A Possible Test to T-Violation and Its Mechanisms

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1. Introduction

- **CP- and T- Violation and the Current Understanding**

CP-Violation: K_L and K_S system (1965)

CPT: invariant

Requirement: local field theory,
Lorentz invarience,
usual spin-statistical relations

T-Violation: direct evidence came many years later in neutral Kaon system (1998)

Current understanding: Standard Model (SM) + Cabibo-Kobayashi-Maskawa matrix (CKM-matrix)

Because there are three quark generations and there are differences between mass eigenstates and interaction eigenstates, these lead to mixing among three generations. The mixing matrix (CKM-matrix) of the three generations naturally carries a complex phase to provide a natural mechenism for a small but non-zero violation of CP conservation.

$$V = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} .(1)$$

- **Spontaneous CP-violation**

  As it is well-known, Higgs mechanism plays an important role in SM. However, it might be difficult to believe that the physical vacuum is as simple as the one in minimal SM. It seems to be natural to ask:

  Could Higgs mechanism play a role in CP(T)-violation?

  The possibility of CP(or T)-violation due to the Higgs sector has been independently studied by

  T. D. Lee, Phys. Rep. **9C** (1974) 143,

  S. Weinberg, Phys. Rev. Lett. **37** (1976) 657..

  Models have been proposed to give CP-violation through both neutral and charged Higgs-boson exchange. Some results are:

  1. Two Higgs doublets +
     Soft breaking of some discrete symmetries in the scalar potential
     $\Rightarrow$ Spontaneous CP(T) breaking through neutral Higgs boson exchanges
     (G. C. Branco, and M. N. Rebelo, Phys. Lett. B **160** (1985),
     S. Weinberg, Phys. Rev. Lett. **63** (1989) 2333,

Phys. Rev. D **42** (1990) 860);

2. Three or more Higgs doublets (discrete symmetries could remain)

   $\rightarrow$ spontaneous CP(T) breaking through both charged and neutral Higgs boson exchanges

   (Weinberg, S., Phys. Rev. Lett. **37** (1976) 657, N. G. and Ma, E., Phys. Rev. D **16** (1977) 1583.).

- **Conzett's null T-violation test**

  As we have said previously, direct observation of T-violation came many years later than CP-violation.

  Difficulties in T-violation measurement:

  – T-violation is small (obvious),

  – no null test of time-reversal invariance (TRI) in $A + B \rightarrow C + D$

  (F. Arash, M. J. Moravcsik, and G. R. Goldstein, Phys. Rev. Lett. **54** (1985) 2649.)

  This theorem indicates that the finding of a nonzero value of any observable can not be taken as a proof of T-violation without some additional condition(s) that must be provided by the interaction dynamics, even for an elastic scattering which is its own inverse reaction. Because of this, the precision of T-violation tests would be difficult to reach to the level of parity violation tests.

The above theorem relies on

$$\sigma \sim |\mathcal{M}|^2 \,. \tag{2}$$

However, there is an optical theorem which relates total cross section to the imaginary part of the linear amplitides in forward scattering,

$$\sigma_{tot} = \frac{4\pi}{k} Im \frac{Tr\rho\mathcal{M}(0)}{Tr\rho} \;. \tag{3}$$

Conzett has shown that a forward-scattering experiment could provide a null test of time-reversal symmetry, which could possibly give high accuracy measurement.

Choose a coordinate system as

$$\mathbf{e_z} = \mathbf{k_z}/k_z \,, \quad \mathbf{e_y} = \mathbf{k} \times \mathbf{k}'/|\mathbf{k} \times \mathbf{k}'| \,, \quad \mathbf{e_x} = \mathbf{e_y} \times \mathbf{e_z} \,, \tag{4}$$

where $\mathbf{k}(\mathbf{k}')$: incident (scattering) momentum. With $\sigma_i = \sigma \cdot \mathbf{e_i}$, we have the following transformation under P and T

	σ_x	σ_y	σ_z
P	$-\sigma_x$	σ_y	$-\sigma_z$
T	$-\sigma_x$	σ_y	σ_z

$$\Rightarrow PNC \quad (TRV) \quad if \quad n_x + n_z \quad (n_x) \quad is \quad odd, \tag{5}$$

where n_x (n_z) is the number of x (z) subscripts.

1. For $\frac{1}{2}+\frac{1}{2} \to \frac{1}{2}+\frac{1}{2}$, the forward-scattering matrix is

$$M(0) = a_{0,0} + a_{0,z}\sigma_0\sigma_z + a_{z,0}\sigma_z\sigma_0 + a_{x,x}(\sigma_x\sigma_x + \sigma_y\sigma_y) + a_{z,z}\sigma_z\sigma_z + a_{x,y}(\sigma_x\sigma_y - \sigma_y\sigma_x), \tag{6}$$

and optical theorem gives,

$$A_{x,y} = Ima_{x,y}/Ima_{0,0} \; (T\, odd, P\, odd)\,. \tag{7}$$

2. For $\frac{1}{2}+1 \to \frac{1}{2}+1$, the forward-scattering matrix is

$$M(0) = a_{0,0} + a_{0,z}\sigma_0 P_z a_{z,0}\sigma_z P_0 + a_{x,x}(\sigma_x P_x + \sigma_y P_y) + a_{z,z}\sigma_z P_z + a_{x,y}(\sigma_x P_y - \sigma_y P_x) + a_{0,zz}\sigma_0 P_{zz} a_{z,zz}\sigma_z P_{zz} + a_{x,xz}(\sigma_x P_{xz} + \sigma_y P_{yz}) + a_{x,yz}(\sigma_x P_{yz} + \sigma_y P_{xz})\,, \tag{8}$$

and the optical theorem gives

$$A_{x,yz} = \frac{3}{4} Ima_{x,yz}/Ima_{0,0} \; (T\, odd, P\, even)\,. \tag{9}$$

The above introduction naturally leads to the following questions:

Questions:

1. Can CKM-matrix well describe CP (or T)-violating systems other than neutral Kaon system, e.g. neutron dipole moment?

   Various approaches to understand the baryon asymmetry of the universe does require much stronger CP-violation than it is suggested in the minimal SM with CKM-matrix.

2. Based on the current understanding, what would Conzett's measurement be?

3. Can we distinguish different CP(T)-violation mechanisms experimentally?

We would like to look into the above questions in the following. We would mainly concentrate on Conzett's null test and would also like to raise the possibility and advantage to carry out the measurement at RHIC.

2. A Few Words About Relativistic Extension of Conzett's Null Test

Conzett's discussion: Non-relativistic

$\Rightarrow$ Relativistic $\Rightarrow$ 4 component spin-1/2 fermion.

(See Stapp, Henry P., Phys. Rev. **103** (1956) 425;

Chou, Kuang-Chao, and Shirokov, M. I., Soviet Phys. JETP, **7** (1958) 851;

Csonka, P. L., Moravcsik, M. J., and Scadron, M. D., Ann. PHYS. **40** (1966) 100.)

I will sum up several points in the above references in the following:

1. The four-component relativistic scattering matrix can be reduced to the two-component formalism provided that the center-of-mass frame (CMS) is used;
2. The reduced Pauli-type scattering matrix possesses the same transformation properties as the non-relativistic scattering matrix under spatial reflections and time reversal;
3. The spin vector of the non-relativistic treatment can be equated to the relativistic spin vector provided

the latter is measured in the particle rest-frame related to the CMS by a Lorentz transformation.

$\Rightarrow$

Conzett's results can be extended to relativistic situation without obvious modification as long as working in the CMS of the system.

We would like to emphasis three points here:

- because of the equivalence of spatial reflection and time reversal transformation between covariant four-component description and two-component description, both four-component and two-component expressions should give the same total spin-correlation, provided the total cross sections have the same transformation properties under spatial reflection and time reversal transformation and their spin vectors are measured in suitable reference frame;
- the amplitude $a_{x,y}$ is T-odd P-odd because $n_x = 1$ and an odd n_x is both P-odd and T-odd;
- since the spin correlations $A_{x,y} = (\sigma_{x,y} - \sigma_{x,-y})/(\sigma_{x,y} + \sigma_{x,-y})$ and $A_{x,yz} = (\sigma_{x,yz} - \sigma_{x,-yz})/(\sigma_{x,yz} + \sigma_{x,-yz})$, the quatities $A_{x,y}(\sigma_{x,y} + \sigma_{x,-y})$ and $A_{x,yz}(\sigma_{x,yz} +$

$\sigma_{x,-yz}$) must only depend on the T-odd P-odd and T-odd P-even parts of the total cross section, respectively.Therefore, one only needs to know the T-odd P-odd and T-odd P-even parts of the total cross section to find $A_{x,y}(\sigma_{x,y} + \sigma_{x,-y})$ and $A_{x,yz}(\sigma_{x,yz} + \sigma_{x,-yz})$.

3. Studies of Two CP-Violation Mechanisms

We will mainly use two mechenisms for the study:

- Minimal SM with CKM-matrix
- $SU(2)_L \times U(1)_Y$ electroweak theory with extra Higgs doublets (spontaneous CP violation)

We use the parton-like model picture in the calculations, namely,

$$\sigma_{had}(AB \longrightarrow CD) = \sum_{a,b,c,d} f_{A,B,C,D}^{a,b,c,d}\, \sigma_{part}(ab \longrightarrow cd) \qquad (10)$$

where $f_{A,B,C,D}^{a,b,c,d}$: weight factors due to both parton distributions and fragmentations.

3.1. Minimal SM with CKM-matrix

In minimal SM with CKM-matrix,

Mixing$\Rightarrow$a complex phase $\delta \Rightarrow \not{CP}(\not{T})$

The part of Lagragian that can possibly lead to CP(T)-violation is given by,

$$\mathcal{L} = \frac{g}{\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}) \ , \qquad (11)$$

where $g = e \sin\theta_W$: coupling constant,

$W^{\pm}_{\mu} = (A^1_{\mu} \mp i A^2_{\mu})/\sqrt{2}$: charged vector bosons,

$J^{\pm}_{\mu} = (J^{-}_{\mu})^{+\dagger} = \frac{1}{2}\overline{U}\gamma_{\mu}(1-\gamma^5)VD$: $SU(2)$ fermionic currents,

V: CKM-matrix,

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \qquad V = \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

Working in the Feynman-'t Hooft gauge, the amplitude for a reaction $ab \longrightarrow cd$ is given by

$$\begin{aligned} \mathcal{M}_W &= \frac{g^2}{8}\frac{1}{k^2 - M_W^2 + i\epsilon} V_{ca} V^*_{db} [\overline{u}_c \gamma^{\mu}(1-\gamma^5)u_a] \\ & \quad [\overline{u}_d \gamma_{\mu}(1-\gamma^5)u_b] \,. \end{aligned} \tag{12}$$

Applying the time reversal and spatial reflection transformation

$$\begin{aligned} \psi'(t') &= \mathcal{T}\psi(t) = T\psi^*(t) \\ \psi'(x') &= \mathcal{P}\psi(x) = \gamma^0\psi(x)\,, \end{aligned} \tag{13}$$

and the time reversal symmetry condition

$$\mathcal{M}_W^T = \mathcal{M}_W^{\dagger}\ , \tag{14}$$

one can seperate $\mathcal{M}$ into T-even P-even (E), T-even P-odd (P), T-odd P-even (T), and T-odd P-odd (TP) parts as the following:

$$\begin{aligned}
\mathcal{M}_W^E &= \frac{g^2}{2}\frac{1}{k^2-M_W^2+i\epsilon}(ReV_{ca}V_{db}^*)\{[\overline{u}_c\gamma^\mu u_a][\overline{u}_d\gamma_\mu u_b] + \\
&\quad [\overline{u}_c\gamma^\mu\gamma^5 u_a][\overline{u}_d\gamma_\mu\gamma^5 u_b]\}\ , \\
\mathcal{M}_W^P &= \frac{-g^2}{2}\frac{1}{k^2-M_W^2+i\epsilon}(ReV_{ca}V_{db}^*)\{[\overline{u}_c\gamma^\mu u_a][\overline{u}_d\gamma_\mu\gamma^5 u_b] + \\
&\quad [\overline{u}_c\gamma^\mu\gamma^5 u_a][\overline{u}_d\gamma_\mu u_b]\}\ , \\
\mathcal{M}_W^T &= \frac{g^2}{2}\frac{1}{k^2-M_W^2+i\epsilon}i(ImV_{ca}V_{db}^*)\{[\overline{u}_c\gamma^\mu u_a][\overline{u}_d\gamma_\mu u_b] + \\
&\quad [\overline{u}_c\gamma^\mu\gamma^5 u_a][\overline{u}_d\gamma_\mu\gamma^5 u_b]\}\ , \\
\mathcal{M}_W^{TP} &= \frac{-g^2}{2}\frac{1}{k^2-M_W^2+i\epsilon}i(ImV_{ca}V_{db}^*)\{[\overline{u}_c\gamma^\mu u_a][\overline{u}_d\gamma_\mu\gamma^5 u_b] + \\
&\quad [\overline{u}_c\gamma^\mu\gamma^5 u_a][\overline{u}_d\gamma_\mu u_b]\}\ .
\end{aligned} \quad (15)$$

The total cross sections that are T-odd P-even and T-odd P-odd should be given by

$$\begin{aligned}
\sigma_{part}^T &\sim Re(\mathcal{M}_W^{E*}\mathcal{M}_W^T) + Re(\mathcal{M}_W^{P*}\mathcal{M}_W^{TP})\ , \\
\sigma_{part}^{TP} &\sim Re(\mathcal{M}_W^{E*}\mathcal{M}_W^{TP}) + Re(\mathcal{M}_W^{P*}\mathcal{M}_W^T)\ .
\end{aligned} \quad (16)$$

It it is easily seen that, at least at tree level,

$$\sigma^T_{part} \; = \; 0.0 \; ,$$

$$\sigma^{TP}_{part} \; = \; 0.0 \; , \qquad (17)$$

and

$$\begin{aligned} \sigma^T_{had} \; &= \; 0.0 \; , \\ \sigma^{TP}_{had} \; &= \; 0.0 \; . \end{aligned} \qquad (18)$$

$\Rightarrow$ **If minimal SM with CKM-matrix is the unique mechanism for CP(T)-violation, the measurements of T-odd cross section should give almost zero result.**

We say "almost zero" because we have not studied higher order contribution yet.

3.2 $SU(2)_L \times U(1)_Y$ EW Theory with CKM-matrix and Extra Higgs Doublets

Assuming: $e = -\frac{1}{3}$ quarks (D) only get masses from ϕ_1-D interactions,

$e = \frac{2}{3}$ quarks (V) only get masses from ϕ_2-V interactions,

due to some discrete symmetry, (in order to prevent excessive rates of K^0, $\bar{K}^0$ or D^0, $\bar{D}^0$ oscillation in a natural way,) the interactions of quarks with scalars are then

$$\begin{aligned} \mathcal{L}_{int} &= -\frac{1}{\lambda_1^*}(\overline{D}_R m_D V^\dagger U_L)\phi_1^{+*} - \frac{1}{\lambda_1^*}(\overline{D}_R m_D D_L)\phi_1^{0*} - \\ & \frac{1}{\lambda_2}(\overline{U}_R m_U U_L)\phi_2^0 + \frac{1}{\lambda_2}(\overline{U}_R m_U V D_L)\phi_2^+ + h.c. \,, \end{aligned} \quad (19)$$

We will study T-odd cross sections given by the above two cases in the following.

3.2.1 Electroweak Theory with Two Higgs Doublets

In this sort of theory, charged-Higgs-bosons do not play a role in CP(T)-violation. Only neutral-Higgs-boson exchanges contribute to CP(T)-violation.

Consider the scalar potential in the form

$$
\begin{aligned}
\mathcal{L}_\phi &= -\partial_\mu \phi_1^\dagger \partial^\mu \phi_1 - \partial_\mu \phi_2^\dagger \partial^\mu \phi_2 - V(\phi_1, \phi_2) \ , \\
V &= \frac{1}{2} g_1 (\phi_1^\dagger \phi_1 - |\lambda_1|^2)^2 + \frac{1}{2} g_2 (\phi_2^\dagger \phi_2 - |\lambda_2|^2)^2 + \\
& \quad g(\phi_1^\dagger \phi_1 - |\lambda_1|^2)(\phi_2^\dagger \phi_2 - |\lambda_2|^2) + g' |\phi_1^\dagger \phi_2 - \lambda_1^* \lambda_2|^2 + \\
& \quad Re[h(\phi_1^\dagger \phi_2 - \lambda_1^* \lambda_2)^2] + \xi \left(\frac{\phi_1}{\lambda_1} - \frac{\phi_2}{\lambda_2} \right)^\dagger \left(\frac{\phi_1}{\lambda_1} - \frac{\phi_2}{\lambda_2} \right), \quad (20)
\end{aligned}
$$

where $\phi_1 = (\phi_1^+, \, \phi_1^0)$,

$\phi_2 = (\phi_2^+, \, \phi_2^0)$,

g_1, g_2, g, g', and ξ: real,

η and h may be complex,

discrete symmetry $\phi_1 \longrightarrow -\phi_1$, $\phi_2 \longrightarrow \phi_2$:

softly broken by $\phi_1^\dagger \phi_2$ and $\phi_2^\dagger \phi_1$, which have

dimentionalities ≤ 3.

This Largrangian conserves CP(T) if and only if h/η^2 is real.

Work in unitary gauge and rewrite (Weinberg, S., Phys. Rev. D **42** (1990) 860):

$$\begin{aligned}
\phi_1^0 &= \frac{\lambda_1}{\sqrt{2}|\lambda_1|}\left(\Phi_1 - \frac{i|\lambda_2|}{\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\Phi_3\right) \ , \\
\phi_2^0 &= \frac{\lambda_2}{\sqrt{2}|\lambda_2|}\left(\Phi_2 - \frac{i|\lambda_1|}{\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\Phi_3\right) \ , \qquad (21)
\end{aligned}$$

so that $< \Phi_j >= 0\,(j = 1, 2, 3)$.

This leads to:

$$\begin{aligned}
\mathcal{L}_\phi &= -\frac{1}{\lambda_1^*}(\overline{D}_R m_D V^\dagger U_L)\phi_1^{+*} - \frac{1}{\sqrt{2}|\lambda_1|}\overline{D} m_D D \Phi_1 \\
&\quad + \frac{i|\lambda_2|}{\sqrt{2}|\lambda_1|\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\overline{D} m_D \gamma^5 D \Phi_3 \\
&\quad - \frac{1}{\lambda_2}(\overline{U}_R m_U V D_L)\phi_2^{+\dagger} - \frac{1}{\sqrt{2}|\lambda_2|}\overline{U} m_U U \Phi_2 \\
&\quad + \frac{i|\lambda_1|}{\sqrt{2}|\lambda_2|\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\overline{U} m_U \gamma^5 U \Phi_3 + h.c.. (22)
\end{aligned}$$

For **CHARGED** particle exchange,

$$\mathcal{M}(charged) = \mathcal{M}_W + \mathcal{M}_{\phi^+} \ ,$$

$$\mathcal{M}_{\phi^+} = V_{ca}V^*_{db} \sum_{i=1,2} \frac{1}{4\,|\lambda_i|^2} < \phi_i^+ \phi_i^{+*} > \cdot$$

$$[\overline{u}_c(1-\gamma^5)u_a][\overline{u}_d(1-\gamma^5)u_b] \ , \qquad (23)$$

and $\mathcal{M}_W$ has the same form as in minimal SM

Because $< \phi_i^+ \phi_i^{+*} >$ is real, CP(T)-violation is due to the real and imaginary parts of $V_{ca}V^*_{db}$.

Applying T and P transformation, one has

$$\sigma^T_{part} = 0.0\,, \quad \sigma^{TP}_{part} = 0.0\,,$$

$$\Rightarrow \sigma^T_{had} = 0.0\,, \quad \sigma^{TP}_{had} = 0.0\,. \qquad (24)$$

For **NEUTRAL** particle exchanges in a reaction, the neutral particles could be γ, Z, H^0's, and even g's. We will not consider g's here. The Lagrangian for quark and neutral particles interactions is given by

$$\mathcal{L}_{NC} = -q_\psi \overline{\psi}\gamma_\mu u A^\mu + \frac{g}{\sin\Theta_W}\overline{\psi}\gamma_\mu(c_V^\psi - c_A^\psi\gamma^5)\psi Z^\mu + \mathcal{L}_{\phi^0} \qquad (25)$$

where $\mathcal{L}_{\phi^0}$: given in eq.22,

q_ψ: the electric charge of ψ,

c_V^ψ and c_A^ψ are given by

c_V^U	$1/4 - (2/3)\sin^2\theta_W$
c_A^U	$1/4$
c_V^D	$-1/4 + (1/3)\sin^2\theta_W$
c_A^D	$-1/4$

one can write down the amplitude for neutral particle exchanges as

(Because of k_0-component in $k_\mu k_\nu/m_Z^2$ in Z boson propagator, the part of Z boson exchange amplitude is more complicate under spatial reflection than a massless photon. However, it can be shown that $k_\mu k_\nu/m_Z^2$ contribution to T-odd cross section is zero due to some symmetry cancelation. Therefore, for studying T-odd cross section, one does not need to consider the $k_\mu k_\nu/m_Z^2$ part of the amplitude and this is the reason why we can seperate $\mathcal{M}_Z$ into $\mathcal{M}_Z^E$ and $\mathcal{M}_Z^P$ in the way shown above. This is also true for charged W bosons, which have $k_\mu k_\nu/m_W^2$ term in its propagator in unitary gauge.)

$$\mathcal{M}(neutral) = \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_{\phi^0}$$

$$= \mathcal{M}_\gamma^E + (\mathcal{M}_Z^E + \mathcal{M}_Z^P) + (\mathcal{M}_{\phi^0}^E + \mathcal{M}_{\phi^0}^{TP}) \ ,$$

$$\mathcal{M}_{\gamma}^{E} = -\frac{e^2}{k^2}(u_c\gamma^{\mu}u_a)(u_d\gamma_{\mu}u_b) \ ,$$

$$\mathcal{M}_{Z}^{E} = -\frac{g^2}{4\cos^2\theta_W}\frac{g_{\mu\nu}-k_{\mu}k_{\nu}/m_Z^2}{k^2-m_Z^2+i\epsilon}[c_V^{u_a}c_V^{u_b}(\overline{u}_c\gamma^{\mu}u_a)$$

$$\cdot(\overline{u}_d\gamma_{\nu}u_b) + c_A^{u_a}c_A^{u_b}(\overline{u}_c\gamma^{\mu}\gamma^5u_a)(\overline{u}_d\gamma_{\nu}\gamma^5u_b)] \ ,$$

$$\mathcal{M}_{Z}^{P} = \frac{g^2}{4\cos^2\theta_W}\frac{g_{\mu\nu}-k_{\mu}k_{\nu}/m_Z^2}{k^2-m_Z^2+i\epsilon}c_V^{u_a}c_A^{u_b}(\overline{u}_c\gamma^{\mu}u_a)$$

$$\cdot(\overline{u}_d\gamma_{\nu}\gamma^5u_b) + c_A^{u_a}c_V^{u_b}(\overline{u}_c\gamma^{\mu}\gamma^5u_a)(\overline{u}_d\gamma_{\nu}u_b)] \ ,$$

$$\mathcal{M}_{\Phi^0}^{E} = X_{ab}^{E}\frac{v_iv_j}{k^2-m_H^2+i\epsilon}(\overline{u}_cm_au_a)(\overline{u}_dm_bu_b)$$

$$+Y_{ab}^{E}\frac{v_kv_l}{k^2-m_H^2+i\epsilon}(\overline{u}_cm_a\gamma^5u_a)(\overline{u}_dm_b\gamma^5u_b) \ ,$$

$$\mathcal{M}_{\Phi^0}^{TP} = X_{ab}^{TP}\frac{iv_iv_j}{k^2-m_H^2+i\epsilon}(\overline{u}_cm_au_a)(\overline{u}_dm_b\gamma^5u_b) +$$

$$Y_{ab}^{TP}\frac{iv_kv_l}{k^2-m_H^2+i\epsilon}(\overline{u}_cm_a\gamma^5u_a)(\overline{u}_dm_bu_b) \quad (26)$$

note:

(1) $u_a(u_b)$ and $u_c(u_d)$ have the same flavor in neutral boson exchanges but different momenta,

(2) the scalar exchange is dorminated by the lighest m_H

of H^0, i.e.

$$< \Phi_a \Phi_b >_k \simeq \frac{v_a v_b}{k^2 - m_H^2 + i\epsilon} \ . \qquad (27)$$

The m_H and v_a can be determined from the mass matrix in a specific scalar potential. (See Weinberg, S., Phys. Rev. D **42** (1990) 860)

X_{UU}^{E}	$1/(2\|\lambda_2\|^2)$
X_{UU}^{TP}	$-\|\lambda_1\|/[2\|\lambda_2\|^2\sqrt{\|\lambda_1\|^2+\|\lambda_2\|^2}]$
X_{UD}^{E}	$1/(2\|\lambda_1\lambda_2\|$
X_{UD}^{TP}	$-1/[2\|\lambda_1\|\sqrt{\|\lambda_1\|^2+\|\lambda_2\|^2}]$
X_{DU}^{E}	$1/(2\|\lambda_1\lambda_2\|$
X_{DU}^{TP}	$-1/[2\|\lambda_2[\sqrt{\|\lambda_1\|^2+\|\lambda_2\|^2}]$
X_{DD}^{E}	$1/(2\|\lambda_1\|^2)$
X_{DD}^{TP}	$-\|\lambda_2\|/[2\|\lambda_1\|^2\sqrt{\|\lambda_1\|^2+\|\lambda_2\|^2}]$
Y_{UU}^{E}	$-\|\lambda_1\|^2/[2\|\lambda_2\|^2(\|\lambda_1\|^2+\|\lambda_2\|^2)]$
Y_{UU}^{TP}	$-\|\lambda_1\|/[2\|\lambda_2\|^2\sqrt{\|\lambda_1\|^2+\|\lambda_2\|^2}]$
Y_{UD}^{E}	$-1/[2(\|\lambda_1\|^2+\|\lambda_2\|^2)]$
Y_{UD}^{TP}	$\mathrm{i} -1/[2\|\lambda_2\|\sqrt{\|\lambda_1\|^2+\|\lambda_2\|^2}]$
Y_{DU}^{E}	$-1/[2(\|\lambda_1\|^2+\|\lambda_2\|^2)]$
Y_{DU}^{TP}	$-1/[2\|\lambda_1\|\sqrt{\|\lambda_1\|^2+\|\lambda_2\|^2}]$
Y_{DD}^{E}	$-\|\lambda_2\|^2/[2\|\lambda_1\|^2(\|\lambda_1\|^2+\|\lambda_2\|^2)]$
Y_{DD}^{TP}	$-\|\lambda_2\|/[2\|\lambda_1\|^2\sqrt{\|\lambda_1\|^2+\|\lambda_2\|^2}]$

$\Rightarrow$

Detail calculation shows that

$$(|\mathcal{M}|^2)^T = 2Re(\mathcal{M}_Z^{P*}\mathcal{M}_{\Phi^0}^{TP})$$

$$= \frac{16\, m_a\, m_b\, g^2}{\cos^2\Theta_W (k^2 - m_Z^2 + i\epsilon)\,(k^2 - m_H^2 + i\epsilon)}\, \varepsilon_{\alpha\beta\mu\nu} \cdot$$

$$[m_a\, v_i\, v_j\, c_V^a\, c_A^b\, X_{ab}\, s_b^\alpha\, p_b^\beta\, (p_a + p_c)^\mu\, p_d^\nu +$$
$$m_b\, v_m\, v_n\, c_A^a\, c_V^b\, Y_{ab}\, s_a^\alpha\, p_a^\beta\, (p_b + p_d)^\mu\, p_c^\nu]$$

$$(|\mathcal{M}|^2)^{TP} = 2Re[(\mathcal{M}_\gamma^{E*} + \mathcal{M}_Z^{E*} + \mathcal{M}_{\Phi^0}^{E})\mathcal{M}_{\Phi^0}^{TP}]$$

$$= \frac{32\, m_a\, m_b\, e_a\, e_b}{(k^2 + i\epsilon)\,(k^2 - m_H^2 + i\epsilon)}\, \varepsilon_{\alpha\beta\mu\nu} \{$$

$$X_{ab}\, v_i\, v_j\, s_a^\alpha\, p_a^\beta\, p_c^\nu\, [\, -(p_b \cdot p_d)\, s_b^\mu + (s_b \cdot p_d)\, p_{b\mu} + m_b^2\, s_b^\mu\,]$$

$$+ Y_{ab}\, v_m\, v_n\, s_b^\alpha\, p_b^\beta\, p_d^\nu\, [-(p_a \cdot p_c)\, s_a^\mu + (s_a \cdot p_c)\, p_{a\mu} + m_a^2\, s_a^\mu\,]\}$$

$$+ \frac{8\, m_a\, m_b\, g^2}{\cos^2\Theta_W (k^2 - m_Z^2 + i\epsilon)\,(k^2 - m_H^2 + i\epsilon)}\, \varepsilon_{\alpha\beta\mu\nu}$$

$$\cdot \{[X_{ab}\, c_V^a\, c_V^b\, v_i\, v_j + Y_{ab}\, c_A^a\, c_A^b\, v_m\, v_n\,] s_a^\alpha\, p_a^\beta\, p_c^\nu\, [$$
$$-(p_b \cdot p_d\, s_b^\mu + (s_b \cdot p_d) p_b^\mu - m_b^2\, s_b^\mu\,]$$

$$+ [X_{ab}\, c_A^a\, c_A^b\, v_m\, v_n + Y_{ab}\, c_V^a\, c_V^b\, v_i\, v_j\,] s_b^\alpha\, p_b^\beta\, p_d^\nu\, [$$
$$-(p_a \cdot p_c\, s_a^\mu + (s_a \cdot p_c) p_a^\mu - m_a^2\, s_a^\mu\,]\, \}\ . \qquad (28)$$

The parton cross section would be

$$\sigma_{part}^{T(TP)} = \int \frac{(|\mathcal{M}|^2)^{T(TP)}}{|\mathbf{v_a} - \mathbf{v_b}| \cdot 2E_a \cdot 2E_b} dQ \ , \tag{29}$$

and the cross section difference between $(s_a,\ s_b)$ and $(s_a,\ -s_b)$ is

$$\Delta\sigma_{part}^{T(TP)} = \int \frac{|\mathcal{M}_{s_a,s_b}|^{2T(TP)} - |\mathcal{M}_{s_a,-s_b}|^{2T(TP)}}{|\mathbf{v_a} - \mathbf{v_b}| \cdot 2E_a \cdot 2E_b} dQ, \tag{30}$$

where

$$dQ = (2\pi)^4 \delta^{(4)}(p_c + p_d - p_a - p_b) \frac{d^3p_c d^3p_d}{(2\pi)^6 4E_c E_d} \ . \tag{31}$$

If we use the parton model, the total cross section difference between $(s_a,\ s_b)$ and $(s_a,\ -s_b)$ would be

$$\Delta\sigma_{had}^{T(TP)} = \sum_{a,b} \int dx_a \, dx_b \, \delta f(x_a) \, \delta f(x_b) \, \Delta\sigma_{part}^{T(TP)} \tag{32}$$

We would like to emphasis a few points here:

- $\Delta\sigma^{TP}(1/2_x + 1/2_y)$: **T-odd P-odd** within their particle rest frames. Since a transversally polarized spin vector does not change under a Lorentz transformation, the two colliding hadrons remain transversally polarized in x and y directions in CMS.

- $\Delta\sigma^T(1/2_x + 1_{yz})$: **T-odd P-even**. The polarizations are measured in CMS.
- A non-zero $\Delta\sigma^T$ or $\Delta\sigma^{TP}$ would indicate that (1) time reversal symmetry is violated; (2) the time reversal symmetry breaking mechanism could be more complicate than the current understanding in minimal SM with CKM-matrix.
- **In t channel:**
  $k^2 = s = 2E_aE_b - 2p_ap_b cos\theta_{ab} + m_a^2 + m_b^2$,
  $k^2 - m^2 \Downarrow$ as $k^2 \Uparrow$, and also
  two p components $\Uparrow$ as $k^2 \Uparrow$,
  $\Rightarrow$ **larger $\Delta\sigma$ at higher energies.**

  In s channel:
  $k^2 = t = -2E_aE_b + 2p_ap_b cos\theta_{ac} + m_a^2 + m_b^2$,
  two p components $\Uparrow$ as $k^2 \Uparrow$, and
  $k^2 - m^2 \Uparrow$ as $k^2 \Uparrow$,
  but because m^2 is a large quatity, $k^2 - m^2$ will increase slowly, at most, to two or three times of m^2 at RHIC energies,
  $\Rightarrow$ **larger $\Delta\sigma$ at higher energies.**

- If neutral-Higgs-boson exchanges contributed to CP(T)-violation, the T-odd amplitude would interfere with photon exchange amplitude (or even gluon exange amplitude under the picture of perturbative QCD), which is larger than the Z boson exchange amplitude in normal situation. Therefore the T-odd P-odd cross section measurements could be more sensitive to the CP(T)-violation effect.

Since beam particles usually mainly contain u and d quarks, one cannot do much to increase the Higgs-quark coupling for neutral particle exchanges. The situation for charged particle exchanges will be different. In this case, the produced quarks could be heavy and the corresponding coupling constants would be larger. This could increase the sensitivity of CP(T) violation detection if charged Higgs bosons existed and contributed to CP(T)-violation. We will consider this case in the following subsection.

3.2.2 Electroweak Theory with Three Higgs Doublets

In this model, both charged-Higgs-bosons and neutral-Higgs-bosons could contribute to CP(T)-violation. Consider the scalar potential that satisfy discrete symmetry $\Phi_i \longrightarrow -\Phi_i$ in the form (see Deshpande, N. G. and Ma, E., Phys. Rev. D **16** (1977) 1583).

$$\begin{aligned} \mathcal{L}_\phi &= -\partial_\mu \phi_1^\dagger \partial^\mu \phi_1 - \partial_\mu \phi_2^\dagger \partial^\mu \phi_2 - V(\phi_2, \phi_3) + \\ &\quad V(\phi_3, \phi_1) + V(\phi_1, \phi_2) \ , \\ V(\Phi_1, \Phi_2) &= -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 + h_1 (\Phi_1^\dagger \Phi_1)^2 + h_2 (\Phi_2^\dagger \Phi_2)^2 \\ &\quad + f_{12} (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \ g_{12} (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ &\quad + k_{12} (\Phi_1^\dagger \Phi_2)^2 + k_{12}^* (\Phi_2^\dagger \Phi_1)^2 \ , \end{aligned} \tag{33}$$

where $\Phi_i = \left(\phi_i^+, \phi_i^0\right)$.

After $SU(2)_L \times U(1)_Y \Rightarrow U(1)_{em}$ spontaneously, the three neutral Higgs fields and the mass eigenstates of three charged Higgs-bosons can be written as

$$\begin{aligned} \phi_j^0 &= \frac{v_j}{\sqrt{2}} (1 + \frac{H_j + i\chi_j}{|v_j|}) \ , \\ \Theta_i &= \Omega_{ij} \phi_j^+ \qquad i, \ j = 1, \ 2, \ 3 \ , \end{aligned} \tag{34}$$

where $v_j/\sqrt{2}$: acquired vacuum expectation values satisfying certain conditions to have no linear H_i, or χ_i term in V, and Ω_{ij} can be obtained from V.

The interactive Lagrangian between quarks and charged bosons is

$$
\begin{aligned}
\mathcal{L}_{CC} \;=\; & \frac{g}{2\sqrt{2}}[\overline{U}\gamma_\mu(1-\gamma^5)VDW^{+\mu}+\overline{D}\gamma_\mu(1-\gamma^5)V^\dagger UW^{-\mu}] \\
& -\frac{1}{2v_1^*}\overline{D}m_D V^\dagger(1-\gamma^5)U\Omega_{j1}\Theta_j^* \\
& -\frac{1}{2v_1}\overline{U}m_D V(1+\gamma^5)D\Omega_{j1}^*\Theta_j \\
& +\frac{1}{2v_2}\overline{U}m_U V(1-\gamma^5)D\Omega_{j2}^*\Theta_j \\
& +\frac{1}{2v_2^*}\overline{D}m_U V^\dagger(1+\gamma^5)U\Omega_{j2}\Theta_j^* \;, \qquad j=2,3\;, \qquad (35)
\end{aligned}
$$

where the Θ_1 has been out in unitary gauge.

Take the effect of charged scalar exchange to be dorminated by the lightest charged boson, the Θ_j can be set to be Θ_3. The amplitudes due to W and Θ_3 boson exchanges for scattering $ab \longrightarrow cd$ are given by

$$
\begin{aligned}
\mathcal{M}_W \;=\; & \frac{g^2}{8}\frac{g_{\mu\nu}-k_\mu k_\nu/m_W^2}{k^2-M_W^2+i\epsilon}V_{ca}V_{db}^* \;\cdot \\
& [\,\overline{u}_a\gamma^\mu\,(1-\gamma^5)\,u_c\,][\,\overline{u}_b\,\gamma^\nu\,(1-\gamma^5)\,u_d\,]\;,
\end{aligned}
$$

$$\mathcal{M}_{\Theta_3} = \frac{1}{k^2 - m_3^2 + i\epsilon}[\,\overline{u}_c\,(A_{ca} + B_{ca}\gamma^5)\,u_a\,] \cdot$$

$$[\,\overline{u}_d\,(A^*_{db} + B^*_{db}\gamma^5)\,u_b\,]\,, \tag{36}$$

where we have let

$$A_{ab} = A^R_{ab} + iA^I_{ab} = (-\frac{m_D\,\Omega^*_{31}}{2\,v_1} + \frac{m_U\,\Omega^*_{32}}{2\,v_2})\,V_{ab}\ ,$$

$$B_{ab} = B^R_{ab} + iB^I_{ab} = -(\frac{m_D\,\Omega^*_{31}}{2\,v_1} + \frac{m_U\,\Omega^*_{32}}{2\,v_2})\,V_{ab}, \tag{37}$$

where all elements in A^R_{ab}, A^I_{ab}, B^R_{ab}, and B^I_{ab} are real.

One can expand the amplitudes into T-even P-even (E), T-even P-odd (P), T-odd P-even (T), and T-odd P-odd parts as

$$\mathcal{M}^E_W = \frac{g^2}{8}\frac{g_{\mu\nu} - k_\mu k_\nu/m^2_W}{k^2 - M^2_W + i\epsilon}(ReV_{ca}V^*_{db}) \cdot$$

$$\left[(\overline{u}_c\gamma^\mu u_a)(\overline{u}_d\gamma^\nu u_b) + (\overline{u}_c\gamma^\mu\gamma^5 u_a)(\overline{u}_d\gamma^\nu\gamma^5 u_b)\right]\ ,$$

$$\mathcal{M}^P_W = \frac{-g^2}{8}\frac{g_{\mu\nu} - k_\mu k_\nu/m^2_W}{k^2 - M^2_W + i\epsilon}(ReV_{ca}V^*_{db}) \cdot$$

$$\left[(\overline{u}_c\gamma^\mu u_a)(\overline{u}_d\gamma^\nu\gamma^5 u_b) + (\overline{u}_c\gamma^\mu\gamma^5 u_a)(\overline{u}_d\gamma^\nu u_b)\right]\ ,$$

$$
\begin{aligned}
\mathcal{M}_W^T &= \frac{g^2}{8}\frac{g_{\mu\nu}-k_\mu k_\nu/m_W^2}{k^2-M_W^2+i\epsilon} i(ImV_{ca}V_{db}^*) \cdot \\
& \left[(\overline{u}_c\gamma^\mu u_a)(\overline{u}_d\gamma^\nu u_b)+(\overline{u}_c\gamma^\mu\gamma^5 u_a)(\overline{u}_d\gamma^\nu\gamma^5 u_b)\right] \ , \\
\mathcal{M}_W^{TP} &= \frac{-g^2}{8}\frac{g_{\mu\nu}-k_\mu k_\nu/m_W^2}{k^2-M_W^2+i\epsilon} i(ImV_{ca}V_{db}^*) \ \cdot \\
& \left[(\overline{u}_c\gamma^\mu u_a)(\overline{u}_d\gamma^\nu\gamma^5 u_b)+(\overline{u}_c\gamma^\mu\gamma^5 u_a)(\overline{u}_d\gamma^\nu u_b)\right] \ , \\
\mathcal{M}_{\Theta_3}^E &= \frac{1}{k^2-m_3^2+i\epsilon}\left[(A_{ca}^R A_{db}^R + A_{ca}^I A_{db}^I)(\overline{u}_c\, u_a)(\overline{u}_d\, u_b)\right. \\
& \left. +(B_{ca}^R B_{db}^R + B_{ca}^I B_{db}^I)(\overline{u}_c\gamma^5 u_a)(\overline{u}_d\gamma^5 u_b)\right] \ , \\
\mathcal{M}_{\Theta_3}^P &= \frac{1}{k^2-m_3^2+i\epsilon}\left[B_{ca}^R A_{db}^{R*} + B_{ca}^I A_{db}^I)(\overline{u}_c\gamma^5 u_a)(\overline{u}_d\, u_b)\right. \\
& \left. +(A_{ca}^R B_{db}^R + A_{ca}^I B_{db}^I)(\overline{u}_c\, u_a)(\overline{u}_d\gamma^5 u_b)\right] \ , \\
\mathcal{M}_{\Theta_3}^T &= \frac{1}{k^2-m_3^2+i\epsilon}\, i\left[(-A_{ca}^R A_{db}^I + A_{ca}^I A_{db}^R)(\overline{u}_c\, u_a)(\overline{u}_d\, u_b)\right. \\
& \left. +(-B_{ca}^R B_{db}^I + B_{ca}^I B_{db}^R)(\overline{u}_c\gamma^5 u_a)(\overline{u}_d\gamma^5 u_b)\right] \ , \\
\mathcal{M}_{\Theta_3}^{TP} &= \frac{1}{k^2-m_3^2+i\epsilon}\, i\left[(-A_{ca}^R B_{db}^I + A_{ca}^I B_{db}^R)(\overline{u}_c\, u_a)(\overline{u}_d\gamma^5 u_b)\right. \\
& \left. +(-B_{ca}^R A_{db}^I + B_{ca}^I A_{db}^R)(\overline{u}_c\gamma^5 u_a)(\overline{u}_d\, u_b)\right] \qquad (38)
\end{aligned}
$$

From the above equation, the $(|\mathcal{M}|^2)^{T(TP)}$ will be

$$
\begin{aligned}
(|\mathcal{M}|^2)^{T} &= 2\,Re(\mathcal{M}_W^{E*}\mathcal{M}_{\Theta_3}^{T} + \mathcal{M}_W^{P*}\mathcal{M}_{\Theta_3}^{TP} + \\
&\qquad \mathcal{M}_W^{T*}\mathcal{M}_{\Theta_3}^{E} + \mathcal{M}_W^{TP*}\mathcal{M}_{\Theta_3}^{P}) \\
= &\ \frac{4g^2}{(k^2 - m_W^2 + i\epsilon)(k^2 - m_3^2 + i\epsilon)}\,\varepsilon_{\alpha\beta\mu\nu}[\\
&s_b^\alpha\, p_b^\beta\, p_a^\mu\, p_d^\nu (m_a C_a + m_c C_c) + s_a^\alpha\, p_a^\beta\, p_b^\mu\, p_c^\nu (m_b C_b + m_d C_d)] \\
(|\mathcal{M}|^2)^{TP} &= 2\,Re(\mathcal{M}_W^{E*}\mathcal{M}_{\Theta_3}^{TP} + \mathcal{M}_W^{P*}\mathcal{M}_{\Theta_3}^{T} + \\
&\qquad \mathcal{M}_W^{T*}\mathcal{M}_{\Theta_3}^{P} + \mathcal{M}_W^{TP*}\mathcal{M}_{\Theta_3}^{E}) \\
= &\ \frac{4g^2}{(k^2 - m_W^2 + i\epsilon)(k^2 - m_3^2 + i\epsilon)}\,\varepsilon_{\alpha\beta\mu\nu}\{ \\
&C_1 s_a^\alpha\, p_a^\beta\, p_c^\nu\, [-s_b^\mu (p_b \cdot p_d) + p_b^\mu (s_b \cdot p_d)] + \\
&C_2 s_b^\alpha\, p_b^\beta\, p_d^\nu\, [p_a^\mu (s_a \cdot p_c) - s_a^\mu (p_a \cdot p_c)] + \\
&(C_{ac} m_a m_c + C_{bd} m_b m_d) s_a^\alpha\, p_a^\beta\, p_c^\nu\, s_b^\mu\}\,,
\end{aligned}
\tag{39}
$$

where $C_a, C_b, C_c, C_d, C_1, C_2, C_{ac}, C_{bd}$ are constants relating to the coefficients in the Lagrangian. It is straight forward to obtain the corresponding cross sections.

$\Rightarrow$ **Remarks:**

- Eq. (40) indicates that, if the Higgs sector does not contribute to CP(T)-violation, i.e.

  $$A_{ab} = c_1 \cdot V_{ab} \ , \qquad B = c_2 \cdot V_{ab} \ , \tag{40}$$

  where c_1 and c_2 are real numbers,

  $\Rightarrow \qquad \sigma^T_{tree} = 0.$ (like the case of SM).

  The zero T-odd cross section is the interference between two CP(T)-violation scheme.

- Since some of the couplings between quarks and charged-Higgs-bosons are proportional to the masses of produced quarks and the masses of produced quarks could be heavy, the measured T-odd cross sections could be more obvious than the ones from neutral-Higgs-boson exchanges if the colliding energy is high enough to produce heavy particles.

- Neutral-Higgs-boson exchanges could also produce T-odd cross sections in this case. The calculation is similar to the case of two-Higgs-doublets except some changes in parameters and more neutral-Higgs-bosons involved. (There are five neutral-Higgs-bosons in the case of three Higgs doublets.) We will not repeat this calculation here.

3.3 Possibly non-Trivial Constraints

Two reasons for raising this possibility:

1. No T-odd P-even term in $1/2 + 1/2 \to 1/2 + 1/2$ forward-scattering matrix $\Rightarrow A_{x,z} = 0$.
2. The detected particles are composite ones.

$\Rightarrow$

1. For two polarized spin-1/2 elementary particles,

$$\sigma(a_x + b_z) - \sigma(a_x + b_{-z}) = 0 \ . \tag{41}$$

2. For two polarized spin-1/2 composite particles,

$$\sum_{a,b,c,d} \Delta_x f_a^1(x_a) \Delta_z f_b^2(x_b) \sigma(a + b \to c + d + X) = 0, \tag{42}$$

$\Rightarrow$ This constrain could become a new sum rule if it is not completely trivial.

(The above cross sections are model dependent. We have given two examples before.)

We only give one example of constraints here. There are other terms in the forward-scattering matrix. One could get other constraints to both un-polarized and polarized structure function by properly choosing the polarizations.

4. Possibility at RHIC

The above discussion shows that:

1. A non-vanishing T-odd cross section could indicate that CP(T)-violation could be due to sources other than CKM-matrix.
2. It also indicates that it might be necessary to extend the minimal SM.
3. The measurement has the potential to be a high precision measurement because it is a null test.

A fixed target experiment with beam energy around 1 GeV has been planed in COSY. (See Eversheim, P. D., Pol. Phen. Nucl. Part. Phys., 7-10, Jan. 1992, Trieste, Italy, Proc. 2nd Adriatico Research Conf., World Scientific,(1993) 142)

As our calculations have shown, higher beam energy would give more sensitive signals to the measurement, and would, in principle, be more appropriate to describe the process with parton model and SM. Since RICH collider has already existed spin physics program and the energy of the beams with satisfied luminosity can reach to as high as 250 GeV. It would be appropriate to carry out the measurement at RHIC. The spin-correlations can be deduced in the following way:

Let $N_1(0)(N_2(0))$: the number of particles at $t = 0$,

$N_1(t)(N_2(t))$: the number of particles at t,

A: cross-sectional area of the colliding beams,

n: the number of bunches in either beam,

σ_t: total cross section,

σ_0: unpolarized cross section,

σ_{loss}: the loss cross section that takes account of the beam losses,

$p_1(p_2)$: polarization of beam 1 (beam 2).

$L(t)$: the luminosity at t and should relate to $N_1(t)$ and $N_2(t)$ by

$$L = f\, n \frac{N_1(t) N_2(t)}{A} \ , \tag{43}$$

One should have:

$$\frac{dN_1}{dt} = -\sigma_t \frac{f\, n}{A} N_1 N_2 \ ,$$
$$\frac{dN_2}{dt} = -\sigma_t \frac{f\, n}{A} N_1 N_2 \ , \tag{44}$$

and

$$\sigma_t = \sigma_0(1 + p_1 p_2 A_{12}) + \sigma_{loss} \tag{45}$$

Fir simplicity, assuming $\sigma_{loss1} = \sigma_{loss2}$, (this may not be true in reality), one would have

$$N_2(t) - N_1(t) = N_2(0) - N_1(0) = 0 \; . \tag{46}$$

$\Rightarrow$

$$\sigma_t = \sigma_0(1 + p_1 p_2 A_{12}) + \sigma_{loss} = \frac{A}{t\,f\,n} \frac{N_1(0) - N_1(t)}{N_1(t)\,N_1(0)} \tag{47}$$

By measuring the sequesce of reaction with flipping polarizartions, one can obtain the spin correlation as

$$\begin{aligned} A_{12} &= \frac{1}{p_1\,p_2} \frac{\sigma_t(+,+) - \sigma_t(+,-)}{\sigma_0} \\ &= \frac{1}{2\,p_1\,p_2} \frac{A N_1(0)}{t\,f\,n} \left[\frac{1}{N_1(t)_{++}} - \frac{1}{N_1(t)_{+-}} \right] \; . \end{aligned} \tag{48}$$

For luminosity of $10^{30}/cm^2\,s$ in COZY, the sensitivity is 10^{-6}. For luminosity of $10^{32}/cm^2\,s$, if $N_1(t)$ is not too low, the sensitivity should match to 10^{-6} within two weeks of running.

- $1/2_x + 1/2_{\pm y} \Rightarrow$ T-odd P-odd A_{xy},
  $1/2_x + 1_{\pm yz} \Rightarrow$ T-odd P-even $A_{x,yz}$.

- $(beam\ \ intensity)_{B.C.}/(beam\ \ intensity)_{A.C.} \quad \Rightarrow$ spin asymmetry A. With more delicate measurements, there is a potential to study more interesting physics.

5. Summary and Future Prospect

We have disscussed the possible physics that could be extract from T-odd cross section measurement and the possibility to carry out the measurement at RHIC. We would like to summarize the discussions as follows:

1. CKM-matrix based minimal standard model will give vanishing T-odd cross section at tree level;
2. a non-vanishing T-odd cross section would indicate additional mechanism of CP(T)-violation besides the CKM mechanism and possible extention of the minimal SM. One possibility of the additional mechanism is due to spontaneous CP(T)-violation.
3. It is possible to carry out the measurement at RHIC. The sensitivity of two weeks of running could match to the sensitivity of 10^{-6}.

More discussions on this subject are still necessary.

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