On averaging inelastic cross sections across experiments

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We average the inelastic cross sections measured by E811 and CDF. The technique employed in this note includes correlations not originally considered in the related CDF analysis.

Introduction

There were three measurements of the inelastic cross section, $\sigma_{inelastic}$, at $\sqrt{s} = 1.8$ TeV. They were made by CDF [?], E710 [?], and E811 [?] and are summarized in Tab. 1. The measurements disagree at the several sigma level. E811, which agrees well with E710, was a follow-up to that experiment. For the purposes of this note, we will drop the E710 measurement from the average.

A weighted average of CDF and E811, assuming no correlations between them and scaling the uncertainties according to the PDG prescription [?], yields

$$\sigma_{inelastic}(1 + \rho^2) = 58.7 \pm 2.3\,\text{mb}.$$

Unfortunately, the two measurements are, in fact, correlated with each other. In this note we investigate the sources of uncertainty in the CDF and E811 inelastic cross section measurements and cross check the averaging methods in Sergei Klimenko's CDF note [?].

Observables and Formulae

The observables are:

$N^{(accepted)}_{elastic}$ — the number of accepted elastic events,

$N^{(accepted)}_{inelastic}$ — the number of accepted inelastic events,

b — the slope of the t distribution which is almost uncorrelated with the other observables.

Experiment	$\sigma_{inelastic}(1+\rho^2)$
E710	56.6 ± 2.2 mb
E811	56.5 ± 1.2 mb
CDF	61.7 ± 1.4 mb

Table 1: Inelastic cross sections measured at $\sqrt{s} = 1.8$ TeV.

The error analysis is simpler if $\sigma_{inelastic}$ is expressed in terms of these independent observables. If we define

$$R = \frac{N^{(total)}_{inelastic}}{N^{(total)}_{elastic}} = \frac{N^{(accepted)}_{inelastic}}{N^{(accepted)}_{elastic}} \times \frac{A_{elastic}}{A_{inelastic}}, \tag{1}$$

then the inelastic cross section is

$$\sigma_{inelastic}(1+\rho^2) = 16\pi(\hbar c)^2 b \frac{R}{(1+R)^2}.$$

(We refer to $\sigma_{inelastic}(1+\rho^2)$ as σ in the rest of this note.) The problem in finding a consistent value of $\sigma_{inelastic}$ is combining measurements from different experiments when both depend on a the common parameter b through the intercept and the acceptance corrections.

b Dependence

R is not an experimental observable. It is corrected for acceptance and is intrinsically b dependent because the acceptance correction, $A_{elastic}$, is b dependent. If we assume the t dependence of the elastic cross section goes as $dN_{elastic}/dt = (N_0/b)e^{-tb}$, then

$$N^{(total)}_{elastic} = \int_0^\infty \frac{dN}{dt} dt = N_0, \text{ and}$$

$$N^{(accepted)}_{elastic} = \int_{t_{min}}^{t_{max}} \frac{dN}{dt} dt = N_0(e^{-bt_{min}} - e^{-bt_{max}}).$$

The acceptance is then

$$A_{elastic}(b) = \frac{N^{(accepted)}_{elastic}}{N^{(total)}_{elastic}} = (e^{-bt_{min}} - e^{-bt_{max}}),$$

and one can define an $R(b)$ that explicitly depends on b and is equal to R when the nominal value of b is used

$$R(b) = R\frac{A_{elastic}(b)}{A_{elastic}(b_0)} = R\frac{(e^{-bt_{min}} - e^{-bt_{max}})}{(e^{-b_0 t_{min}} - e^{-b_0 t_{max}})}.$$

The inelastic acceptance uncertainties should be uncorrelated between the two experiments and are either included in the R uncertainty or in the residual uncertainty of the inelastic cross section.

	CDF	E811
$dN_{elastic}/dt\big\vert_{t=0}$ [GeV^{-2}]	1336532 ± 40943	$(86282 \pm 603) \times 10^2$
b [GeV^{-2}]	16.98 ± 0.25	16.98 ± 0.22
t_{min} [GeV^2]	0.04	0.0045
t_{max} [GeV^2]	0.29	0.036
$A_{elastic}$	0.495	0.384
$N_{elastic}^{(total)}$	78691 ± 1463	$(5081 \pm 35) \times 10^2$
$N_{inelastic}^{(total)}$	240982 ± 2967	$(17995 \pm 572) \times 10^2$
R	3.062 ± 0.068	3.542 ± 0.113

Table 2: Experimental parameters used in this analysis. $N_{elastic}^{(total)}$ and $N_{inelastic}^{(total)}$ are corrected for detector and acceptance effects. R is the ratio defined in Eq. 1.

Numerical Estimates

For CDF the elastic acceptance is 49.5%, for E811 it is 38.4%. However, the b dependence for the two measurements is very different.

For CDF, if b increases by one standard deviation (we use the CDF uncertainty of 0.25 throughout for consistency) or +1.5%, the elastic acceptance **decreases** by 0.9%. For E811, the elastic acceptance **increases** by 1.0%. This is because the missing acceptance for CDF is at mostly at low b, while the missing acceptance for E811 is at high b.

Carrying this additional b dependence through to the cross section calculation, a one standard deviation increase in b causes a 1.2 mb (+1.9%) increase in the CDF inelastic cross section but only a 0.5 mb (+0.9%) increase in the E811 result. This is because the acceptance and slope terms in the cross section are positively correlated in the final estimate for CDF and anti-correlated for E811 because of the difference in the b dependence of the acceptance.

A one standard deviation increase in R from the experimental uncertainties due to measuring the numbers of accepted events, as considered in Sergei's memo, changes the inelastic cross section for CDF by -0.7 mb (-1.1%) and the inelastic cross section for E811 by -1.0 mb (-1.8%).

This can be used to construct approximate error matrices:

$$\epsilon_R^2 = \begin{bmatrix} 0.492 & 0 \\ 0 & 1.070 \end{bmatrix} \text{mb}^2, \text{ and}$$

$$\epsilon_b^2 = \begin{bmatrix} 1.401 & 0.614 \\ 0.614 & 0.269 \end{bmatrix} \text{mb}^2.$$

The estimated uncertainty in the inelastic cross sections is 1.37 mb for CDF and 1.15 mb for E811 combining the b uncertainty and the other uncertainties in R in quadrature. The officially quoted uncertainties are 1.4 and 1.2 mb for the CDF and E811 respectively so the b and R uncertainties represent most of the uncertainty.

We can find the appropriate weighting between the results by forming the average as

$$\sigma(b, R_{CDF}, R_{E811}; f) = f\sigma_{CDF}(b, R_{CDF}) + (1-f)\sigma_{E811}(b, R_{E811}),$$

where f is the unknown weight. The χ^2 is defined as

$$\chi^2 = \sum_{i,j} (\sigma_i - \sigma(f))(\sigma_i - \sigma(f))W_{ij},$$

where W_{ij} is the inverse of the error matrix. When f is varied to minimize the χ^2 the resulting weighting is $f = 0.39$ and the central value is

$$\sigma_{inel}(1 + \rho^2) = 58.8 \pm 0.9\,\mathrm{mb}$$

with a χ^2 of 9.2.

The uncertainty estimate does not include the additional small systematic uncertainty components beyond b and R and is not PDG scaled [**?**]. If the uncertainties are inflated to yield a $\chi^2/d.o.f.$ of 1 (PDG method), the result is

$$\sigma_{inelastic}(1 + \rho^2) = 58.8 \pm 2.7\,\mathrm{mb},$$

where the uncertainty is due to b and R alone. Sergei agrees with this result and calls this method C. Sergei's method B has PDG weighting applied to the R part of the error matrix but not the b part, as a result the relative weight f changes from 0.39 to 0.65 and the result is

$$\sigma_{inelastic}(1 + \rho^2) = 60.1 \pm 2.3\,\mathrm{mb}.$$

The relative weight necessary to achieve a reasonable $\chi^2/d.o.f.$ is

$$\epsilon^2 = \mathbf{13} \times \epsilon_R^2 + \epsilon_b^2$$

which essentially removes the b component in the uncertainty.