Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 649 (2007) 241–242
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Reply to: "Comment on: 'Chiral anomalies and rooted staggered fermions' [Phys. Lett. B 649 (2007) 230]" [Phys. Lett. B 649 (2007) 235]

Michael Creutz

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Received 11 April 2007; accepted 12 April 2007
Available online 14 April 2007
Editor: B. Grinstein

Abstract

I respond to the Bernard et al. comment on my Letter "Chiral anomalies and rooted staggered fermions".

In their comment on my Letter "Chiral anomalies and rooted staggered fermions" [1], Bernard, Golterman, Shamir, and Sharpe [2] do not address the main point that the chiral symmetry group for the rooted theory has a higher rank than the target theory. This immediately calls into question whether the theories are in the same universality class. This misunderstanding is clear in their item 4 when they say "This kind of phenomenon … is common whenever the lattice theory has less symmetry than the continuum theory". The issue is the reverse: the lattice theory has too much symmetry. To have a symmetry suddenly disappear in the continuum limit is certainly not common.

Much of their discussion concerns whether "taste" symmetry is restored. Indeed, if the unrooted staggered theory did reduce to four uncoupled but equivalent fermions in the continuum limit, one might expect rooting to work. This is especially true in perturbation theory, where taking the fourth root of the determinant just multiplies all fermion loops by one quarter.

But taste restoration is a considerably more complicated issue when non-perturbative effects are taken into account. Fermion doubling generically arises from momenta in various corners of the Brillouin zone. These corners divide such that the various tastes appear with differing physical chirality, i.e. their low-energy modes use gamma matrices that differ by signs. The result is that the exact chiral symmetry of staggered fermions represents a non-singlet symmetry amongst the tastes. The staggered determinant, even in the continuum limit, does not correspond to the fourth power of a single fermion theory. The problem with rooting appears because the procedure effectively averages over the different chiralities. This is inconsistent with the index theorem that says the one-flavor theory should have a zero mode of a single chirality when the gauge field has non-vanishing winding. The Bernard et al. comparison of rooted staggered and overlap theories (in the paragraph following their Eq. (5)) is misleading in this respect because four flavors of overlap or Wilson fermions are forced by construction to have all eigenvalues in identical quartets with identical chirality.

As mentioned in my Letter, the eigenvalue matching that numerically suggests taste symmetry restoration, such as seen in Ref. [3], must break during transitions between topological sectors. On passing from zero to unit winding, the four nearly zero eigenmodes must have evolved from two eigenvalues dropping down in the complex plane from above and two symmetrically rising from below.

The underlying issue lies in the structure of the 't Hooft vertex [4], which the algorithm does not treat properly. Before rooting this is a multilinear fermionic operator that strongly couples all tastes. To obtain the one flavor theory this should be converted to a simple fermion bilinear. However the exact chi-

DOI of original article: 10.1016/j.physletb.2007.03.065.
DOI of comment: 10.1016/j.physletb.2007.04.018.
E-mail address: creutz@bnl.gov.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2007.04.017

ral symmetries of the unrooted theory are retained in the rooting process and forbid the appearance of the correct target form.

The numerous numerical studies by the staggered community have shown that rooted staggered quarks can quite accurately describe many physical processes where the ’t Hooft vertex does not play a major role. But any processes where these non-perturbative effects are important will not be reproduced correctly by the algorithm.

Acknowledgements

This manuscript has been authored under contract number DE-AC02-98CH10886 with the US Department of Energy. Accordingly, the US Government retains a non-exclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for US Government purposes.

References

[1] M. Creutz, Phys. Lett. B 649 (2007) 230, hep-lat/0701018.
[2] C. Bernard, M. Golterman, Y. Shamir, S.R. Sharpe, Phys. Lett. B 649 (2007) 235.
[3] E. Follana, A. Hart, C.T.H. Davies, Q. Mason, HPQCD Collaboration, Phys. Rev. D 72 (2005) 054501, hep-lat/0507011.
[4] G. ’t Hooft, Phys. Rev. D 14 (1976) 3432;
G. ’t Hooft, Phys. Rev. D 18 (1978) 2199, Erratum.