Calculation of t-acceptances and Edge Corrections

Cedric M. Guss

31 October, 1995

Abstract

We report here the method for obtaining t-acceptances of the E811 detector and edge corrections for dN/dt points near the common solid angle edge of the elastic combinations. The common t-range covered by the detector during low-t data taking was 0.001 to 0.014 GeV/c^2 and during high-t running was 0.007 to 0.035 GeV/c^2. The t-acceptance coverage ranged from 25 % to 44 % at low-t and from 3 % to 29 % at high-t. The edge corrections, measured as a function of t, ranged from 76 % to 97 % at low-t and from 65 % to 98 % at high-t.

I. Calculation of t-acceptances

To calculate t-acceptance, we first need to switch to effective angular space between the left side detector and right side detector; i.e. the angular coordinate system that is common to both sides for a particle scattering at the E0 intersection, passing through the detector on either side, and measured at some angle, after accounting for x and y shifts in traversal due to magnetic field effects which are different on each side. Each side thus has associated with it an effective x and y length from E0; we used 45.747 m and 80.195 m for x and y at D47, respectively, and 31.784 m and 73.345 m for x and y at E14, respectively. Each detector heel counter edge was determined with respect to its physical position from the center of the beam by using known pot positions, the distance from the bottom of the flat surface of the scintillation fiber detector to the center of the beam for the run in question, and the known heel counter calibrated position with respect to the scintillation fiber detector itself. Since the heel counter was in the shape of a T, there were the distances of 4 x edge lines and 4 y edge lines to find with respect to the beam center, defined as x=y=0. By dividing these edge line measurements by the corresponding effective x and y lengths at each side, we could find these edges in effective space. Finally, a comparison of each of these edge lines with the corresponding edge lines on the other side was done in order to find the LEAST common distance between the two sides in effective space, and hence the LEAST common overlapping area of the two sides that particles scattering off each other at the same angle at E0 would enter into on each side, thus constituting a heel$_{left}$ * heel$_{right}$ elastic trigger. An event found inside this common area is thus said to be accepted by the detector at its Mandelstam t value; and this region accepted by the E811 detector divided by that accepted by a 4π detector is called the t-acceptance of the E811 detector for this value of t.

In effective space, it becomes easy to visualize that the line traced out by any scattered angle (θ_{sc}) at E0 projected onto the detector's z-plane is a circle when integrating over 2π; it's length is simply 2πr, where r is the radius of the circle for this particular scattered angle. Now the common E811 detector region for this scattered angle is defined by the least common x versus y area of the two sides; in x and y effective space, this common area is translated accordingly into common x and y effective angles, θ_x and θ_y. The left EDGE angle is defined by $\theta_{left} = \sqrt{\theta_x^2 + \theta_y^2}$, and the same for the right. Hence, the common angular acceptance of the E811 detector is $\theta_{common} = \mid \theta_{right}$ - $\theta_{left} \mid$, and the t-acceptance is given by the length of the t-line across this angular acceptance (rθ_{common}) divided by that for a 4π detector (2πr), or simply $\theta_{common}/2\pi$ for t = p$^2\theta_{sc}^2$, where p is the beam particle momentum and θ_{sc} is its physical scattering angle at the E0 interaction point. In practice, one also has to subtract out the amount of θ_{common} that extends above the high y common edge of the two sides, if any; and the edge angles on each side must be taken from the smallest of 2 x points: the x common edge or the x point where t line enters above the low y common edge, where the y edge angle is obtained from the y value of the x edge point used. In order to subtract out regions of common angle where the t-line leaves the bottom "fat" part of the heel T area and then reenters the "thin" part above it, θ_{common} was calculated for the "fat" and "thin" heel common regions separately, and then the two added together for the final divison by 2π in order to get the t-acceptance of a t-line. For each t-bin used in the analysis, a t-acceptance was calculated for the high-t and low-t lines of the bin separately, and the two were averaged in order to obtain the final t-acceptance of the t-bin itself.

Table 1 shows the calculated t-acceptances for t-bins used in the low-t analysis of runs 743-747 and high-t analysis of runs 748-750 (1-4 combination).

low-t		high-t	
starting t	t-acceptance	starting t	t-acceptance
0.0007 GeV/c^2	0.291	0.007 GeV/c^2	0.042
0.0014	0.329	0.008	0.089
0.0021	0.357	0.009	0.129
0.0028	0.377	0.010	0.161
0.0035	0.392	0.011	0.186
0.0042	0.403	0.012	0.206
0.0049	0.411	0.013	0.223
0.0056	0.418	0.014	0.237
0.0063	0.423	0.015	0.249
0.0070	0.427	0.016	0.259
0.0077	0.431	0.017	0.268
0.0084	0.434	0.018	0.276
0.0091	0.437	0.019	0.283
0.0098	0.439	0.020	0.290
0.0105	0.410	0.021	0.291
0.0112	0.366	0.022	0.287
0.0119	0.323	0.023	0.281
0.0126	0.284	0.024	0.274
0.0133	0.251	0.025	0.260
		0.026	0.243
		0.027	0.226
		0.028	0.209
		0.029	0.186
		0.030	0.145
		0.031	0.106
		0.032	0.073
		0.033	0.046
		0.034	0.026

Table 1. t-acceptances of the t-bins used in the E811 low-t and high-t run analyses (1-4 combination).

II. Edge Corrections

For every elastic particle found on one side, it must have had a partner on the other. If it isn't found, one of two things may have happened. First, the detector may have been inefficient in the region where the particle hit, and hence must be accounted for in dN/dt with left*right spacially dependent cluster efficiency corrections, weighted to the elastic distribution function within the common detector. The other thing that could have happened is that the particle was near an edge on one side and not picked up on the other side by either the software or the hardware itself. In order to correct for this latter effect, we make what are called edge corrections to dN/dt.

We first make the assumption that the distribution of the elastic partner on the other side (the pattern side) is a two-dimensional Gaussian function for all particles centered at some t-point on one side (the looking side). The widths of this Gaussian are determined by several factors; foremost are the x, y, and z distribution widths of the interaction point at E0 (z being along the beam axis). Also, there are the x and y betatron oscillation widths of the two beams and the x and y resolution widths of the detectors on both sides. Eleven distribution widths in all are folded into the total width of this 2-D Gaussian, which projections along x and y can be seen as the width of the xx and yy diagonal plots of the elastic signal, σ_x and σ_y respectively. These sigmas can be obtained by fitting a Gaussian to the (x_{left} - x_{right}) and (y_{left} - y_{right}) distributions for all the elastic data in the run. By using these sigmas and by using the x and y position of the particle on the looking side as the x and y means of the distributions on the pattern side (μ_x and μ_y respectively), one can project the expected distribution of events at (x,y) on the looking side to the pattern side, and hence find what percentage of the projected distribution on the pattern side lies INSIDE the pattern side detector. This percentage is called the edge correction at point (x,y). The x and y pattern projection distributions are done separately, and the Error Function is used to integrate the Gaussians (using μ_x and σ_x, and the same for y), starting from the tail that is fully contained within the pattern side detector to its nearest x edge, and the same for y. This integral, normalized so that $-\infty$ to $+\infty$ equals 1, gives the edge correction in x (ϵ_x), and the same for y (ϵ_y).

In practice, these x and y edge corrections are calculated for the "fat" part and "thin" part of the T-shaped heel region of the pattern side detector separately. Then the total edge correction at the pattern side for point (x,y) on the looking side is

$$\epsilon_{x,y} = (\epsilon_x \epsilon_y)_{fat} + (\epsilon_x \epsilon_y)_{thin},$$

where "fat" and "thin" refer to the contribution from integrals inside the

the "fat" part and "thin" part of the T-shaped heel region of the pattern side detector. And by reversing the role of the 2 sides, one can also find the edge correction of point (x,y) looking the other way. For any set of events making up a dN/dt bin which t value is defined as the average of the measured t of the particle in the left side detector with that in the right, we have

$$dN/dt = [(dN/dt/\epsilon_{x,y})_{left} + (dN/dt/\epsilon_{x,y})_{right}]/2,$$

where dN/dt is the average of dN/dt on one side looking at the other, corrected for edge effects on the pattern side, with dN/dt on the other side looking back, and point (x,y) on the left side isn't necessarily quite the same as point (x,y) on the right side. Therefore, we define the edge correction of the AVERAGED (x,y) point between the 2 sides in common angular effective space (see above) for events binned at this (xavr,yavr) point to be

$$1/\epsilon_{xavr,yavr} = [(1/\epsilon_{x,y})_{left} + (1/\epsilon_{x,y})_{right}]/2.$$

For all such AVERAGED (x,y) points (xavr,yavr) that belong to some t-bin, we can now write the summed edge correction for the t-bin to be

$$1/\epsilon_t = \sum_{xavr,yavr}^{N} (1/\epsilon_{xavr,yavr})/N,$$

where N is the total number of elastic events and (xavr,yavr) belongs to the t-bin such that t = $p^2(xavr^2+yavr^2)$. We sum over the number of elastic data points in order to take into account the proper weighting of the elastic distribution function within the t-bin at the edges. Finally, the fully edge-corrected dN/dt of a t-bin of some width Δt centered at t equals N/Δt/ϵ_t.

Table 2 lists the bin by bin summed edge corrections versus t calculated for low-t run 745 and high-t run 748 (1-4 combination). One can notice how the highest and lowest t in each set has the most correction (meaning away from 1.0, defined as no correction), and how the middle t-bins have the least correction. This is simply because the edge t-bins have the highest percentage of data near the edge than those nearer the center of the t range of the common elastic detector.

low-t			high-t	
starting t	edge correction		starting t	edge correction
0.0007 GeV/c^2	0.860		0.007 GeV/c^2	0.694
0.0014	0.925		0.008	0.709
0.0021	0.932		0.009	0.826
0.0028	0.942		0.010	0.910
0.0035	0.946		0.011	0.970
0.0042	0.965		0.012	0.942
0.0049	0.971		0.013	0.976
0.0056	0.974		0.014	0.967
0.0063	0.966		0.015	0.975
0.0070	0.971		0.016	0.975
0.0077	0.961		0.017	0.968
0.0084	0.951		0.018	0.969
0.0091	0.945		0.019	0.975
0.0098	0.907		0.020	0.966
0.0105	0.879		0.021	0.935
0.0112	0.846		0.022	0.970
0.0119	0.822		0.023	0.949
0.0126	0.781		0.024	0.954
0.0133	0.765		0.025	0.932
			0.026	0.906
			0.027	0.888
			0.028	0.883
			0.029	0.803
			0.030	0.781
			0.031	0.689
			0.032	0.651
			0.033	0.648
			0.034	0.650

Table 2. Edge corrections versus t for low-t and high-t data (1-4 combination).