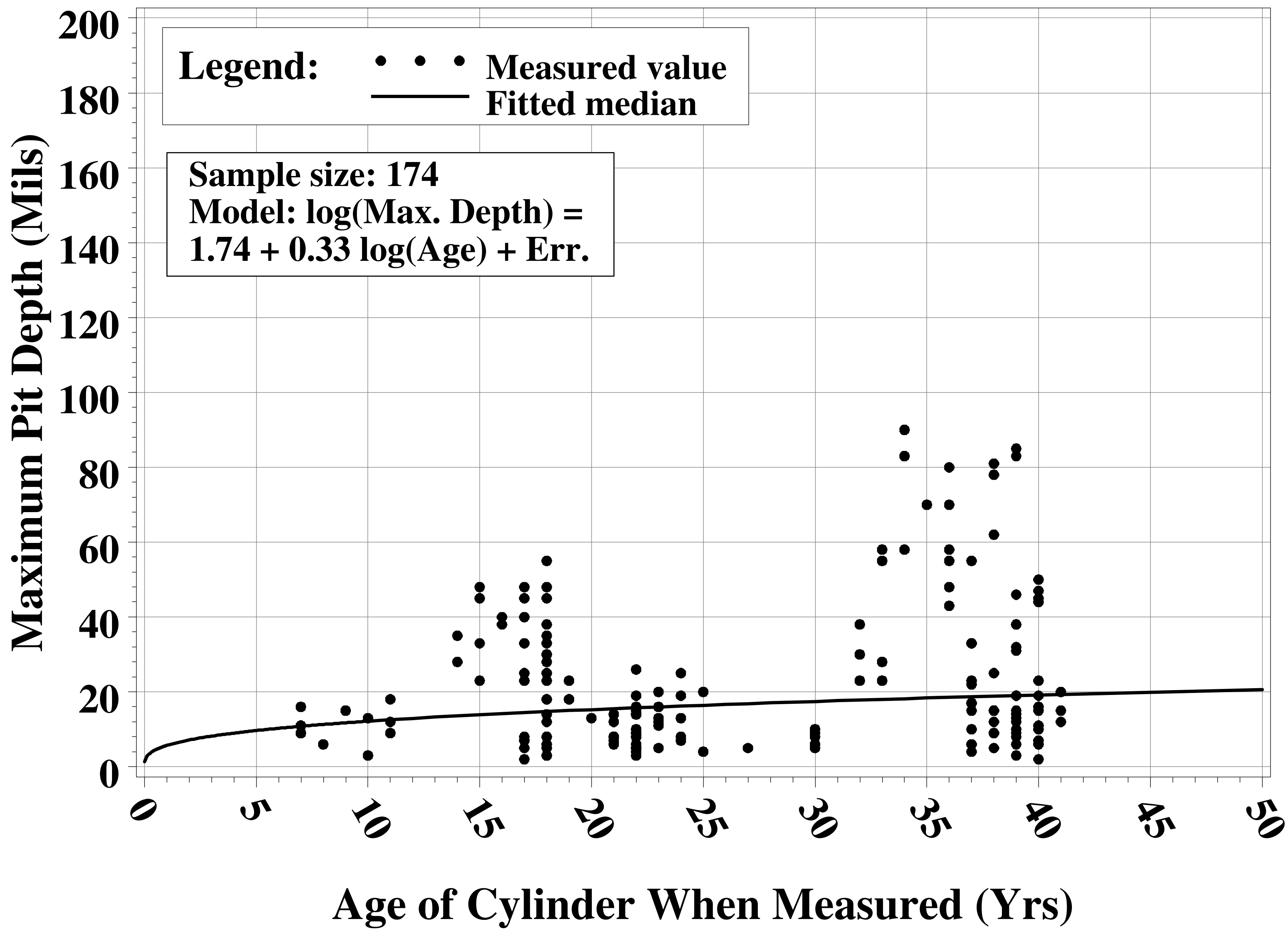

Figure 4. Pit Depths for PGDP bottom, except G-yard.

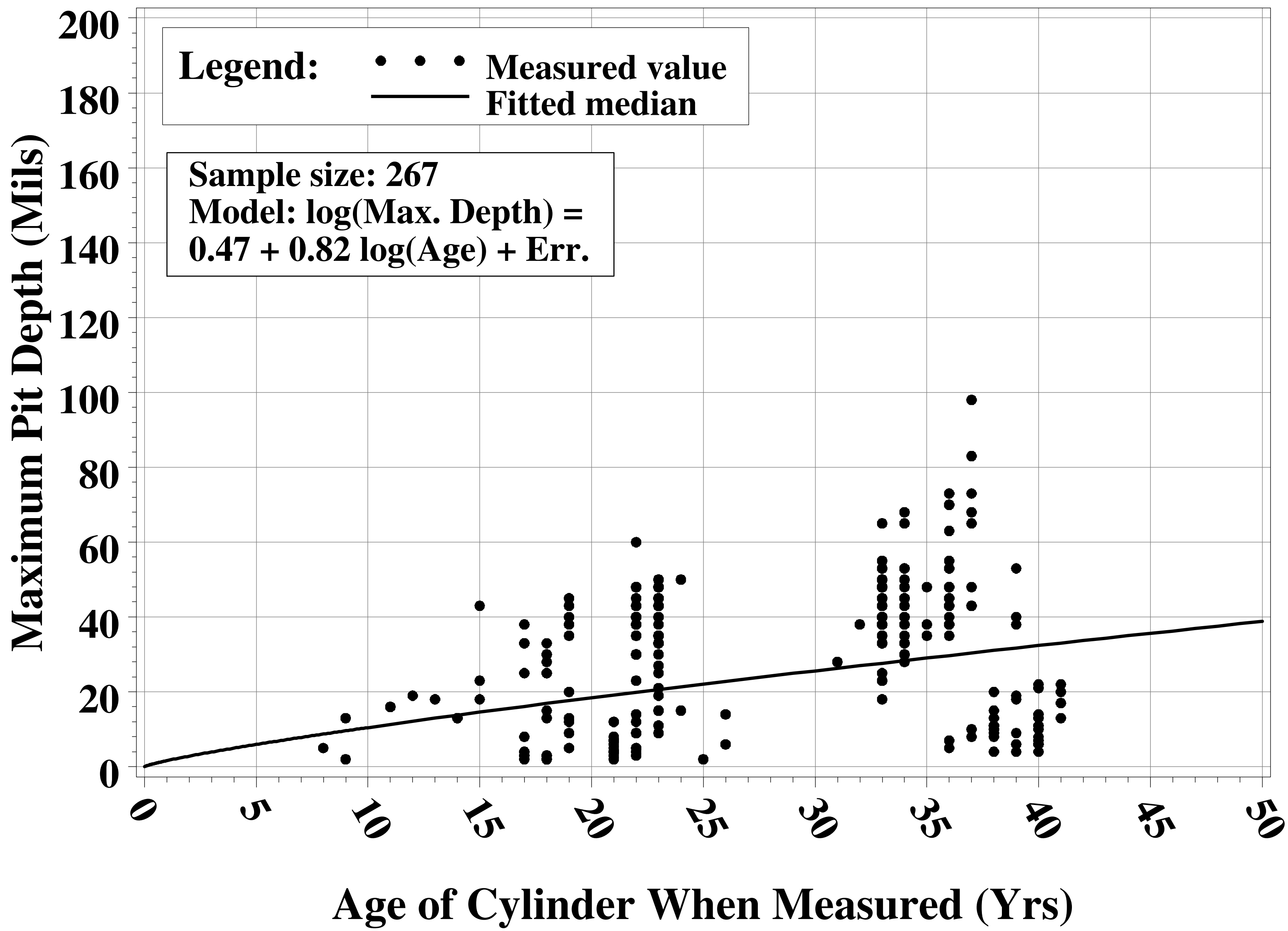

Figure 5. Pit Depths for PGDP top.

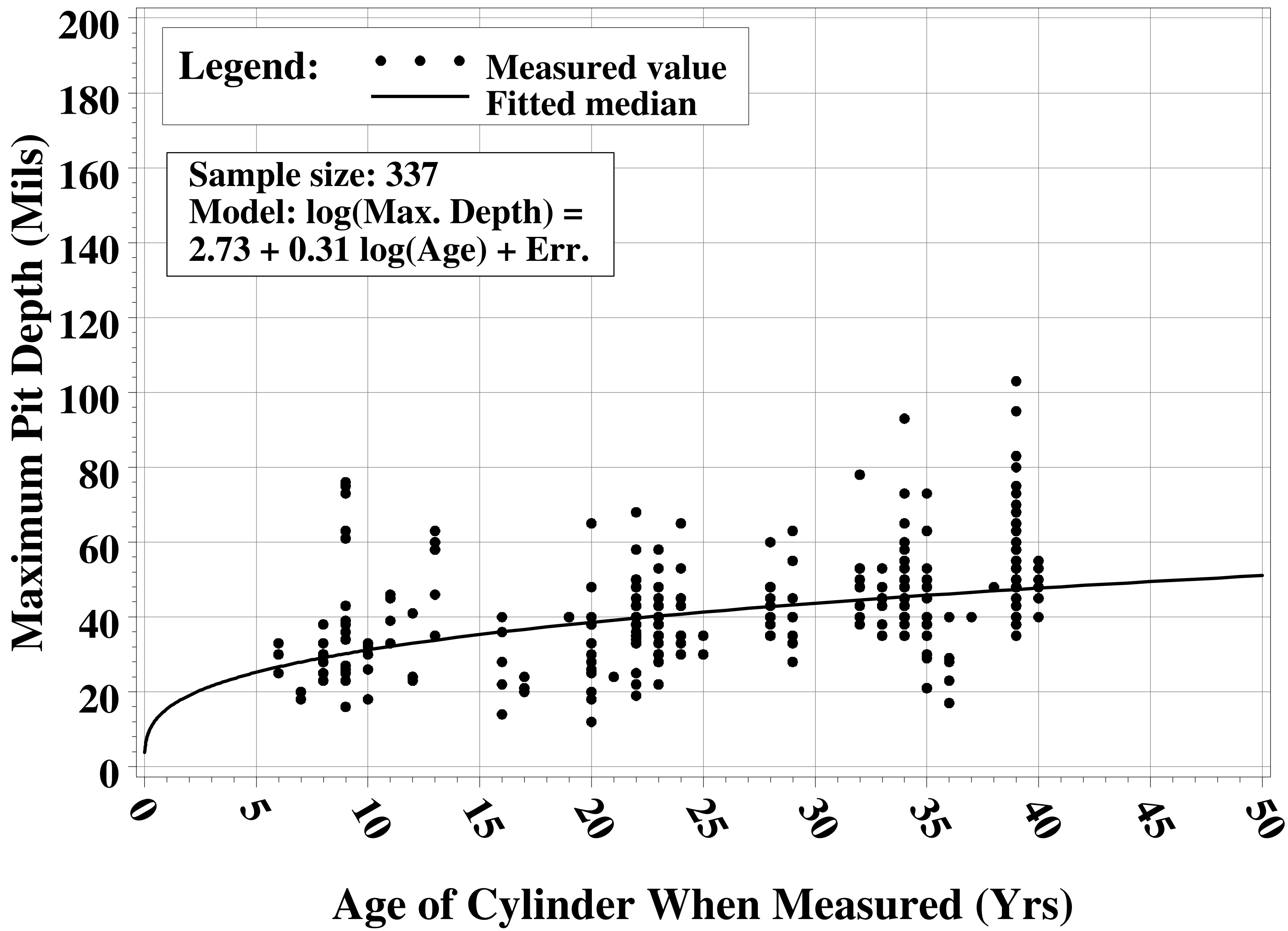

Figure 6. Pit Depths for PORTS, top, pre-FY99.

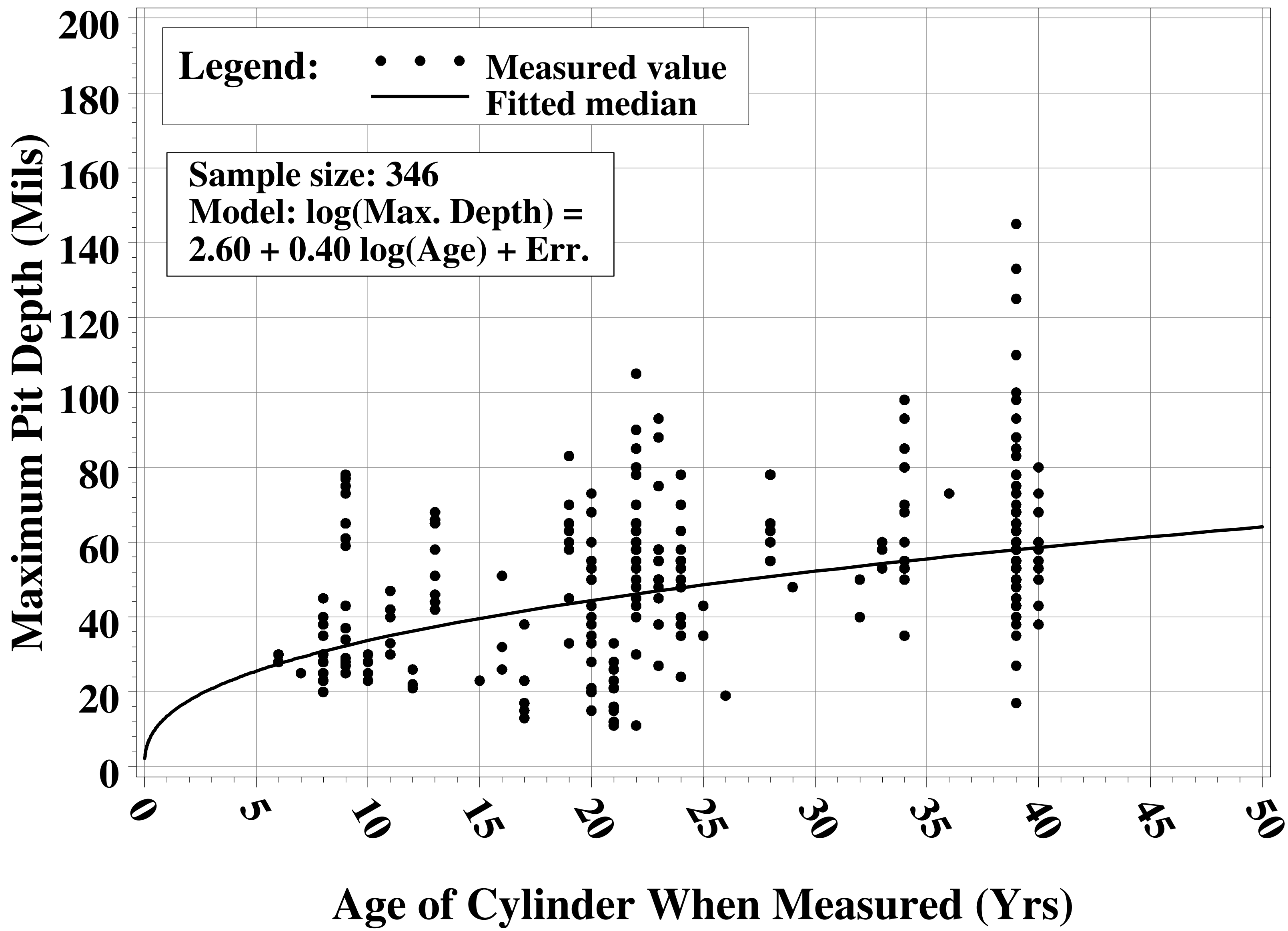

Figure 7. Pit Depths for PORTS bottom, pre-FY99.

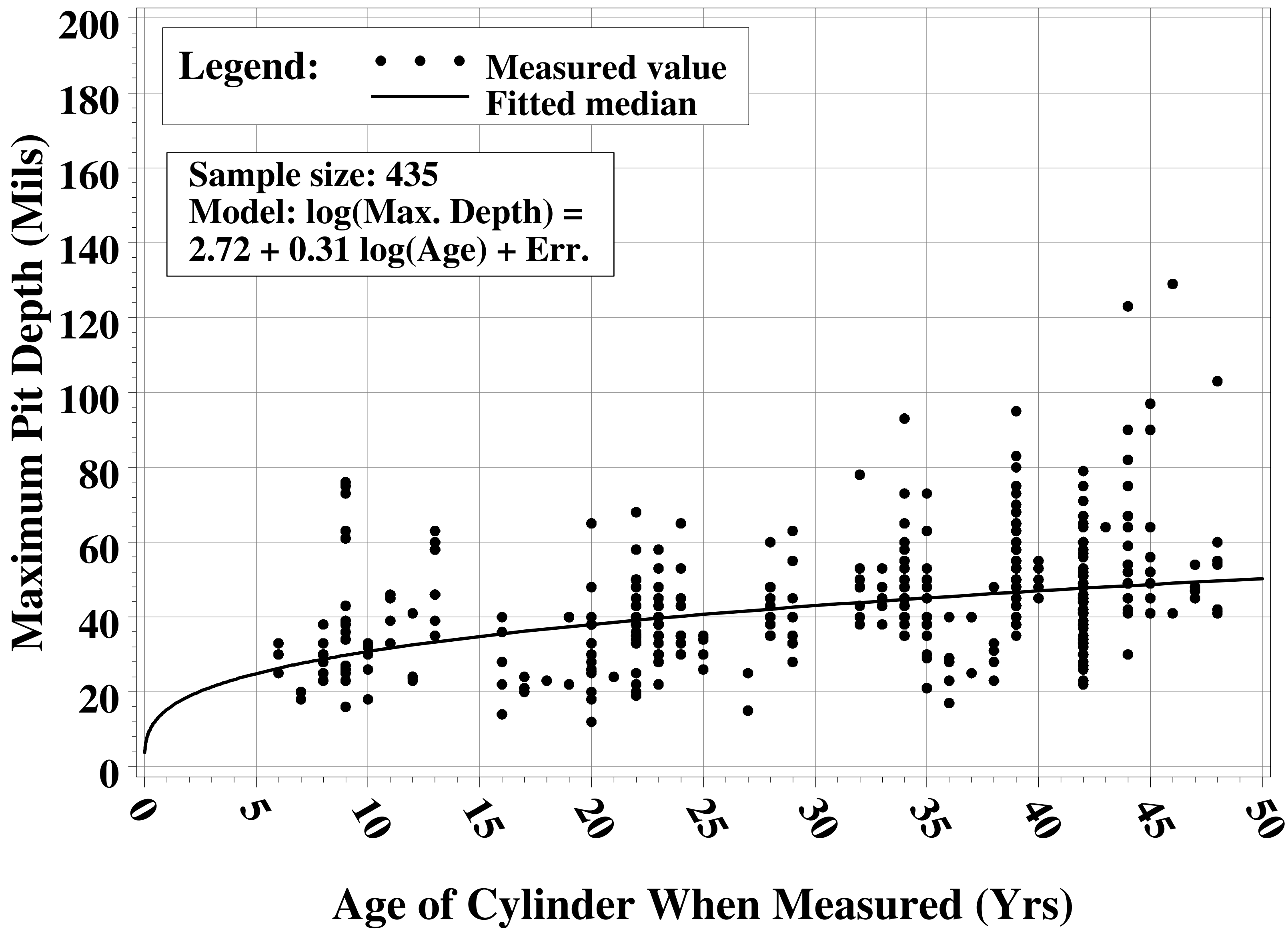

Figure 8. Pit Depths for PORTS Thin and PORTS and PGDP Thick, top.

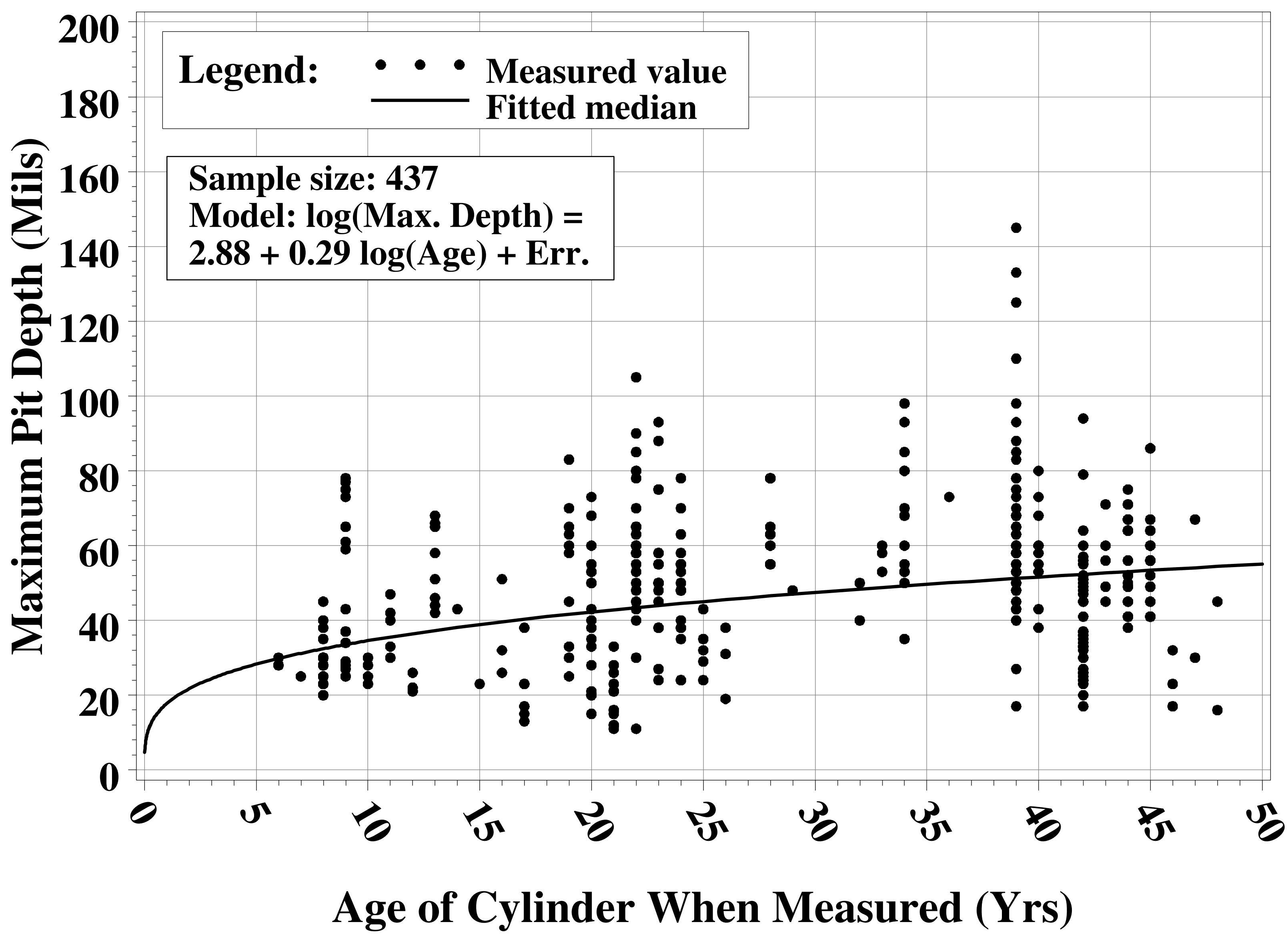

Figure 9. Pit Depths for PORTS Thin and PORTS and PGDP Thick, bottom.

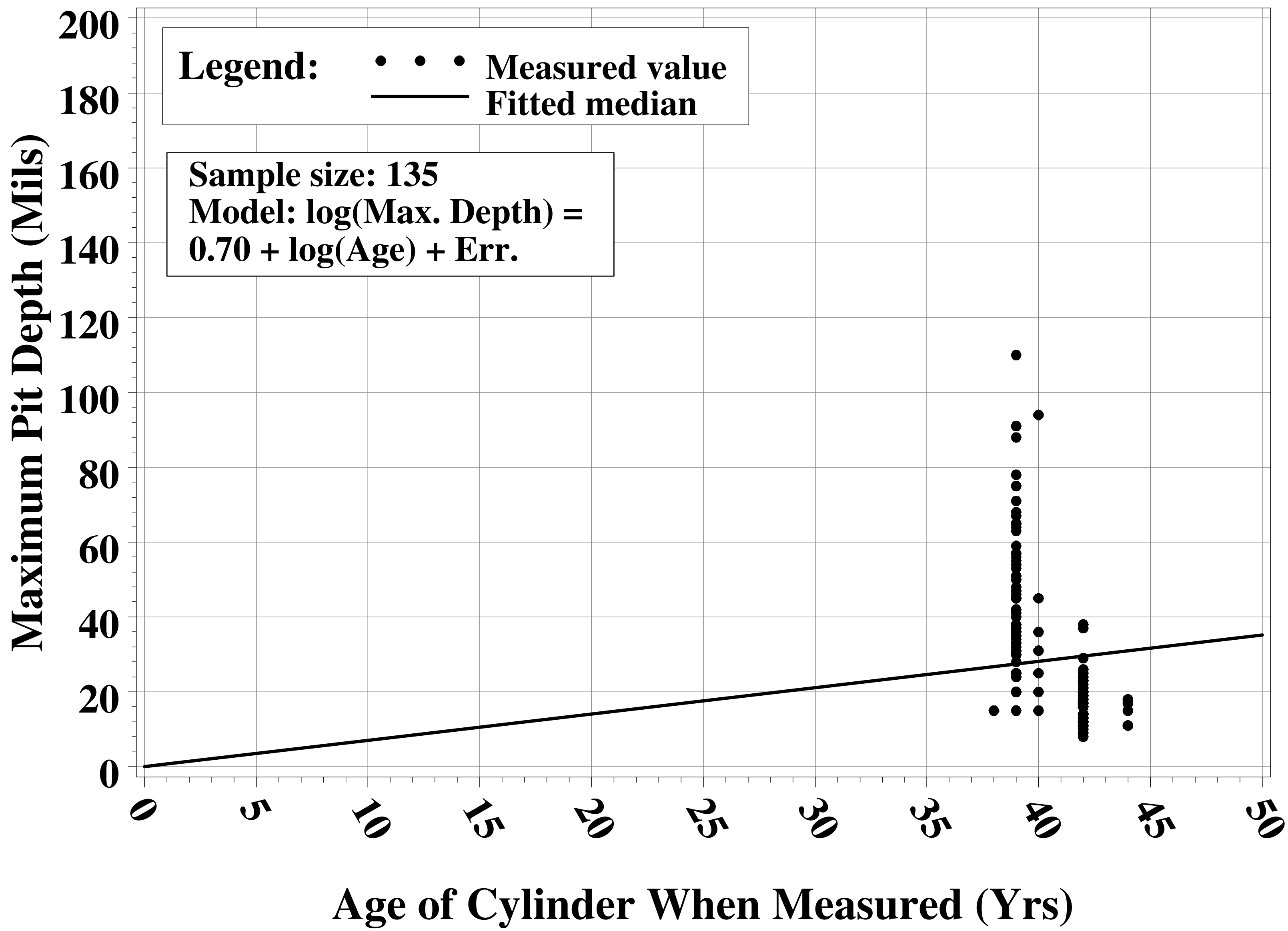

Figure 10. Pit Depths for PORTS Thin, skirted, top.

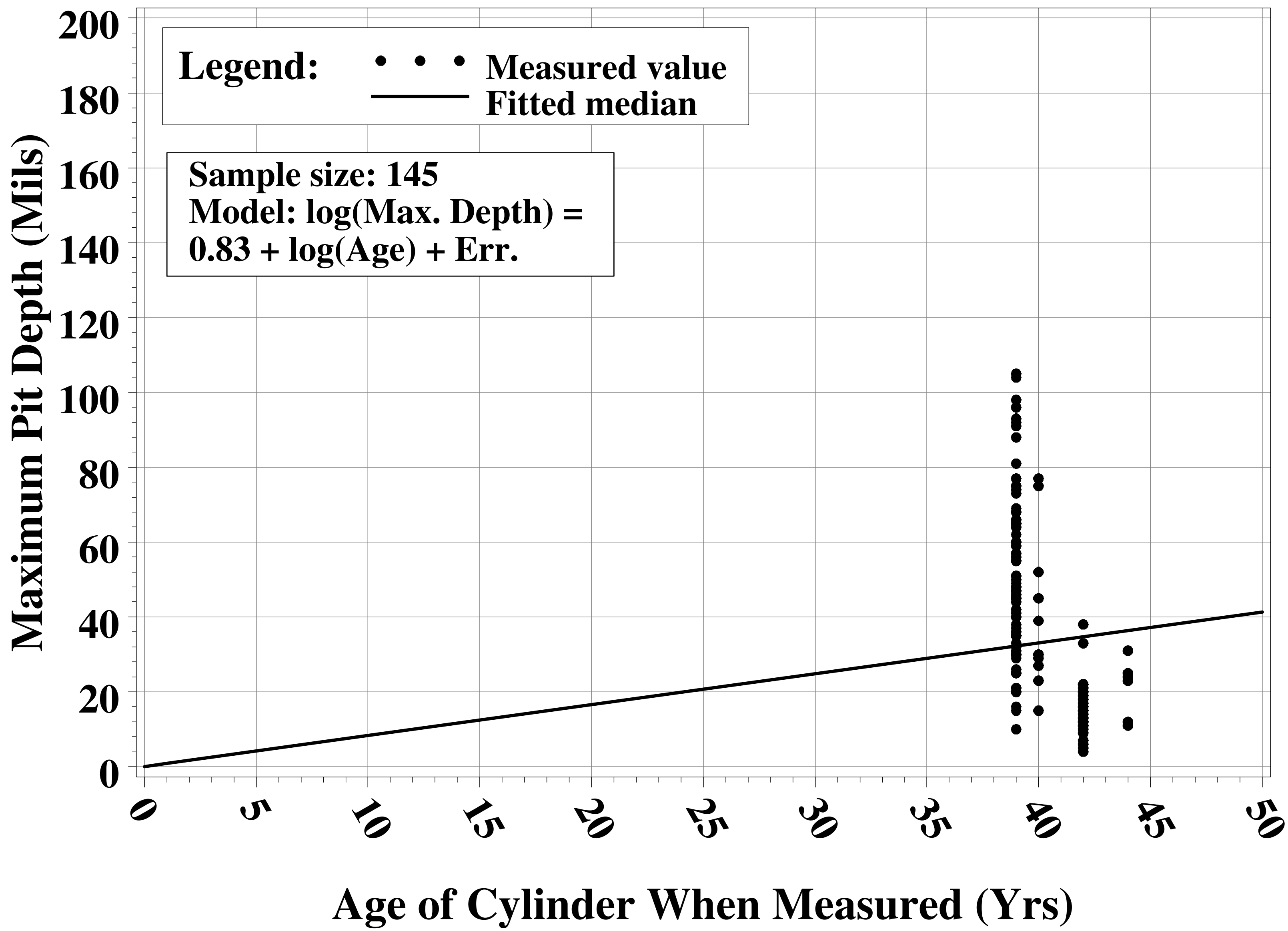

Figure 11. Pit Depths for PORTS Thin, skirted, bottom.

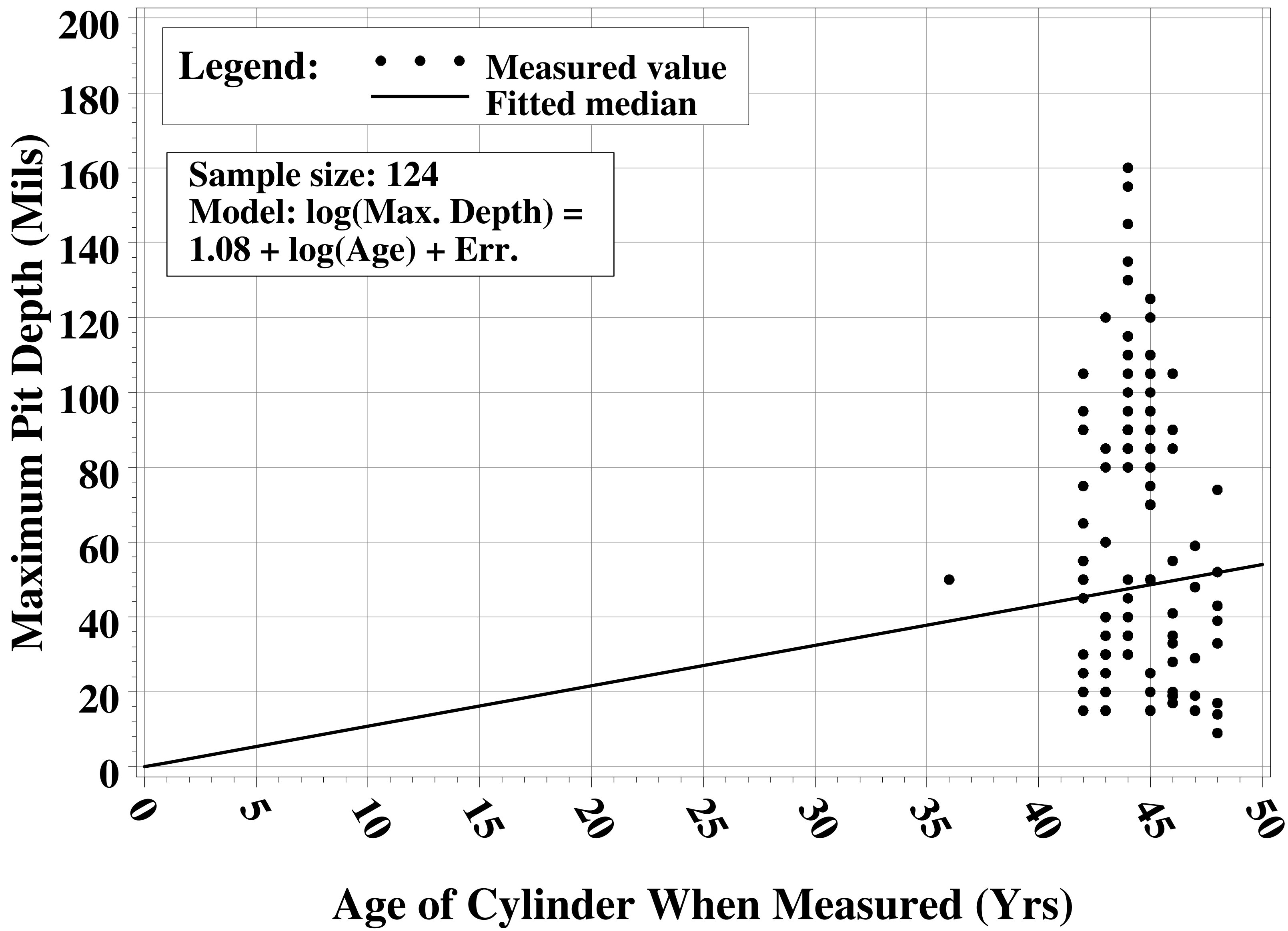

Figure 12. Pit Depths for PORTS Thick, skirted, top and bottom.

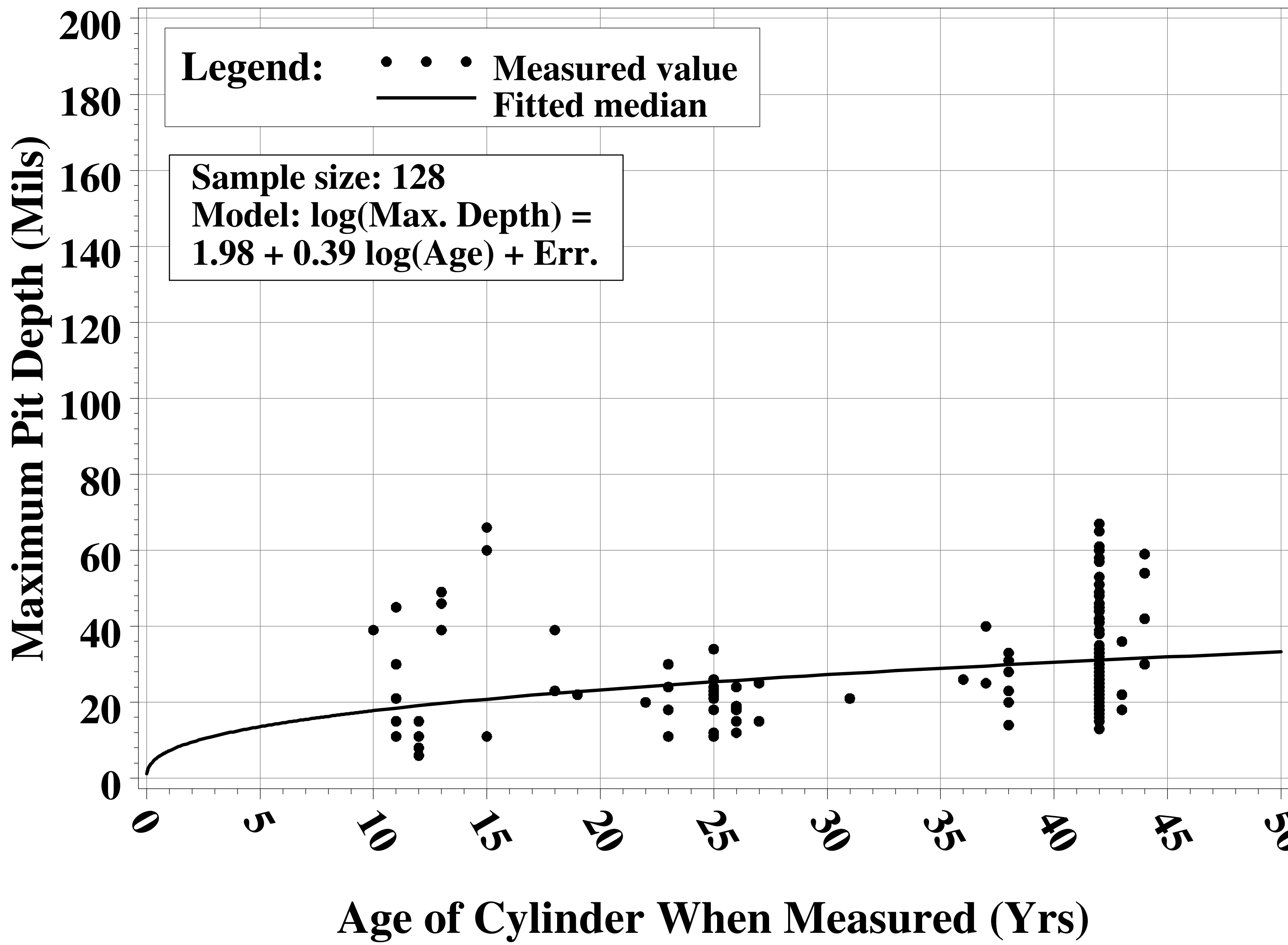

Figure 13. Pit Depths for PORTS Thin, Top, FY99 and later.

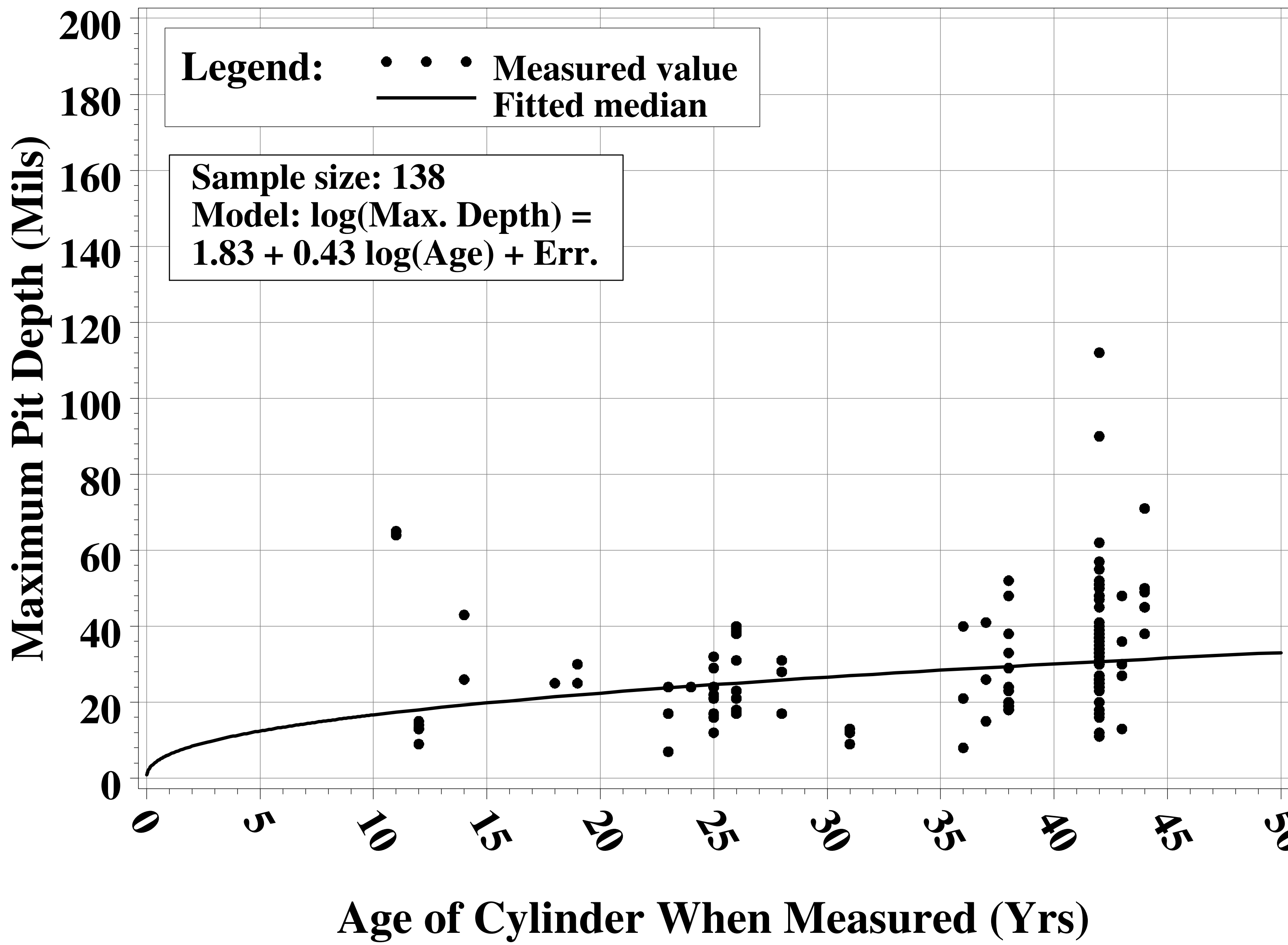

Figure 14. Pit Depths for PORTS Thin, Bottom, FY99 and later.

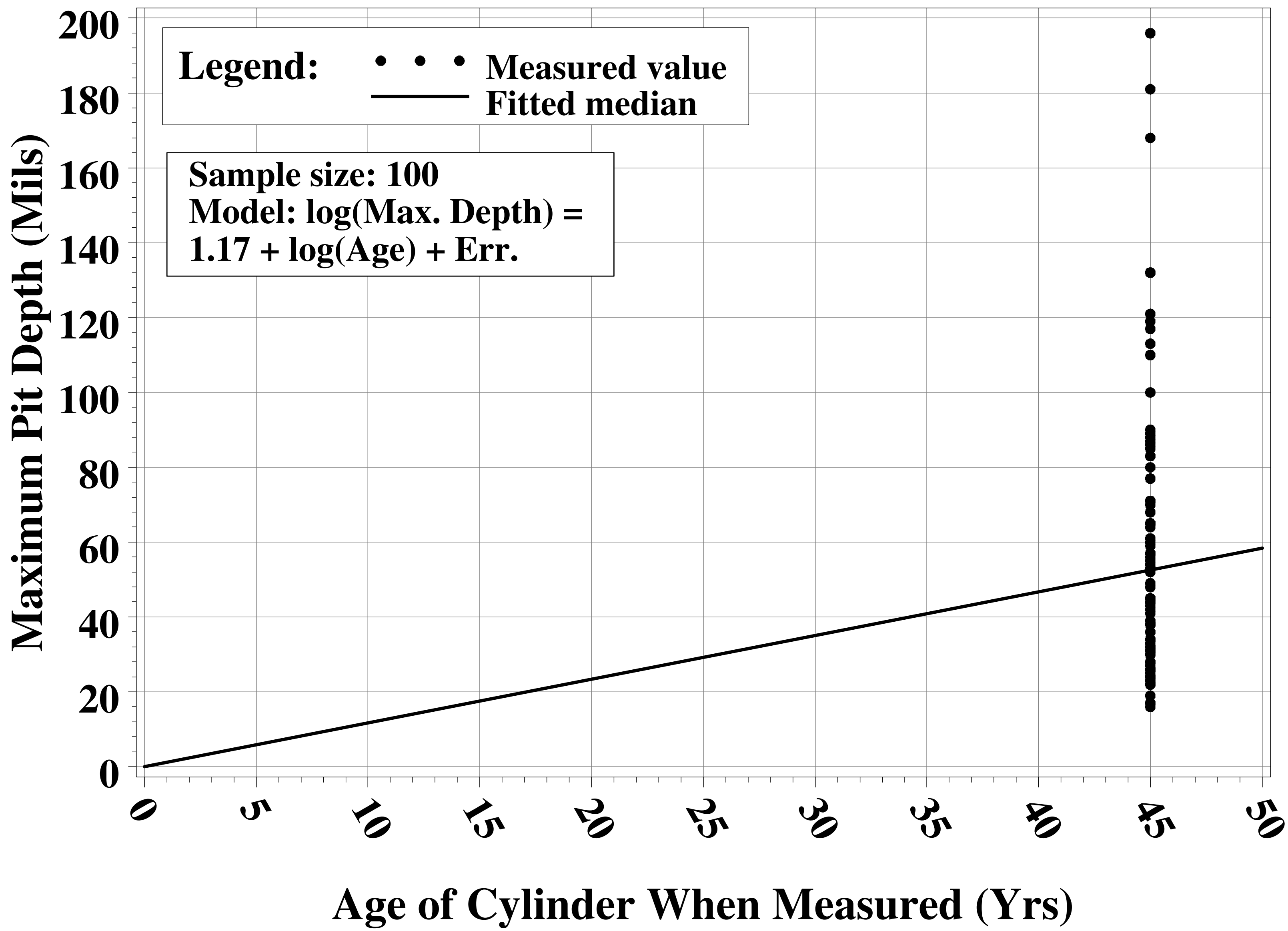

Figure 15. Pit Depths for Paducah 30As, evaluated FY99.

APPENDIX B: METHODS

B.1. Calculating the cumulative distribution function for the difference of two distributions

All of the methods discussed in this report are based on the model

$$M(t) = C_0 - P(t) \tag{1}$$

where $M(t)$ is the minimum wall thickness at cylinder age t, $P(t)$ is the amount of corrosion that results in the minimum wall thickness, and C_0 is the initial thickness where the minimum wall thickness occurs. Both $P(t)$ and C_0 are taken as random, and calculation of the number of cylinders that have a minimum thickness below a certain thickness z requires calculating the probability that $M(t)<z$. Under model (1), this is equivalent to calculating the probability that $C_0-P(t)<z$. Since C_0 and $P(t)$ are both random, calculation of this probability is not as straightforward as calculating probabilities for C_0 and $P(t)$ separately, except for certain special cases (e.g., when $P(t)$ and C_0 are both normal distributions, in which case the difference is also a normal distribution). In this section, the method of calculating the needed probabilities are developed.

General Formula

If the random variable W is defined by $W=X-Y$, where X and Y are independent random variables, then any sample w from W can be written in the form (not necessarily uniquely)

$$w = F^{-1}(p) - G^{-1}(q)$$

where p, q are in $[0,1]$, and F^{-1} and G^{-1} are the inverse cumulative distribution functions for X and Y, respectively. Determination of the probability that $W<z$ is then equivalent to evaluating

$$\iint_{A(z)} dp\, dq \tag{2}$$

where $A(z)$ is the set defined by $A(z)=\{(p,q) \mid F^{-1}(p)-G^{-1}(q) < z\}$. Since F^{-1} and G^{-1} are inverse cumulative distribution functions, they are both nondecreasing functions, and so the function $h(p,q)=F^{-1}(p)-G^{-1}(q)$ is a nondecreasing function of p and nonincreasing function of q. This makes evaluation of the integral in Eq. 3 relatively straightforward. First,

$$\begin{aligned} A(z) &= \{ (p,q) \mid F^{-1}(p) - G^{-1}(q) < z \} \\ &= \{ (p,q) \mid F^{-1}(p) < z + G^{-1}(q) \} \\ &= \{ (p,q) \mid p < F(z + G^{-1}(q)) \} \end{aligned}$$

and so

$$\int\int_{A(z)} dp\, dq = \int_0^1 \int_0^{F(z+G^{-1}(q))} dp\, dq$$
$$= \int_0^1 F(z+G^{-1}(q))\, dq$$

Therefore,

$$Prob\{W = X - Y < z\} = \int_0^1 F(z+G^{-1}(q))\, dq \qquad (3)$$

Alternatively, the probability (3) can be written as

$$\int_{-\infty}^{\infty} (1 - G(z-x))\, dF(x). \qquad (4)$$

This follows because

$$Prob(X - Y < z) \;=\; Prob(Y > z - X) \;=\; \int_{x=-\infty}^{\infty} \int_{\{y > z-x\}} dG(y)\, dF(x) \;=\; \int_{x=-\infty}^{\infty} (1 - G(z-x))\, dF(x).$$

For G lognormal with log-scale mean and variance μ and σ^2, (4) is

$$\int_{-\infty}^{\infty} \left(1 - N\left(\frac{\log(z-x) - \mu}{\sigma} \right) \right) dF(x) \;=\; \int_{-\infty}^{\infty} N\left(\frac{\mu - \log(z-x)}{\sigma} \right) dF(x). \qquad (5)$$

where N denotes the standard normal distribution function.

The integrals (3-5) can be evaluated using the adaptive quadrature method described in Burden and Faires (1989). With this method, subintervals are determined so that the integral is approximated with the desired accuracy using Simpson's rule on each subinterval. This method is generally faster than simpler integration methods to achieve the same accuracy because the ultimate subdivision that is used need not be uniformly spaced over the entire interval of integration; the subintervals can be selected based on the desired accuracy and the variability of the function to be integrated.

Application

In this report, F is the cumulative distribution function (cdf) for the initial thickness C_0 which has a truncated normal distribution, and G is the cdf for the penetration depth $P(t)$ at a fixed time t which has a lognormal distribution with mean of the logarithm of the values of $\mu_L(t)$ and standard deviation of the logarithm of the values of σ_L.

Let *N(u)* denote the cdf for the standard normal distribution (this is the normal distribution with mean 0 and standard deviation 1), and denote the *q*th quantile of the standard normal distribution by n_q. Then by the formula above it follows that

$$Prob\{C_0 - P(t) < z\} = \int_0^1 F\left(z + e^{\mu(t) + n_q \sigma(t)};\ \mu,\sigma \right) dq$$

where

$$F_{[a,b]}(x;\mu,\sigma) = \begin{cases} 0 & if\ x<a \\ \dfrac{N(x;\mu,\sigma) - N(a;\mu,\sigma)}{N(b;\mu,\sigma) - N(a;\mu,\sigma)} & if\ a<x<b \\ 1 & if\ x>b \end{cases}$$

where *N(x;m,s) = N((x-m)/s)*, where *N(z)* is the standard normal distribution.

B.2. Calculation of Upper Confidence Limits

In the methods used in this report, the maximum penetration depth *P(t)* is modeled using a lognormal distribution, with either $P(t) \sim Log(\mu_L, \sigma_L)*t$ (slope set to 1) or $P(t) \sim Log(\log A + n \operatorname{Log} t, \sigma_L)$, and the parameters are fit with the data available. The number of cylinders with a minimum thickness below a certain thickness *z* by a given time *T* is calculated by a sum of the form

$$\sum_i Prob(\ C_0 - P(t_i) < z\ ) \times \left\{ Number\ of\ cylinders\ of\ age\ t_i\ at\ time\ T \right\} \qquad (6)$$

where the sum is over all age classes for the cylinder population of interest.

Given the initial thickness and penetration distributions, the probabilities in (6) can be computed using Simpson's rule, as discussed above. However, the initial thickness and penetration distributions have to be estimated. In this section confidence bounds for (6) are developed to account for uncertainty in the estimates of the penetration distribution. The uncertainty in the initial thickness distribution is assumed to be negligible.

A Departure from the FY2000 Report

The method used in this report to compute point estimates of the number of cylinders that will fail to meet various depth specifications is exactly the same as the method used for the FY2000 report. The method for computing upper confidence limits (UCLs) differs, however. This subsection elaborates on the differences between the new and previous approaches. Although exactly the same assumptions underlie both methods, slightly different mathematics leads to slightly better confidence limits for the new approach. A table at the end of this subsection illustrates the differences. Documentation of the previous method is maintained so that the rationales underlying the two approaches can easily be compared. Tighter UCLs are important because they mean that sampling error is less costly than previously thought. Each sample actually buys more precision for understanding and predicting the corrosion process,.

In the FY2000 version of this report, the approach taken to calculating a UCL for the sum (6) is based on the Bonferroni inequality, which can be used to determine a value α such that if an upper $100\alpha\%$ confidence limit is used for each term in the sum, the final sum will be bounded with at least 95% confidence. This term-by-term approach is conservative because there is structure in the model that is not exploited.

The required value of α for each individual UCL in the term-by-term approach depends on the number of individual UCLs, which is the number of terms (age classes) in (6). The Bonferroni approach becomes ever more conservative as the number of individual UCLs increases. In the term-by-term approach, separate UCLs are required for each age class. For the cylinder populations considered in this report, the number of age classes ranges from 6 to 25. However, although expression (6) may have up to 25 terms, the statistical distributions of all of the terms depend on just three parameters—the intercept, slope, and standard deviation from the regression of log-depth on log-age (with uncertainty in the initial thickness distribution assumed negligible.) Therefore, joint confidence bounds for the penetration at each age represented in (6) can also be computed from joint confidence bounds for the three parameters. This suggests that a more efficient use of the Bonferroni approach would be to use it to derive joint confidence bounds for the three parameters, rather than joint confidence bounds for all of the terms in (6).

Furthermore, a refinement of the three-parameter Bonferroni approach is possible. Joint confidence bounds for the intercept and slope can be used to derive joint UCLs for the penetration log-scale means $\mu(t_i) = a + b \log(t_i)$ for each age t_i. But joint confidence bounds for the intercept and slope imply joint confidence bounds for **all** points on the curve $\mu(t) = a + b \log(t)$, including, for example, points for ages such as t = 10,000 years or t = -10,000 years. In the cylinder modeling, however, the only confidence bounds for points on the regression line that are needed are confidence bounds for points corresponding to ages of concern—in the range of about 0 to 75 years. As Figure 16 illustrates, the line that interpolates UCLs for the regression line at the endpoints of a range of interest is in fact a joint UCL for all points on the regression line in that range. Because their range is restricted, joint UCLs based on the line restricted to the interval, tend to be tighter than UCLs for the whole line, based on confidence bounds for the intercept and slope.

Combining equation (5) and expression (6) gives

$$\sum_i \left\{ \textit{Number of cylinders of age } t_i \textit{ at time } T \right\} \times \int_{-\infty}^{\infty} N\left(\frac{\mu(t_i) - \log(z - x)}{\sigma} \right) dF(x) \qquad (7)$$

for the number of cylinders at time T for which the thickness specification z is violated. It is straightforward to show that expression (7) is increasing in each $\mu(t_i)$. Therefore, a UCB for the entire expression can be obtained by substituting UCLs for the individual $\mu(t_i)$—if an appropriate limit is also substituted for σ. That limit for σ is discussed next.

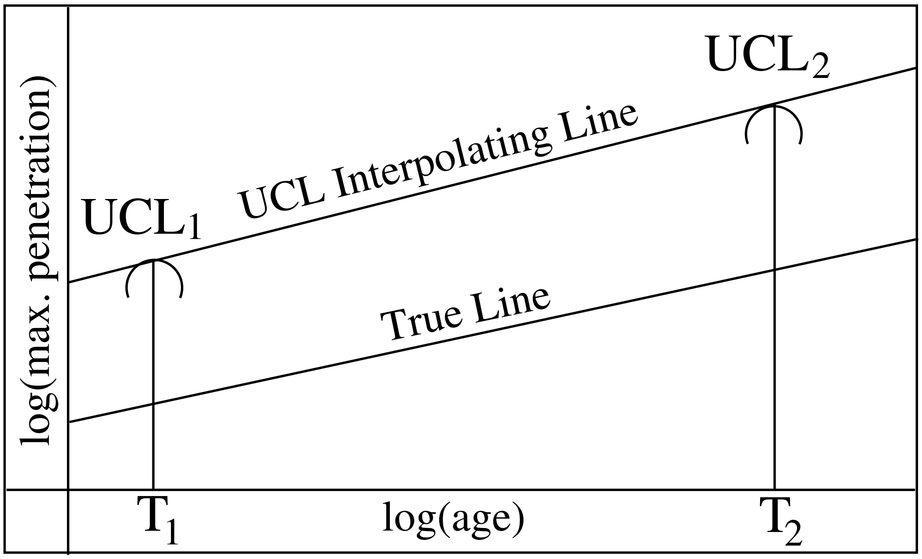

Figure 16. Example of a joint confidence line over an interval, based on two joint UCL's computed at endpoints T1 and T2 of the interval.

In a lognormal regression with d degrees of freedom (d = number of observations - 2 for a simple line model), the mean squared error (MSE) is an unbiased estimate of σ^2, and $d \times MSE / \sigma^2$ has a chi-square distribution with d degrees of freedom. It follow that $d{\times}MSE/X^2_\alpha$ and $d{\times}MSE/X^2_{1-\alpha}$ are upper and lower confidence limits for σ^2, where X^2_α and $X^2_{1-\alpha}$ denote the α and 1-α percentiles of the chi-square distribtution with d degrees of freedom. The square roots of the confidence bounds are confidence bounds for σ.

Because σ is positive, each term in (7) is increasing or flat or decreasing in σ, depending on whether $\mu(t_i) - \log(z{-}x)$ is, respectively, negative or zero or positive. If $\mu(t_i) - \log(z - x) > 0$ for every age t_i, then a UCL for expression (7) can be obtained by taking σ to be its smallest acceptable value (i.e., its lower confidence limit). On the other hand, if $\mu(t_i) - \log(z - x) < 0$ for every age t_i, then the UCL for (7) can be obtained by taking σ to be its largest acceptable value (UCL). If $\mu(t_i) - \log(z{-}x)$ is negative for some of the t_i and positive for other t_i, then the UCL for (7) can be determined by a one-dimensional grid search from the lower to upper confidence limits for σ. The adequacy of the grid step size can be guaranteed, because a bound can be determined for the derivative of (7) as a function of σ. (Note that this grid step size is separate from the step size for the numerical integration discussed above. Because, a bound can be determined for the fourth derivative of the of the integrand, the adequacy of the numerical integration step size can also be guaranteed.)

Table 14 contains point estimates and UCLs comparable to estimates and UCLs in Tables 10-13 of the FY2000 report. The confidence levels for all of the UCLs are the same, 95%. The values in Table 14 are predictions for years 2000 to 2024 in eight-year intervals and for the largest and smallest (point breach) thickness specification used in the FY2000 report. The values in the FY2000 reports are actually for years 2000 to 2024 in four-year intervals and for more than two thickness specifications (e.g., for thin-walled cylinders, 250, 197, 62.5, and 0 mils), but conclusions based on the complete set of comparisons are the same as the conclusions based on the results shown here.

Table 14. FY2000 Estimates and UCLs Computed Using FY2001 Method

					Projected Number of Cylinders Below Spec. Min. Thickness							
					2000		2008		2016		2024	
Cylinder Grouping	**Population**	**Spec. Value**	**Thick-ness**	**Model**	**Esti-mate**	**95% UCB**	**Esti-mate**	**95% UCB**	**Esti-mate**	**95% UCB**	**Esti-mate**	**95% UCB**
Thin-Walled Cylinder Populations	K-1066-K, top and bottom, pre-1998	250	Thin	Slope Set to 1	1,310	1,513	1,690	1,912	1,969	2,171	2,162	2,328
		0	Thin	Slope Set to 1	1	9	4	22	11	47	25	83
	K-1066-K, evaluated 1998-2000	250	Thin	Slope Set to 1	314	575	462	742	616	898	770	1,042
		0	Thin	Slope Set to 1	3	26	6	44	11	67	18	93
	C-745-G, bottom	250	Thin	Fitted Slope	862	1,158	1,156	1,499	1,397	1,730	1,581	1,882
		0	Thin	Fitted Slope	0	3	1	10	2	25	4	51
	PGDP bottom, except G-yard	250	Thin	Fitted Slope	445	1,623	866	2,368	1,465	3,328	2,187	4,501
		0	Thin	Fitted Slope	0	1	0	3	0	8	0	18
	PGDP top	250	Thin	Fitted Slope	96	305	221	623	436	1,086	762	1,764
		0	Thin	Fitted Slope	0	0	0	0	0	0	0	0

Table 14 (cont'd). FY2000 Estimates and UCLs Computed Using FY2001 Method

					Projected Number of Cylinders Below Spec. Min. Thickness							
					2000		2008		2016		2024	
Cylinder Grouping	**Population**	**Spec. Value**	**Thick-ness**	**Model**	**Esti-mate**	**95% UCB**	**Esti-mate**	**95% UCB**	**Esti-mate**	**95% UCB**	**Esti-mate**	**95% UCB**
Thin-Walled Cylinder Populations	PORTS, top, pre-1999	250	Thin	Fitted Slope	87	169	184	331	339	565	554	882
		0	Thin	Fitted Slope	0	0	0	0	0	0	0	0
	PORTS bottom, pre-1999	250	Thin	Fitted Slope	543	845	951	1,354	1,452	1,939	1,994	2,571
		0	Thin	Fitted Slope	0	0	0	0	0	0	0	0
	PORTS Thin, Top, 1999	250	Thin	Fitted Slope	0	3	0	7	0	16	1	33
		0	Thin	Fitted Slope	0	0	0	0	0	0	0	0
	PORTS Thin, Bottom, 1999	250	Thin	Fitted Slope	8	71	20	148	44	264	86	403
		0	Thin	Fitted Slope	0	0	0	0	0	0	0	1
Thick-Walled Cylinder Populations	PORTS Thin and Thick, and C-745-C Thick, top, pre-1999	500	Thick	Fitted Slope	0	0	0	1	0	2	1	4
		0	Thick	Fitted Slope	0	0	0	0	0	0	0	0

Table 14 (cont'd). FY2000 Estimates and UCLs Computed Using FY2001 Method

					Projected Number of Cylinders Below Spec. Min. Thickness							
					2000		2008		2016		2024	
Cylinder Grouping	Population	Spec. Value	Thick-ness	Model	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB	Esti-mate	95% UCB
Thick-Walled Cylinder Populations	PORTS Thin and Thick, and C-745-C Thick, bottom, pre-1999	500	Thick	Fitted Slope	2	7	4	12	7	19	11	28
		0	Thick	Fitted Slope	0	0	0	0	0	0	0	0
Skirted Cylinder Populations	PORTS Thin, skirted, top	250	Thin	Slope Set to 1	161	338	313	551	515	793	752	1,050
		0	Thin	Slope Set to 1	0	0	0	1	0	3	1	7
	PORTS Thin, skirted, bottom	250	Thin	Slope Set to 1	318	533	562	820	854	1,128	1,167	1,434
		0	Thin	Slope Set to 1	0	1	0	3	1	8	2	15
	PORTS Thick, skirted, top and bottom	500	Thick	Slope Set to 1	22	69	39	105	62	146	91	192
		0	Thick	Slope Set to 1	0	2	0	3	1	5	1	9
Model 30A Cylinders	Paducah 30As, evaluated 1999	100	30As	Slope Set to 1	3	21	7	34	12	50	19	69
		0	30As	Slope Set to 1	1	10	3	18	5	27	8	38

The point estimates in Table 14 are identical to the corresponding estimates in the FY2000 report, with the following minor exceptions: (1) for the population "K-1066-K, Evaluated 1998 and 1999," for the 250 mil spec only, the Table 14 estimates for the years 2000 and 2016 are 314 and 616, as opposed to 315 and 617 in the FY2000 report; (2) for the model 30A cylinders, the estimates for the years 2000, 2008, 2016, and 2024 are (a) for the 100 mil spec: 3, 7, 12, and 19, as opposed to 4, 8, 14, and 22 in the FY2000 report, and (b) for the 0 mil spec: 1, 3, 5, and 8 as opposed to 1, 3, 6, and 9 in the FY2000 report. The identity of the other point estimates establishes that the point estimation procedures are essentially the same. The differences in the K-1066-K yard results are due to rounding differences. The difference in the 30A cylinder results are probably due to differences in the special treatment of this case, which requires special treatment because all of the 30A cylinders are the same age.

Each 95% UCL in Table 14 is smaller than its corresponding 95% UCL in the FY2000 report. For example, the first UCL in Table 14, which is for the K-1066-K, top and bottom, pre-1998 population, is 1,513. The corresponding FY2000 value is 1,887. However, the UCL itself is not as good a measure of the statistical uncertainty of an estimate as the **difference** between the UCL and the estimate. For the K-1066-K, top and bottom pre-1998 population, the point estimates are the same: 1,310 cylinders. The Table 14 UCL (1,513) is 203 cylinders above the point estimate, and the FY2000 UCL (1,887) is 577 above its point estimate, which is 2.8 times farther. According to the FY2000 UCL the uncertainty is 2.8 times greater. Yet both the Table 14 and FY2000 UCLs have the same confidence level (95%), have the same corresponding point estimate, and are premised on exactly the same set of assumptions. The difference is due to more efficient use of the Bonferroni and other approximations in the Table 14 UCLs.

Details of the FY2000 (Previous) Approach

Because a distribution is assumed for the initial thickness C_0, it is involved to calculate confidence limits for the terms $Prob\{C_0 - P(t) < z\}$, even for a fixed time t. This is because one must obtain an upper confidence limit on an integral of the form

$$\int_0^1 F(z+G^{-1}(q,t))\,dq$$

where $F(z)$ is the cumulative distribution function for the initial thickness and $G(z,t)$[4] is the cumulative distribution function for the penetration depth at time t (i.e., $G(z,t) = Prob\{P(t) < z\}$).

In order to obtain an upper confidence limit on the integral above, it is not sufficient to use the confidence limits for $G^{-1}(q,t)$ for a fixed q. In particular, neglecting the uncertainty in the initial thickness distribution, a curve $H(q;q_1,q_2)$ must found such that

$$Prob\left\{G^{-1}(q,t)\ <\ H(q;q_1,q_2)\ \ for\ \ q \in (q_1,q_2)\right\}\ =\ \alpha$$

where α is the desired confidence level (e.g., 0.95 for an upper 95% confidence limit). The details of calculating the function $H(q;q_1,q_2)$ when $P(t)$ is either normally or lognormally distributed are provided in Appendix C.

[4] This is a slight abuse of notation that hopefully will help more than confuse the reader.

It is not possible to obtain simultaneous confidence limits that hold for q in the closed interval $[0,1]$ if $P(t)$ is normally or lognormally distributed. However, for arbitrary q_1 and q_2, it can be concluded with at least $100\alpha\%$ confidence that

$$\int_0^1 F(z+G^{-1}(q,t))\,dq \;<\; \int_0^{q_1} F(z+G^{-1}(q,t))\,dq + \int_{q_2}^1 F(z+G^{-1}(q,t))\,dq + \int_{q_1}^{q_2} F(z+H(q;q_1,q_2))dq$$

Since $F(z) \le 1$,

$$\int_0^1 F(z+G^{-1}(q,t))\,dq \;<\; q_1+(1-q_2)+\int_{q_1}^{q_2} F(z+H(q;q_1,q_2))dq$$

If it is assumed that $P(t) \sim Log(lnA + n\ ln\ t,\ \sigma)$, then

$$H(q;q_1,q_2) \;=\; \hat{A}\, t^{\hat{n}} e^{S_L z_q}\, e^{S_L r_U(t,\alpha,q_1,q_2)}$$

where $\ln \hat{A}$ and $\hat{n}$ are the least squares estimates of lnA and n, respectively, S_L is an unbiased estimate of σ_L, z_q is the qth percentile of the standard normal distribution, and $r_U(t,\alpha,q_1,q_2)$ is the factor that makes the curve $H(q;q_1,q_2)$ an upper confidence limit valid over the entire interval (q_1,q_2). In the end, one can conclude with at least $100\alpha\%$ confidence that

$$Prob\left\{C_0-P(t)<z\right\} \;<\; q_1+(1-q_2)+\int_{q_1}^{q_2} F\left(z+\hat{A}t^{\hat{n}}e^{S_L z_q}e^{S_L r_U(t,\alpha,q_1,q_2)}\right)dq$$

Note that the practical effect is to "increase" the term $\hat{A}$, and this facilitates calculation of the confidence limits because previously implemented integration routines can be employed, after replacing $\hat{A}$ with $\hat{A}\, e^{S_L r_U(t,\alpha,q_1,q_2)}$, to calculate the integral.

The limits q_1 and q_2 are completely arbitrary, and can be chosen so as to minimize the upper confidence limit on the integral. The term $q_1 + (1-q_2)$ becomes smaller the closer (q_1,q_2) is to the whole interval (0,1). However, this results in an increase of the term $H(q;q_1,q_2)$ since the interval is wider. At the present, time constraints prevented a two-dimensional search algorithm for finding q_1 and q_2. Instead, q_1 is fixed and q_2 is found by finding where the derivative with respect to q_2 is zero.

B.3. Statistical Tests

Kolmogorov-Smirnov Goodness-of-Fit Test

The test used to determine if it was reasonable to assume that the distribution of average corrosion rates were lognormally distributed was a variant of the Kolmorogov-Smirnov test. This test uses the difference between the empirical (or sample) cumulative distribution function and that of the hypothesized distribution. If the difference is too large, then one rejects the hypothesis that the sample came from the hypothesized distribution.

The empirical cumulative distribution function evaluated at a given value r is simply the fraction of the samples with value less than or equal to r. In particular, given N samples $r_1, r_2, \ldots r_N$ from a distribution R, the empirical cumulative distribution function $F_N(r)$ is defined by

$$F_N(r) = \frac{\#\ samples\ \leq r}{N}$$

Let $F(r)$ denote the cumulative distribution of the hypothesized distribution. The Kolmogorov-Smirnov test statistic D_N is defined by

$$D_N = \max_r \left| F_N(r) - F(r) \right| \tag{8}$$

If the cdf $F(r)$ is continuous, then this statistic is independent of the hypothesized distribution. A closed form has been derived for the distribution of this statistic (see, e.g., Vincze 1970), and so the calculated value D_N can be compared to critical values of its distribution to perform hypothesis tests at a prescribed significant level; i.e., one can test the null hypothesis that R has a specified distribution with cumulative distribution function $F(r)$ versus the alternative hypothesis that the distribution of R is other than that specified. One accepts the null hypothesis if $D_N < D^*(\alpha)$, where $D^*(\alpha)$ is such that $Prob\{D_N > D^*(\alpha)\} = \alpha$. This test is called the Kolmogorov-Smirnov goodness-of-fit test (K-S test).

In contrast to the χ^2 goodness-of-fit test, the K-S test is applicable for all sample sizes, and avoids the problem of the grouping of the data. However, the K-S test in the above form is not applicable if any of the parameters of the hypothesized distribution have been estimated from the data. For the standard χ^2 test, this is addressed by reducing the degrees of freedom of the χ^2 distribution used (one for each parameter estimated from the data). For the K-S test, it is recognized (e.g., Massey 1951) that if the parameters of the hypothesized distribution are estimated from the data, then the critical value $D^*(\alpha)$ should be reduced, but the amount of the reduction necessary is not known in general. This means that if the standard $D^*(\alpha)$ values are used, then one is more likely to falsely accept the null hypothesis that the distribution has the hypothesized form. Using Monte Carlo calculation, Lilliefors (1967) estimated the appropriate critical values $D^*(\alpha)$ for the case when the hypothesized distribution is a normal distribution with mean and variance estimated from the data. These values can be applied for lognormal distributions as well since the statistic in equation (8) has the same value whether or not the data are transformed. As expected, the estimated critical values in Lilliefors (1967) are smaller than those for the more general K-S test. Table 15 shows the critical values $D^*(\alpha)$ from Lilliefors (1967), as well as the standard values (from Massey 1951).

Table 15. Critical values for the statistic used for goodness-of-fit tests.

Sample size N	Critical $D^*(\alpha)$ for α=0.05[A]	Standard K-S Critical Value for α=0.05[B]
4	0.38	0.62
5	0.34	0.57
6	0.32	0.52
7	0.30	0.49
8	0.29	0.46
9	0.27	0.43
10	0.26	0.41
11	0.25	0.39
12	0.24	0.38
13	0.23	0.36
14	0.23	0.35
15	0.22	0.34
16	0.21	0.33
17	0.21	0.32
18	0.20	0.31
19	0.20	0.30
20	0.19	0.29
25	0.18	0.27
30	0.16	0.24
Over 30	$\sim 0.886\ N^{-1/2}$	$\sim 1.36\ N^{-1/2}$

[A] Calculated from Monte Carlo analysis in Lilliefors (1967).
[B] Massey (1951).

The K-S test, using the appropriate critical values from Lilliefors (1967), was utilized here. If the sample size was not one of those in Table 15, then the critical value for the next highest sample size in Table 15 was used for sample sizes < 30, and the asymptotic approximation for sample sizes above 30.

B.4. T-Test with Unequal Variances

Before combining certain cylinder populations, statistical tests were performed to determine if it was likely that the populations were too "different" in a statistically meaningful sense. To this end, the *t*-test with unequal variances was used to determine if the medians were equal (the usual *t*-test requires that the variances of the populations being compared are equal). When the results of the goodness-of-fit tests indicated that it was reasonable to assume that the distribution of rates was lognormally distributed for each subpopulation considered, the *t*-test with unequal variances is applicable to the logarithm of the corrosion rates. The *t*-test is actually a test for means, but the median is equal to the mean for normal distributions. Thus, if we can accept or reject the hypothesis that the medians of the logarithms are not equal, then we can make the same conclusion for the exponential of the random variables.

For two normally distributed random variables X and Y, let m_X and m_Y denote the sample means and s_X and s_Y denote the sample standard deviations from samples of sizes n_X and n_Y from the populations X and Y, respectively. The purpose of the *t*-test is to test the null hypothesis that the means μ_X and μ_Y of X and Y are equal, versus the alternative hypothesis that they are not equal. The statistic used to test this hypothesis is given by

$$t' = \frac{m_X - m_Y}{\sqrt{s_X^2/n_X + s_Y^2/n_Y}}$$

By Satterthwaite's approximation (see, e.g., Casella and Berger 1990), this random variable can be approximated by a t-distribution with degrees of freedom $\hat{\nu}$ given by

$$\hat{\nu} = \frac{\left(s_X^2/n_x + s_Y^2/n_Y\right)^2}{\dfrac{s_X^4}{n_X^2(n_X-1)} + \dfrac{s_Y^4}{n_Y^2(n_Y-1)}}$$

For a given confidence level α, the hypothesis that the means are equal is rejected if t' is too large or too small. In particular, the hypothesis is rejected if $t' < -t_{\hat{\nu}}(\alpha/2)$ or $t' > t_{\hat{\nu}}(\alpha/2)$, where $t_{\hat{\nu}}(\alpha/2)$ is the upper $\alpha/2$ percentile of the t-distribution with $\hat{\nu}$ degrees of freedom; i.e.,

$$\int_{t_{\hat{\nu}}(\alpha/2)}^{\infty} f(t,\hat{\nu})\, dt = \alpha/2,$$

where

$$f(t,\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} (\pi\nu)^{-1/2}(1+t^2/\nu)^{-(\nu+1)/2}.$$

The slightly different test of testing the null hypothesis that the means are equal versus the alternative hypothesis that one mean is larger than the other can be performed using the one-tail probabilities from the t-distribution. For example, we would reject the hypothesis that the means are equal versus the alternative hypothesis that $\mu_X > \mu_Y$ if $t' > t_{\hat{\nu}}(\alpha)$.

B.5. References

Burden, R.L. and J.D. Faires (1989). *Numerical Analysis*, Fourth Edition, PWS-Kent Publishing Company, Boston.

Casella, G. And R.L. Berger (1990). *Statistical Inference*. Wadsworth and Brooks/Cole Advanced Books and Software, Pacific Grove, California.

Evans, M., N. Hastings, and Brian Peacock (1993). *Statistical Distributions*. Second Edition, John Wiley

& Sons, New York.

Lilliefors, H.W. (1967). On the Kolomogorov-Smirnov test for normality with mean and variance unknown. *American Statistical Association Journal*, pp. 399-402, June 1967.

Massey, F.J., Jr. (1951). The Kolomogorov-Smirnov test for goodness of fit. *American Statistical Association Journal*, Vol. 46, pp. 68-79.

Vincze, I. (1970). On Kolmogorov-Smirnov Type Distribution Theorems, in *Nonparametric Techniques in Statistical Inference* (ed. M.L Puri). Cambridge at the University Press.

APPENDIX C: SIMULTANEOUS CONFIDENCE LIMITS ON THE PERCENTILES OF A NORMAL DISTRIBUTION (FOR FY2000 APPROACH)

In order to calculate upper confidence limits for the case when both the penetration depth *P(t)* and the initial thickness C_0 are treated as a distribution, it is necessary to find a curve *h(q)* such that the percentiles P_q of *P(t)* satisfy $P_q<h(q)$ for all *q* in a given interval *[a,b]* for a specified confidence. In this case, it is assumed that *P(t)* is lognormally distributed, and so the *q*th percentile of *P(t)* is of the form $exp[\mu_L(t) + \sigma_L z_q]$, where z_q is the *q*th percentile of the standard normal distribution. Since the exponential function is an increasing function, it is sufficient to find a curve that bounds the term $\mu_L(t) + \sigma_L z_q$ for all *q* in a given interval *[a,b]* for a specified confidence. In this appendix, how this curve is determined is described, first for the special case when the mean $\mu_L(t)$ is constant, and then in the case for linear regression when $\mu_L(t)$ is assumed to be a linear function of *t*. The former case reduces to the problem of determining simultaneous confidence limits for the percentiles of a normal distribution. The latter case follows from the former with only a few modifications based on differences in the relevant sampling distributions.

C.1. Simultaneous Confidence Limits for the Percentiles of a Normal Distribution

If μ and σ are the mean and standard deviation of a normal distribution, then the *p*th percentile of the distribution is $\mu + \sigma z_p$, where z_p is the *p*th percentile of the standard normal distribution. If *m* and *s* are a sample mean and standard deviation from this distribution with a sample size of *n*, then an upper 100α% confidence limit for $\mu + \sigma z_p$ is given by $m + s(z_p + e_p(\alpha))$, where $e_p(\alpha)$ is calculated from the percentage points of the noncentral *t*-distribution (Owen 1968). A lower 100α% confidence limit for $\mu + \sigma z_p$ is the same as an upper 100(1-α)%; i.e., $m + s(z_p + e_p(1-\alpha))$ These confidence limits are not simultaneous in *p*; i.e., one cannot state with 100α% confidence that these bounds hold for a subinterval *[a,b]*. Here we describe determination of exact confidence limits that hold uniformly over a fixed interval. In particular, we describe how to determine numbers $\varepsilon_U(\alpha,a,b)$ and $\varepsilon_L(\alpha,a,b)$ such that

$$Prob\left\{\mu + \sigma z_p \le m + s(z_p + \varepsilon_U(\alpha,a,b))\ for\ all\ p \in [a,b]\right\} = \alpha \tag{1}$$

and

$$Prob\left\{\mu + \sigma z_p \ge m + s(z_p + \varepsilon_L(\alpha,a,b))\ for\ all\ p \in [a,b]\right\} = \alpha \tag{2}$$

The numbers $\varepsilon_U(\alpha;a,b)$ and $\varepsilon_L(\alpha;a,b)$ can be determined using the distributions

$$T_U = \max_{p \in [a,b]} \left[\frac{(\mu + \sigma z_p) - (m + s z_p)}{s}\right] \tag{3}$$

and

$$T_L = \min_{p \in [a,b]} \left[\frac{(\mu + \sigma z_p) - (m + s z_p)}{s}\right] \tag{4}$$

respectively, because

$$Prob\left\{\mu+\sigma z_p \le m+s(z_p+\varepsilon_U(\alpha,a,b)) \text{ for all } p\in[a,b]\right\} = Prob\left\{\max_{p\in[a,b]}\left[\frac{(\mu+\sigma z_p)-(m+sz_p)}{s}\right] \le \varepsilon_U(\alpha,a,b)\right\}$$
$$= Prob\left\{T_U \le \varepsilon_U(\alpha,a,b)\right\} \tag{5}$$

$$Prob\left\{\mu+\sigma z_p \ge m+s(z_p+\varepsilon_L(\alpha,a,b)) \text{ for all } p\in[a,b]\right\} = Prob\left\{\min_{p\in[a,b]}\left[\frac{(\mu+\sigma z_p)-(m+sz_p)}{s}\right] \ge \varepsilon_L(\alpha,a,b)\right\}$$
$$= Prob\left\{T_L \ge \varepsilon_L(\alpha,a,b)\right\} \tag{6}$$

It is shown below that

$$Prob\left\{T_U < t\right\} = F_n(t;a,b)$$
$$Prob\left\{T_L < t\right\} = F_n(t;b,a) \tag{7}$$

where

$$F_n(t;a,b) = \int_0^\infty H_n(u;t,a)\,du + \int_0^{\sqrt{\nu}}\left[H_n(u;t,b)-H_n(u;t,a)\right]du \tag{8}$$

the integrand is defined by

$$H_n(u;t,a) = \frac{1}{\Gamma(\nu/2)2^{\nu/2-1}}G\left(\frac{u}{\sqrt{\nu}}(t+z_a)\sqrt{n}-z_a\sqrt{n}\right)e^{-u^2/2}u^{\nu-1} \tag{9}$$

$\nu=n-1$, and $G(u)$ is the cumulative distribution function for the standard normal distribution, given by

$$G(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{w} e^{-w^2/2}\,dw \tag{10}$$

Therefore, the numbers $\varepsilon_U(\alpha;a,b)$ and $\varepsilon_L(\alpha;a,b)$ are found by solving the equations

$$F_n(\varepsilon_U(\alpha,a,b);a,b) = \alpha$$
$$F_n(\varepsilon_L(\alpha,a,b);b,a) = 1-\alpha \tag{11}$$

Note that this implies, at least formally, that

$$\varepsilon_L(\alpha,a,b) = \varepsilon_U(1-\alpha,b,a) \qquad (12)$$

When $a=b=p$ these limits reduce to the usual factors used to calculate confidence limits on the percentile of a normal distribution, and

$$F_n(t;p,p) \;=\; Prob\left\{t_{n-1}(z_p\sqrt{n}) < (t+z_p)\sqrt{n}\right\} \qquad (13)$$

where $t_\upsilon(\delta)$ is the noncentral *t*-distribution with noncentrality parameter δ. Denoting the upper αth percentile by $t^{(\alpha)}{}_\upsilon(\delta)$, we have that

$$\begin{aligned} \varepsilon_U(\alpha,p,p) &= t_{n-1}^{(\alpha)}(z_p\sqrt{n})/\sqrt{n} \;-\; z_p \\ \varepsilon_L(\alpha,p,p) &= t_{n-1}^{(1-\alpha)}(z_p\sqrt{n})/\sqrt{n} \;-\; z_p \end{aligned} \qquad (14)$$

in which case the upper and lower confidence 100α% limits on $\mu+\sigma\, z_p$ reduce to $m+s\; t_{n-1}^{(\alpha)}(z_p\sqrt{n})/\sqrt{n}$ and $m+s\; t_{n-1}^{(1-\alpha)}(z_p\sqrt{n})/\sqrt{n}$, respectively.

Finally, using the elementary fact that $G(-w)=1-G(w)$, we have that $F_n(t;a,b) = 1-F_n(-t;1-a,1-b)$, and hence

$$\varepsilon_L(\alpha,a,1-a) = -\varepsilon_U(\alpha,a,1-a) \qquad (15)$$

that is, the upper and lower confidence limits are symmetric about the curve $m+s\; z_p$ when $b=1-a$.

C.2. Numerical Methods

The first integral in Eq. 8 is that for the cumulative distribution of the noncentral *t*-distribution. In particular,

$$\int_0^\infty H_n(u;t,a)\; du = Prob\left\{t_\nu\left(z_a\sqrt{n}\right) < t+z_a\sqrt{n}\right\} \qquad (16)$$

where $t_\upsilon(\delta)$ is the noncentral *t*-distribution with noncentrality parameter δ. This integral is evaluated using the method discussed in Owen (1968).

The second integral in Eq. 8 is evaluated using adaptive quadrature. However, this integral is first split into several pieces and adaptive quadrature is applied to each piece. This is done because the integrand has a maximum value near the upper limit of integration and is close to zero over most of the integration range. As a result, the adapative quadrature integration routine will terminate prematurely unless the endpoints of the first few subintervals are close to where the maximum occurs. For this reason, the integral is broken up into integrals over the regions $[0,\sqrt{\nu}/2]$, $[\sqrt{\nu}/2,\sqrt{\nu}-1]$, and $[\sqrt{\nu}-1,\sqrt{\nu}]$.

A combination of the bisection method and the secant method is used to solve the equation

$$F_n(\varepsilon,a,b) = \alpha \tag{17}$$

Two initial guesses are required for the secant method. Since $z_b > z_a$, the inequality $H_n(u;t,b) - H_n(u;t,a) \le 0$ holds for $0 \le u \le \sqrt{\nu}$. Using Eq. 8, this means that $\varepsilon_U(\alpha,a,b) > L = \zeta_\alpha/\sqrt{n} - z_a$, where $Prob\{t_{n-1}(z_a\sqrt{n}) < \zeta_\alpha\} = \alpha$, and so L can serve as a lower bound for $\varepsilon_U(\alpha,a,b)$. An upper bound U is determined by adding 0.2 to the lower limit L until $F_n(U,a,b) > \alpha$. In this manner, $L < \varepsilon_U(\alpha,a,b) < U$. Similarly, $\varepsilon_L(\alpha,a,b) < \zeta_{1-\alpha}/\sqrt{n} - z_a$, and a lower bound is determined by subtracting 0.2 from the upper bound until an interval is found that contains the root $\varepsilon_L(\alpha,a,b)$. After an interval is found containing the root, the secant method is then used to generate the next approximation. If the approximation falls outside of the interval containing the root, then the bisection method is used to generate the next approximation. This procedure is continued until an approximation $\hat{\varepsilon}$ satisfies

$$\left| F_n(\hat{\varepsilon},a,b) - \alpha \right| < Tolerance \tag{18}$$

for the upper confidence limit, and

$$\left| F_n(\hat{\varepsilon},b,a) - (1-\alpha) \right| < Tolerance \tag{19}$$

for the lower confidence limit.

C.3. Derivation of Formula for Relevant Distribution

In this section we derive the integral representation of the cumulative distribution for the functions T_U and T_L as shown in Eq. 8 above.

The function defined by

$$\frac{(\mu + \sigma z_p) - (m + s z_p)}{s} \tag{20}$$

has its maximum and minimum on $[a,b]$ at one of the endpoints since its derivative with respect to p never vanishes (unless $s=\sigma$, in which case it is a constant, an event with probability 0). Further, this function is an increasing function of p if $\sigma > s$ and decreasing if $\sigma < s$ because z_p is an increasing function of p. Thus, the maximum will occur at the left endpoint if σ is overestimated by s, and *vice versa*. This means that

$$\begin{aligned}
\max_{p\in[a,b]} \left[\frac{(\mu + \sigma z_p) - (m + s z_p)}{s} \right] &= \frac{\sigma}{s}\left(\frac{\mu - m}{\sigma} \right) + \left(\frac{\sigma}{s} - 1 \right) \begin{cases} z_b & if\ \sigma/s > 1 \\ z_a & if\ \sigma/s < 1 \end{cases} \\
\min_{p\in[a,b]} \left[\frac{(\mu + \sigma z_p) - (m + s z_p)}{s} \right] &= \frac{\sigma}{s}\left(\frac{\mu - m}{\sigma} \right) + \left(\frac{\sigma}{s} - 1 \right) \begin{cases} z_a & if\ \sigma/s > 1 \\ z_b & if\ \sigma/s < 1 \end{cases}
\end{aligned} \tag{21}$$

This shows that once we have found the distribution for the maximum, the distribution for the minimum can be obtained by simply interchanging a and b. Subsequent derivations are only shown for the maximum.

The random variable $W=(\mu - m)/\sigma$ is normally distributed with mean 0 and standard deviation $1/\sqrt{n}$, and the random variable $X=\sigma/s$ is distributed as $1/\sqrt{\chi^2_\nu/\nu}$, where $\nu = n-1$, and the distributions W and X are independent. Let $G(w)$ denote the cumulative distribution function for the standard normal distribution (= $\sqrt{n}\,W$) and $f(x)$ denote the probability density function for X. Then

$$G(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{w} e^{-w^2/2}\, dw$$

$$f(x) = \frac{2\nu}{x^3} \frac{1}{\Gamma(\nu/2)2^{\nu/2}} (\nu/x^2)^{\nu/2-1} e^{-\nu/(2x^2)}$$

Therefore we have that

$$Prob\left\{\frac{\sigma}{s}\left(\frac{\mu-m}{\sigma}\right) + \left(\frac{\sigma}{s}-1\right)\begin{Bmatrix} z_b \ if\ \sigma/s>1 \\ z_a \ if\ \sigma/s<1\end{Bmatrix} < t\right\} = Prob\left\{XW+(X-1)\begin{Bmatrix} z_b if\ X>1 \\ z_a if\ X<1\end{Bmatrix} < t\right\}$$

$$= Prob\left\{XW\sqrt{n}+(X-1)\begin{Bmatrix} z_b\sqrt{n} if\ X>1 \\ z_a\sqrt{n} if\ X<1\end{Bmatrix} < t\sqrt{n}\right\}$$

$$= \int_0^1 f(x) G\left(\frac{t\sqrt{n}-(x-1)z_a\sqrt{n}}{x}\right) dx + \int_1^\infty f(x) G\left(\frac{t\sqrt{n}-(x-1)z_b\sqrt{n}}{x}\right) dx \quad (23)$$

$$= \int_0^\infty f(x) G\left(\frac{t\sqrt{n}-(x-1)z_a\sqrt{n}}{x}\right) dx + \int_1^\infty f(x)\left[G\left(\frac{t\sqrt{n}-(x-1)z_b\sqrt{n}}{x}\right) - G\left(\frac{t\sqrt{n}-(x-1)z_a\sqrt{n}}{x}\right) dx\right]$$

Making the substitution $u=\sqrt{\nu}/x$ yields

$$Prob\left\{\frac{\sigma}{s}\left(\frac{\mu-m}{\sigma}\right) + \left(\frac{\sigma}{s}-1\right)\begin{Bmatrix} z_b \ if\ \sigma/s>1 \\ z_a \ if\ \sigma/s<1\end{Bmatrix} < t\right\} = \int_0^\infty H_n(u;t,a)\, du + \int_0^{\sqrt{\nu}} \left[H_n(u;t,b) - H_n(u;t,a)\right] du \quad (24)$$

where

$$H_n(u;t,a) = \frac{1}{\Gamma(\nu/2)2^{\nu/2-1}} G\left(\frac{u}{\sqrt{\nu}}(t+z_a)\sqrt{n} - z_a\sqrt{n}\right) e^{-u^2/2} u^{\nu-1}$$

C.4. Application to Linear Regression

Assume that we have pairs of samples p_i at various ages t_i: (t_i,p_i), $I=1,...,N$, and we assume that $p(t)=N(at+b,\sigma)$. Estimates of the regression coefficients a and b are given by

$$\hat{a} = \frac{\sum_{i=1}^{N} p_i t_i - \frac{1}{N}\left(\sum_{i=1}^{N} p_i\right)\left(\sum_{i=1}^{N} t_i\right)}{\sum_{i=1}^{N} t_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} t_i\right)^2}$$

$$\hat{b} = \bar{p} - \hat{a}\,\bar{t}$$

where $\bar{p} = \frac{1}{N}\sum_{i=1}^{N} p_i$ and $\bar{t} = \frac{1}{N}\sum_{i=1}^{N} t_i$. An unbiased estimate of the standard deviation σ is

$$S^2 = \frac{1}{N-2}\sum_{i=1}^{N}(p_i - \hat{a}t_i - \hat{b})^2$$

Assuming that $p(t)$~$N(at+b,\sigma)$, the following is known about the sampling distributions for the estimates of the parameters (Casella and Berger, 1990; pp. 569-575):

$$\hat{a} + t\,\hat{b} \sim N\big(a+bt, \sigma\, e_N(t)\big)$$

$$S^2 \sim \sigma^2\,\frac{\chi^2_{N-2}}{N-2}$$

where we define $e_N(t)$ by

$$e_N(t) = \sqrt{\frac{1}{N} + \frac{(t-\bar{t})^2}{S_{tt}}}$$

and

$$S_{tt} = \sum_{i=1}^{N}(t_i - \bar{t})^2$$

and χ^2_ν is the χ-squared distribution with ν degrees of freedom.

Based on similarities in the sampling distributions, the preceding discussion can be applied with only slight modifications to derive simultaneous confidence limits for the pth percentile for $N(at+b, \sigma)$ valid uniformly for p in the range (q_1,q_2). In particular, all that changes is that we replace $1/\sqrt{n}$ with $e_n(t)$ and $\upsilon=n-2$ instead of $\upsilon=n-1$. The result is that an upper and lower 100 α% confidence limit on the pth percentile for $N(at+b, \sigma)$ are given by

$$\left\{\textit{Upper } 100\alpha\% \textit{ confidence limit on } at+b+\sigma z_p, p\in(q_1,q_2)\right\} = \hat{a}t+\hat{b}+S(z_p+r_U(\alpha,q_1,q_2))$$

$$\left\{\textit{Lower } 100\alpha\% \textit{ confidence limit on } at+b+\sigma z_p, p\in(q_1,q_2)\right\} = \hat{a}t+\hat{b}+S(z_p+r_L(\alpha,q_1,q_2))$$

where

$$F_n^{(reg)}\left(r_U(\alpha,q_1,q_2);q_1,q_2\right)=\alpha$$

$$F_n^{(reg)}\left(r_L(\alpha,q_1,q_2);q_2,q_1\right)=\alpha$$

$$F_n^{(reg)}(\omega;a,b) = \int_0^\infty R_n(u;\omega,a)\, du + \int_0^{\sqrt{n-2}} \left[R_n(u;\omega,b)-R_n(u;\omega,a)\right] du \tag{33}$$

and

$$R_n(u;\omega,a)=\frac{1}{\Gamma((n-2)/2)2^{(n-2)/2-1}} G\left(\frac{u}{\sqrt{n-2}}(\omega+z_a)/e_n(t)-z_a e_n(t)\right) e^{-u^2/2}u^{n-2-1} \tag{34}$$

Finally, if in the case that $\ln p(t) \cong N(a \ln(t)+b, \sigma)$, i.e., $p(t)$ is lognormally distributed, then this same method can be applied, replacing t with $\ln(t)$ and taking the exponential of the upper confidence limits.

C.5. References

Lawless, J.F. (1982). *Statistical Models and Methods for Lifetime Data*. John Wiley and Sons, New York.

Owen, D.B. (1968). A survey of properties and applications of the noncentral *t*-distribution. *Technometrics*, Vol. 10, No.3.

APPENDIX D: STATISTICAL ISSUES IN COMBINING TOP AND BOTTOM ROWS AND INCLUSION OF BREACHED CYLINDERS FOR 1994 DATA FOR K-1066-K YARD

A large portion of the cylinders located at K-1066-K yard were in ground contact for extended periods while they were in a previous yard (K-1066-G yard). When using data from 1994, there are several issues that require assumptions: (1) how to incorporate the two cylinders discovered in 1992 that were deemed to breach due to external corrosion (Barber et al. 1994), and (2) whether or not the top and bottom row populations should be modeled separately. In this report, the breaches are included with the data collected in 1994, and the top and bottom rows are combined. It is not clear, however, how the two breached cylinders discovered in 1992 should best be incorporated in the analyses. One of these cylinders was in the top row and one was in the bottom row at the time they were discovered, but the prior location of these cylinders in K-1066-G yard is not known. Because these two cylinders were not evaluated as part of a random sample, it is natural to deem them as the most extreme cases, and hence exclude them. However, this then omits potentially important information about this population of cylinders; namely, that extremely high corrosion rates did occur. For conservative purposes, it is considered appropriate to include these cylinders in the analysis, and that is what is assumed here.

An analysis of the top and bottom row cylinders using the 1994 data suggests that there is little statistical difference between the two populations, although the average corrosion rate is slightly higher for the bottom row cylinders. The main reason that this difference is not larger may be a reflection of the nature of the relocation efforts that were conducted when K-1066-G yard was moved to K-1066-K yard in 1982. In particular, if the top and bottom row cylinders were not kept together, then "different" populations (from a corrosion standpoint) may have been effectively "shuffled", thereby obscuring any row-effects that may have been present.

The statistics for the corrosion rates for the different subpopulations within K-1066-K yard are shown in the table below.

Table 16. Comparison of statistics for corrosion rate for K-1066-K subpopulations.

		Sample Size	Log of Age-Averaged Corrosion Rate Mean, St.Dev.
Top Row	not including breach	60	0.48, 0.40
	including breach	61	0.51, 0.47
Bottom Row	not including breach	55	0.53, 0.39
	including breach	56	0.56, 0.45
Top and Bottom Row	not including breaches	115	0.50, 0.39
	including breaches	117	0.53, 0.46

Using the *F*-test (see, e.g., Snedecor and Cochran 1978, pp.116-117), one can accept that the variances of the logarithms of the pit depths for the top and bottom row populations are equal with 5% significance, whether or not the breaches are included. Similarly, using the same test as described in Lyon (1996), namely the t-test with unequal variances (Casella and Berger 1990), one can conclude that the medians of the distributions are the same; i.e., there is not a statistically significant difference between the medians of the distributions for the top and bottom row cylinders. Both of these results support treating the top and bottom rows as a single population, whether or not the breaches are included.

DISTRIBUTION

1. ORNL Central Research Library
2. ORNL Laboratory Records–OSTI
3. Mike Taylor (Bechtel Jacobs Company)
4. Bob Shelton
5. Carolyn Moser
6. Randy Curlee
7. Bill Craddick
8. Kim Grubb