A Calculation of the Dynamic Aperture of the LHC *

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Abstract

The dynamic aperture for version 5.1 of the Large Hadron Collider (LHC) lattice has been estimated using the tracking code TEVLAT. This version of the lattice has a (full) crossing angle of 300μr at IP1 and IP5.

The dynamic aperture calculated here is due to the magnetic field errors in the High Gradient Quadrupoles (HGQ) in the two low-β interaction regions at IP1 and IP5. No errors were assigned to the magnets in the arc regions of the lattice nor were beam-beam effects incorporated.

A physical aperture of ± 30mm was imposed on the particles during the tracking. The dynamic aperture is expressed in terms of the σ of the beam corresponding to the beam emittance of 3.75mm mr.

With only short term tracking the combined effect of the multipoles and the crossing angle is to yield an average (over a 100 set of multipole cofficients) dynamic aperture of $\approx 11.7 \pm 1.2\sigma$.

1 INTRODUCTION

It is necessary, with any new lattice, to try and calculate the emittance of the beam that can be accepted by the lattice and to verify that the acceptance, the *dynamic aperture*, is large enough for the intended beam. The physical aperture of the magnets, of course, sets the absolute upper limit to the dynamic aperture but with superconducting magnets the dynamic aperture is typically limited by the high order multipoles (the *magnets errors*) used to describe the field of the magnet.

The methodology for calculating the dynamic aperture is fairly standard:

- Construct a computer model of the lattice;
- Adjust the tunes and chromaticites to the nominal values;
- Introduce the magnetic field errors;
- Track a series of test particles for a given number of turns;
- Identify the emittance of the test particles which survive for the required number of turns.

While well defined, this procedure can only give an upper limit to the dynamic aperture since:

- The model of the lattice is an approximation to the true lattice and all the unknown errors which exist in an actual lattice are not incorporated in to the model;
- The actual magnet errors are not known until the magnets are built and measured;
- A particle can be realistically tracked for a number of turns which is far smaller than the number of turns it will need to survive in the actual lattice.

With these caveats, results are presented on the dynamic aperture for version 5.1 of the lattice of the Large Hadron Collider (LHC). The lattice has the beams crossing at IR1 and IR5 with a 300μr full crossing angle. At IR1 the crossing is vertical plane and at IR5 horizontal. Tracking was done using the code TEVLAT [1].

The lattice has regions of very high-$\beta(\approx 4700m)$ in the high gradient quadrupoles (HGQ), in which the beam is off axis, which generate the low-β insertions at the interaction points. The corresponding maximum β in the arcs is only $\approx 180m$. The effect of magnetic field errors increases with β so it is expected that the errors in the HGQ will dominate the calculation of the dynamic aperture. For that reason only the magnetic field errors in the HGQ will be considered and incorporated into this calculation.

2 MAGNET ERRORS

The low-β insertions at each IR are formed by a quadrupole triplet with four quadrupoles. [2] Two different designs of the coils for the HGQ will be used in the LHC, those built at FERMILAB and those built at KEK. In the model the outer members of the triplet are KEK magnets while the two inner magnets are from FERMILAB. The field errors for each are expressed in terms of multipole coefficients given at a reference radius of 17mm. The values for the multipole coefficients are given in Table 1 and Table 2.[3]. One hundred different sets of multpole errors were generated from these tables.

3 METHOD AND RESULTS

A particle was launched at a point in phase space $(x, x' = 0), (y, y' = 0)$. For each value of x, which ranged from $x = 0$ to $x = 20 \cdot \sigma_x$ [4] in steps of σ_x, the maximum value of y was found where the particle completed 1024 turns. The stable points, from the the hundred sets of multipole errors, are plotted in Figure 1.

It is apparent that the dynamic aperture depends on the particular values of the multipole coefficients. That is, the size of the random component of the multipoles is large

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Table 1
FNAL High Gradient Quad Reference Harmonics at Collision. ver. 2
Reference radius= 17.0mm

FNAL n	CERN n	$< b_n >$	$\sigma(b_n)$	$< a_n >$	$\sigma(a_n)$
2	3	.000	.800	.000	.800
3	4	.000	.800	.000	.800
4	5	.000	.300	.000	.300
5	6	.000	.600	.000	.100
6	7	.000	.060	.000	.060
7	8	.000	.050	.000	.040
8	9	.000	.030	.000	.020
9	10	.000	.030	.000	.030

Table 2
KEK Reference Harmonics at Collision. (Modified.)
Reference radius= 17.0mm

FNAL n	CERN n	$< b_n >$	$\sigma(b_n)$	$< a_n >$	$\sigma(a_n)$
2	3	.000	1.00	.000	1.00
3	4	.000	.570	.000	.570
4	5	.000	.380	.000	.380
5	6	.000	.190	.000	.190
6	7	.000	.060	.000	.060
7	8	.000	.030	.000	.010
8	9	.000	.010	.000	.010
9	10	-0.250	.010	.000	.010

enough that the dynamic aperture can change by a significant factor depending on the value in a given magnet.

In order to provide a more concise representation of the dynamic aperture, though a less complete one, for each set of the multipoles errors , the average radius of the stable points was computed. The results are histogrammed in Figure 2. [5]

The average dynamic aperture, that is the average of the average radius for the 100 distributions is 11.7σ with a width of $\approx 1.2\sigma$. Depending on the set of multipoles found in the actual magnets the dynamic aperture could be as small as $\approx 8\sigma$ or as large as $\approx 14\sigma$.

The dynamic aperture also depends on whether the initial displacement was primarily along x or along y. Figure 3 plots the average dynamic aperture as a function of $\epsilon_x/(\epsilon_x+\epsilon_y)$ It is quite clear that the dynamic aperture when a particle is launched along the y direction is smaller then if it were launched along the x direction.

It is instructive to compute the dynamic aperture with only some of the multipoles present. Figure 4 shows the effect on the dynamic aperture when the highest order of the multipoles is reduced. The results are summarized in Table 3.

If it were possible to remove the b_{10} and a_{10} mulipoles from the magnets we would gain $\approx 1.6\sigma$ in the dynamic aperture. There are no other significant improvements until we remove the b_6 and a_6 from the magnets.

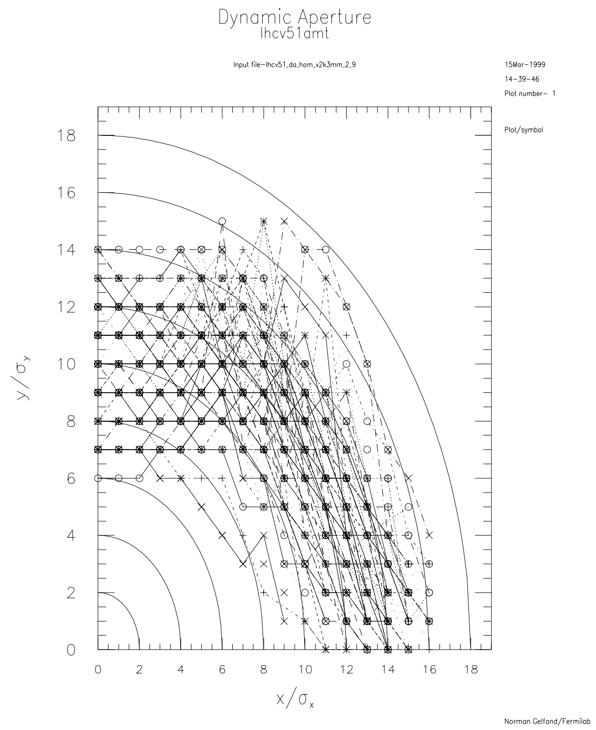

Figure 1: Stable Points for 100 Different Sets of Multipole Errors.

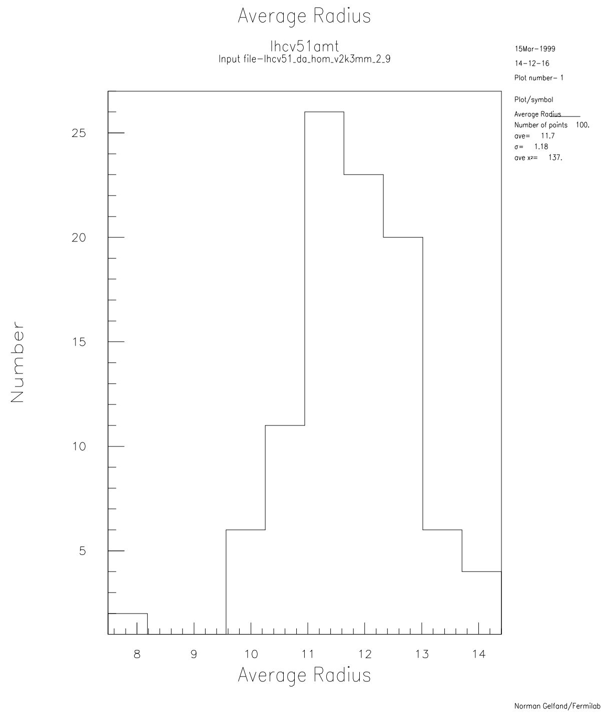

Figure 2: Distribution of the Radius for the Stable Points

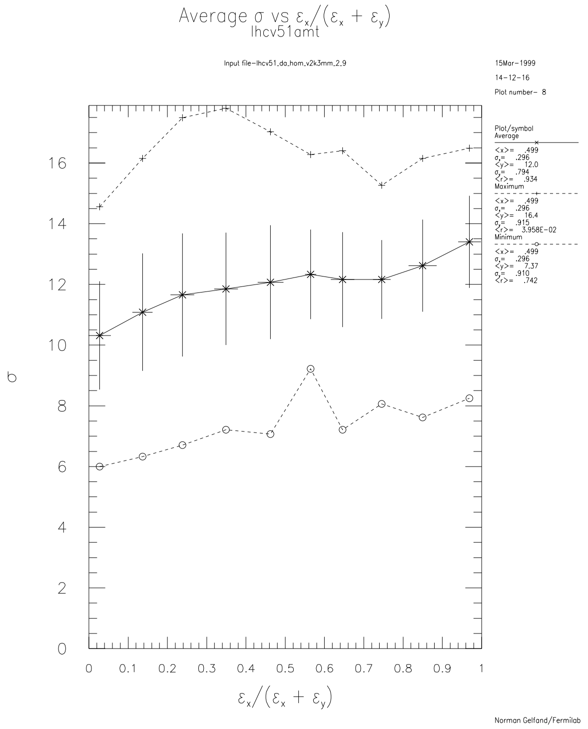

Figure 3: Dynamic Aperture vs $\epsilon_x/(\epsilon_x + \epsilon_y)$
The upper and lower curves are the maximum and minimum values coming from the different sets of the moments.

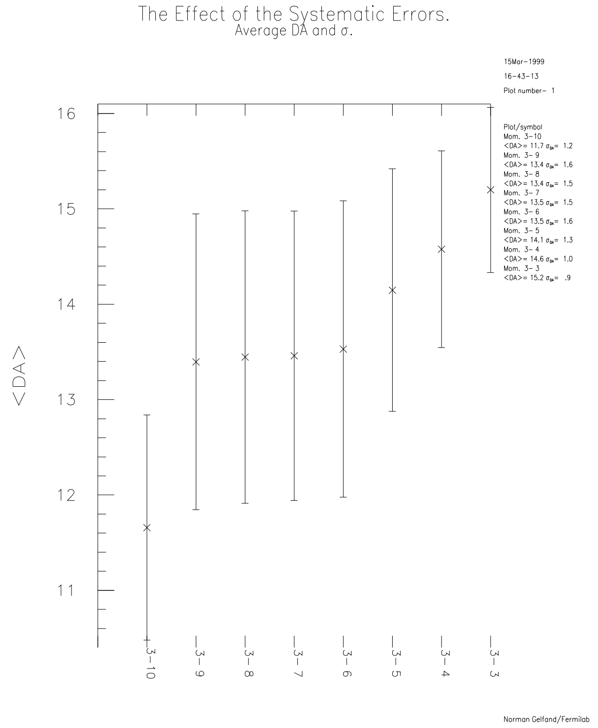

Figure 4: The Variation in the Dynamic Aperture with the Order of the Multipoles

Table 3
The Effect of Removing High Order Multipoles on the Dynamic Aperture

Lowest Order (CERN)	Highest Order (CERN)	Average Dynamic Aperture	Variance
3	10	11.659	1.181
3	9	13.396	1.551
3	8	13.446	1.533
3	7	13.459	1.518
3	6	13.530	1.554
3	5	14.147	1.272
3	4	14.576	1.032
3	3	15.199	.867

The variance of the distribution of the dynamic apertures decreases when the octupole contribution to the magnetic field is removed. The octupole moment contributes to an amplitude dependent tune shift. A smaller random octupole moment would reduce the spread in the dynamic aperture seen in Figure 1 and Figure 2.

4 CONCLUSION

With the existing error tables there is a large variation in the computed dynamic aperture depending on the particular set of harmonics coefficents used for the HGQ. This suggests a need to reduce the random component of the magnet errors in order to have a greater degree of confidence that the LHC will perform satisfactorily.

The dynamic aperture appears sensitive to the highest order multipole specified in the reference tables. Tracking has also been done with errors in all magnets derived from an earlier version of the Fermilab error table. This error table included estimates for the moments up to b_{14} and a_{14}. Those calculations showed a decrease in the dynamic aperture when the moments beyond b_{10} were included. It is important to check that the dynamic aperture is not sensitive to the multipoles which will present in the magnets but, because they are not defined, are not included in the calculations described here.

5 REFERENCES

[1] A. Russell, private communication.

[2] The two middle magnets are used as a single optical element.

[3] The uncertainty in the average multipoles, db_n and da_n were not included in the calculations.

[4] σ_x and σ_y are defined by the values of the lattice functions at the launch point and the normalized invariant emittance of the beam. For the LHC $\epsilon_{nx} = \epsilon_{ny} = 3.75$mmmr.

[5] The radius of a stable point is defined as
$r = \sqrt{(x/\sigma_x)^2 + (y/\sigma_y)^{2)}}$