Observations on Factorization

George Sterman
Physics Department, Brookhaven National Laboratory,
Upton, NY 11973, U.S.A.

C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook
Stony Brook, NY 11794 – 3840, U.S.A.

This talk began with a review of the formalism for collinear factorization, [1] which applies to inclusive hard scattering cross sections involving one or more hard scales. Such cross sections, including those for Drell-Yan, jet and heavy quark production, are convolutions in partonic longitudinal momentum fractions, between parton distribution functions, $\phi_{a/A}$, and hard scattering functions. For inclusive single-particle production at high transverse momentum, fragmentation functions are included. For the latter case, the differential cross section is thus

$$\begin{aligned} d\sigma_{AB\to C(p)} &= \sum_{ijk}\int dx\, dy\, dz\; \phi_{i/A}(x,\mu_F)\; \phi_{j/B}(y,\mu_F) \\ &\qquad \times D_{C/k}(z,\mu_F)\; d\hat{\sigma}_{ijk}\left(xp_A, yp_B, p_C/z, \mu_F, \alpha_s(\mu_F)\right)\,, \end{aligned} \tag{1}$$

where μ_F is the factorization scale, and where $\hat{\sigma}$ is perturbatively calculable. This formalism applies at leading power in the hard scale, for both polarized and unpolarized cross sections. Corrections to the factorization formula itself are suppressed by powers of the hard scale (p_T, etc.).

Parton distributions and fragmentation functions may be expressed as expectation values of nonlocal operators in hadronic states. [2] For example, for the fragmentation function of a quark (referred to the plus direction for convenience)

$$D_{C/f}(z,\mu_F) = \int \frac{d\lambda}{2\pi}\, \mathrm{e}^{-i\lambda(p_C^+/z)}\; \sum_X \langle 0|q(0)|C,X\rangle\, \Gamma\, \langle C,X|\bar{q}(\lambda,0^+,0_\perp)|0\rangle \tag{2}$$

where information on polarization can be incorporated through the choice of the Dirac matrix Γ. These expectations require a renormalization, corresponding to a maximum relative transverse momentum for the observed hadron and parton, or equivalently a cutoff on the transverse distance to the light-cone. The freedom to choose this scale, which corresponds to the factorization scale, leads directly to calculable evolution for the parton distributions.

Factorization may be understood heuristically in terms of the Lorentz transformation properties of classical fields, by an analysis familiar from electrodynamics. [1] The physical electromagnetic field strengths are Lorentz contracted even more strongly than a scalar field, suggsting that factorization holds even to the first nonleading power. [3] In the case of polarized cross sections, however, some of this suppression may be lost, due to the mechanical origin of the orbital angular momentum.

The most basic extension of collinear factorization is to include the transverse momentum degrees of freedom of the partons. [5, 4] The corresponding expectation values are very closely related to the normal parton distributions, so that for quark q in polarized hadron A we have the general form [5]

$$\mathcal{P}_{q/A}(x,k_\perp,s) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, \mathrm{e}^{-ixp^+y^- + ik_\perp\cdot y_\perp}\, \langle A(p,s)|\bar{q}(0^+,y^-,y_\perp)\,\Gamma\, q(0)|A(p,s)\rangle\,, \tag{3}$$

where again the choice of Γ determines the spin content of the distribution. In the case of parton distributions at measured k_T, a good deal is known about the summation of logarithms of the transverse momentum. [5]

Dependence on the transverse momentum of partons is power suppressed in inclusive cross sections with *only* hard scales in the final state. When there is in addition a second scale, the k_T distribution of initial-state partons, or final-state hadrons in fragmentation, may become crucial. At the same time, if the cross section remains otherwise inclusive, factorization is not lost. The most familiar example is the Drell-Yan cross section at measured Q_T, but another example is found in the Collins effect, [6] where the expectation of the triple product $\langle s \cdot p_J \times p_\pi \rangle$ acts as a small scale, with p_J and p_π a jet and pion momentum, respectively.

Finally, power suppressed effects may be important, when leading power contributions are small or absent altogether. Examples are multiple scattering in nuclei and chiral-even single-spin asymmetries. [7] Such contributions are, as indicated above, often amenable to collinear factorization. Parton k_T-effects are likely to be important in these cases as well.

References

[1] J.C. Collins, D.E. Soper and G. Sterman, in *Perturbative quantum chromodynamics*, ed. A.H. Mueller (World Scientific, Singapore, 1989).

[2] J.C. Collins and D.E. Soper, *Nucl. Phys.*, **194**, 445 (1982).

[3] J.-W. Qiu and G. Sterman, *Nucl. Phys.* **B353**, 137 (1991).

[4] E. Laenen, G. Sterman and W. Vogelsang, hep-ph/0010080.

[5] J.C. Collins and D.E. Soper, *Nucl. Phys.* **193**, 381 (1981).

[6] J. Collins, *Nucl. Phys.* **420**, 565 (1994), hep-ph/9305309.

[7] J.-W. Qiu and G. Sterman, *Phys. Rev.* **D59**, 014004 (1999), hep-ph/9806356.