Fermi National Accelerator Laboratory

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Abstract

Electron cooling in the presence of intrabeam scattering is studied for the Recycler Ring, with an initial batch of *unused* antiprotons and a batch fresh antiprotons added every hour. The dynamical equation is derived. The cooling process is modeled for different scenarios.

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1 Bunch beam Dynamics in Recycler

The Recycler is intended to recycle *unused* antiprotons from the Tevatron and accumulate cooled antiprotons from the Accumulator. To optimize the operational scenarios, the initial state of the antiproton beam is listed in the first column of Table I. The amount of cooled antiprotons from the Accumulator added every hour is listed in the second column. The goal of the cooling is shown in the third column.

Table I: Properties of antiproton beams in the Recycler.

	Recycled antiprotons	antiprotons Accumulator	goal
N (10^{12})	3		6
N (10^{12}) h^{-1}		0.4	
$\mathcal{A}$ (eV-s) (95%)	150	10	50
ϵ (π mm-mrad) (95%)	30	10	10

In the Recycler, the growth rate of the transverse emittance due to the intrabeam scattering is small provided that the 95% transverse emittance is larger than 10 π mm-mrad. When the slow intrabeam-scattering growth is neglected, the equation for instantaneous emittance cooling is given by

$$\frac{d\epsilon}{dt} = -\alpha_\perp \epsilon \ , \tag{1.1}$$

where $\alpha_\perp$ is the emittance cooling rate. If we assume a storage time of 6 h, the required transverse cooling rate is $\alpha_\perp = 0.18\ \mathrm{h}^{-1}$.

On the other hand, the longitudinal momentum spread growth rate due to the intrabeam scattering is much faster. The growth rate has been shown to be well approximated by [1]

$$\alpha_{\mathrm{ibs}} = k_0 \left(\frac{\sigma_p}{p}\right)^{-3} , \tag{1.2}$$

where

$$k_0 = 1.3 \times 10^{-11} \left(\frac{N}{7 \times 10^{12}}\right) \quad \mathrm{h}^{-1} \ , \tag{1.3}$$

N is the number of antiprotons in the Recycler. Note that the above growth rate derives from results of simulations by letting the bunch particles occupying the whole Recycler ring uniformly. If the bunch is squeezed to $1/n$ of the ring circumference, k_0 should be replaced by $k = nk_0$

Assuming linear cooling, the dynamical equation for the rms beam momentum spread δ of the beam then becomes

$$\frac{d\delta}{dt} = -\alpha_\| \delta + \alpha_{\rm ibs} \delta = -\alpha_\| \delta + \frac{k}{\delta^2} \;, \tag{1.4}$$

where $\alpha_\|$ is the longitudinal electron cooling rate. The asymptotic rms momentum spread is therefore given by

$$\delta_\infty = \left(\frac{k}{\alpha_\|}\right)^{1/3} . \tag{1.5}$$

For example, if $\alpha_\| = 1$ h^{-1}, the final 95% phase space area of the beam with 6×10^{12} antiprotons that spreads evenly in the Recycler will be 88.9 eV-s, which is too large. Here we have assumed the total momentum spread to be four times the rms value. Thus, a proper rf manipulation is needed to attain the goal of 50 eV-s for 6×10^{12} antiprotons.

The transient solution of Eq. (1.4) is given by

$$\left(\frac{\delta}{\delta_\infty}\right)^3 = 1 + \left[\left(\frac{\delta_i}{\delta_\infty}\right)^3 - 1\right] e^{-3\alpha_\| t} \;, \tag{1.6}$$

where δ_i is the initial rms momentum spread of the beam.

To study the dynamics of electron cooling, we first examine properties of cooling electrons and antiprotons in the Recycler. The solid curve in Fig. 1 shows the longitudinal temperature of cooling electrons T_e vs the rms momentum spread. The solid straight line shows the 95% phase-space area vs the rms momentum spread of antiprotons if the beam fills the entire Recycler. Similarly, the dotted and the dashed straight lines show the 95% phase-space area vs the rms momentum spread if the beam fills $\frac{1}{2}$ and $\frac{1}{4}$ of the Recycler respectively. The longitudinal electron temperature is defined as the spread in kinetic energy at the rest frame of the electron beam, or

$$T_e = \frac{1}{2} m_e c^2 \beta^2 \delta^2 \;, \tag{1.7}$$

where m_e is the electron mass, βc the average velocity of the electron or antiproton beam, and δ the rms momentum spread of the electron beam in the laboratory frame. Since β is fixed by the energy of the antiproton beam, the velocity spread in the rest frame is directly proportional to the momentum spread in the lab frame. We note that, when an electron beam with temperature above ~ 0.15 eV is used, the velocity spread of antiprotons is inside the corresponding velocity spread of cooling electrons if the beam fills more than $\frac{1}{2}$ of the Recycler at 150 eV-s. Thus the linear-cooling-force model of Eq. (1.4) for the rms momentum spread is valid.

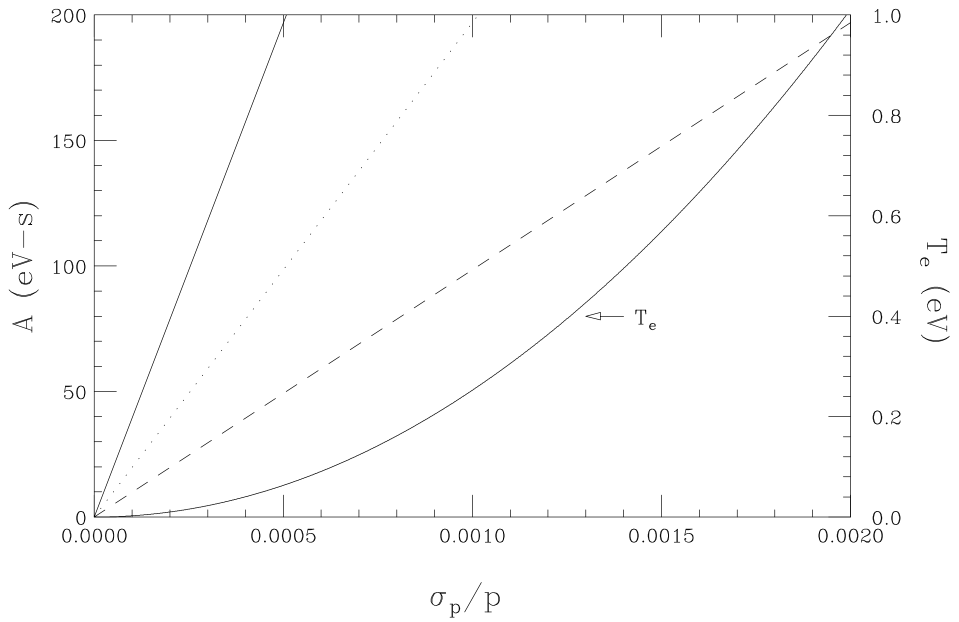

Figure 1: The rms momentum spread of the antiproton beam in the Recycler is compared with the momentum spread of cooling electrons. When the velocity spread of antiprotons is smaller than that of cooling electrons, the linear cooling model is a good approximation to the equation of motion for antiprotons.

2 Modeling of electron cooling in the Recycler

We will examine the antiproton bunching structure in order to attain efficient antiproton cooling. The antiproton bunching structure is achieved by barrier rf buckets. To simplify our discussion, we examine three models with different bunching structures. Model A corresponds to a coasting antiproton beam, model B corresponds to the antiproton beam filling $\frac{1}{2}$ of the Recycler, and model C corresponds to the antiproton beam filling $\frac{1}{4}$ of the Recycler. In both cases, we assume linear cooling with a cooling rate of $\alpha_{\parallel} = 1\ \mathrm{h}^{-1}$. The accumulation rate of the accumulator is assumed to be 4×10^{11} antiprotons per hour.

2.1 The first hour

At the first hour, there are 3×10^{12} antiprotons with a 95% phase-space area 150 eV-s and a 95% emittance 30 π mm-mrad. The beam is cooled in the Recycler for 1 hour. According to the transient solution of Eq. (1.6), the resulting phase-space areas are, respectively, 79 eV-s for the model A, 63 eV-s for the model B, and 57 eV-s for model C.

2.2 The second hour

After the first hour of antiproton recycling, 0.4×10^{12} newly accumulated antiprotons in the Accumulator are ready for transfer. The pre-cooled antiprotons in the Accumulator are of 10 eV-s and 10 π mm-mrad. Thus the density of the newly accumulated antiprotons is about the same as the recycled cooled antiprotons

The newly accumulated antiprotons are merged with the recycled antiprotons with the aid of barrier rf waves. The resulting phase space-area is simply the arithmetic sum of individual phase space areas. Thus we have 89 eV-s, 69 eV-s, and 66 eV-s for 3.4×10^{12} antiprotons for the three bunching structures. After another hour of cooling, the resulting phase space areas are 74 eV-s, 48 eV-s, and 34 eV-s for models A, B, and C, respectively.

2.3 The final states

The procedure can be continued readily and the final phase space areas are plotted in Fig. 2 for a total cooling time of 10 hours, along with the amount of antiprotons in the Recycler. We see that the phase-space area in each of the three bunching structures drops to a minimum and increases slowly thereafter. This is due to the fact that more antiprotons from the Accumulator are being injected every hour, thus increasing the growth rate due intrabeam scattering. At the end of 8 hours, with the accumulation of 5.8×10^{12} antiprotons, the phase-space areas are 87 eV-s for model A, 55 eV-s for model B, and 35 eV-s for model C. Therefore, in order to attain a design goal of roughly 50 eV-s for $\sim 6 \times 10^{12}$ antiprotons, we should bunch the antiprotons to $\frac{1}{2}$ of the Recycler Ring during the cooling stage. For this bunching structure, the antiproton density at the beginning is 0.020×10^{12}/eV-s, and is cooled to 0.048, 0.071, 0.079, 0.084, 0.090, 0.095, 0.100, and 0.104×10^{12}/eV-s, at the end of the next consecutive 8 hours. The additional antiprotons injected from the Accumulator has the density of 0.04×10^{12}/eV-s. We see from Fig. 2 that the cooling scenario of model C brings the antiproton batch to a much lower phase-space area. However, we must bear in mind that the antiprotons start out with a much larger momentum spread and linear cooling may not apply to all the antiprotons in the beam. As a result, Eq. (1.4) may not apply and the actual *effective* cooling rate will be much slower. Also, it is not required to cool the antiproton batch to a phase-space area much below 50 eV-s, because a much lower bunch emittance may lead to collective instabilities.

3 Rough Estimation

In fact, the final bunch area at a certain time can be readily estimated using the asymptotic rms momentum spread given by Eq. (1.5). For example, if the final bunch is squeezed to $1/n$ of the ring circumference, the equilibrium bunch area will become

$$\mathcal{A}_\infty = 4\beta^2 E\delta\frac{\tau_0}{n} \ , \tag{3.1}$$

where $E = 8.93827$ GeV is the total main energy of the antiprotons which corresponds to a relativistic beta of $\beta = 0.99448$, and $\tau_0 = 11.134$ μs is the revolution period of the Recycler. After 8 hours, which is one hour after the last batch of 0.4×10^{12} antiprotons has been transferred, equilibrium will be reached approximately if the cooling rate is $\alpha_\parallel \sim 1$ h^{-1}. Therefore, we can replace the rms momentum spread δ in Eq. (3.1) by the asymptotic value δ_∞ given by Eq. (1.5). If a final bunch has a restricted area of $\mathcal{A}_\infty = 50$ eV-s and occupies only $\frac{1}{2}$ $(n = 2)$ of the ring, the electron cooling rate in the

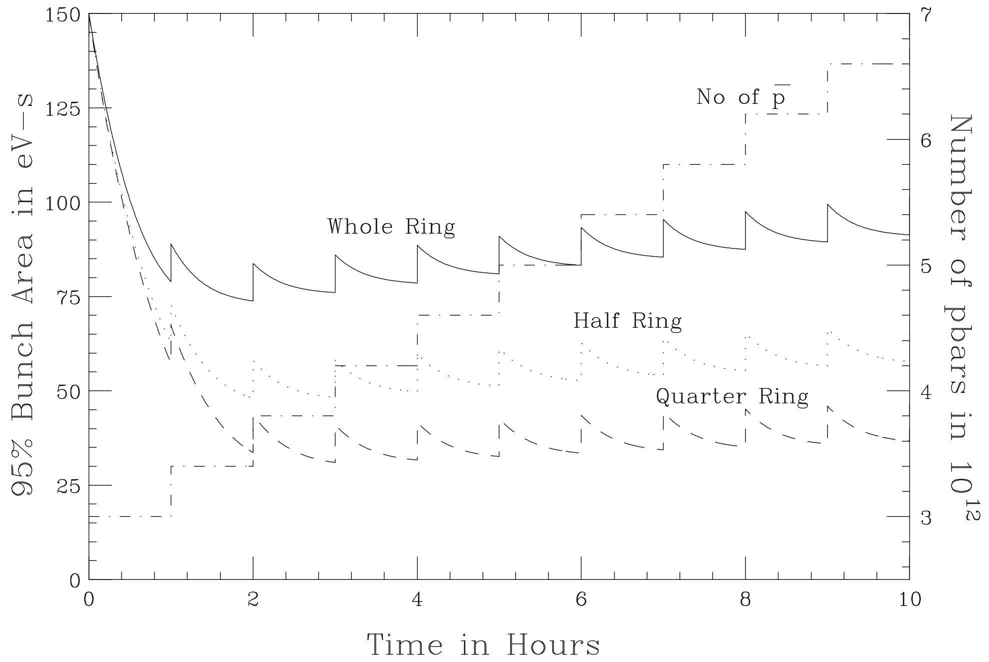

Figure 2: The phase-space area of the antiproton batch in 10 hrs, with 3×10^{12} antiproton in 150 eV-s to start with and an additional 0.4×10^{12} in 10 eV-s transferred from the Accumulator every hour. A cooling rate of 1 h^{-1} and an intrabeam-scattering growth rate inversely proportional to the cube of the momentum spread are assumed. The bunching structures are: model A (solid) with particles filling the whole ring, model B (dots) filling $\frac{1}{2}$ of the ring, and model C dashes) filling $\frac{1}{4}$ of the ring.

longitudinal direction must satisfy

$$\alpha_\parallel > \frac{k_0}{n^2}\left(\frac{4\beta^2 E\tau_0}{\mathcal{A}_\infty}\right)^3 \;, \tag{3.2}$$

or 1.31 h^{-1}, where the total number of antiprotons, $N=5.8\times10^{12}$, has been used. The transient solution of the dynamical cooling equation of Eq. (1.4) gives $\alpha_\parallel > 1.42$ h^{-1}.

On the other hand, if the cooling rate remains at $\alpha_\parallel = 1$ h^{-1} and a final bunch area of 50 eV-s is desired, the amount of bunch squeezing is

$$n > \left(\frac{k_0}{\alpha_\parallel}\right)^{1/2}\left(\frac{4\beta^2 E\tau_0}{\mathcal{A}_\infty}\right)^{3/2} \;, \tag{3.3}$$

or $n > 2.29$. In other words, the bunch must be squeezed to less than 0.436 of the Recycler circumference. The transient solution of Eq. (1.4) gives $n > 2.33$. It is important to understand why more bunch squeezing is necessary if a smaller final bunch area is desired. A larger bunch width implies a smaller momentum spread or a lower longitudinal temperature at the expense of the transverse temperature. When the longitudinal temperature is low enough, the effect of electron cooling will be minimal. On the other hand, if the antiproton bunch is squeezed to a much shorter length, the momentum spread will be larger and so is the longitudinal temperature. The effect of electron cooling will be more efficient.

References

[1] See for example, contribution of C. Bhat, P. Colestock, and L. Spentzouris in G. Jackson, *Fermilab Recycler Ring Technical Design Report*, Revision 1.2, November, 1996, Fermilab TM-1991, or later edition.