Aeroelastic Deflection of NURBS Geometry

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**6th International Conference on
Numerical Grid Generation in Computational Field Simulation
University of Greenwich, Avery Hill Campus, London, UK
July 6th−9th, 1998**

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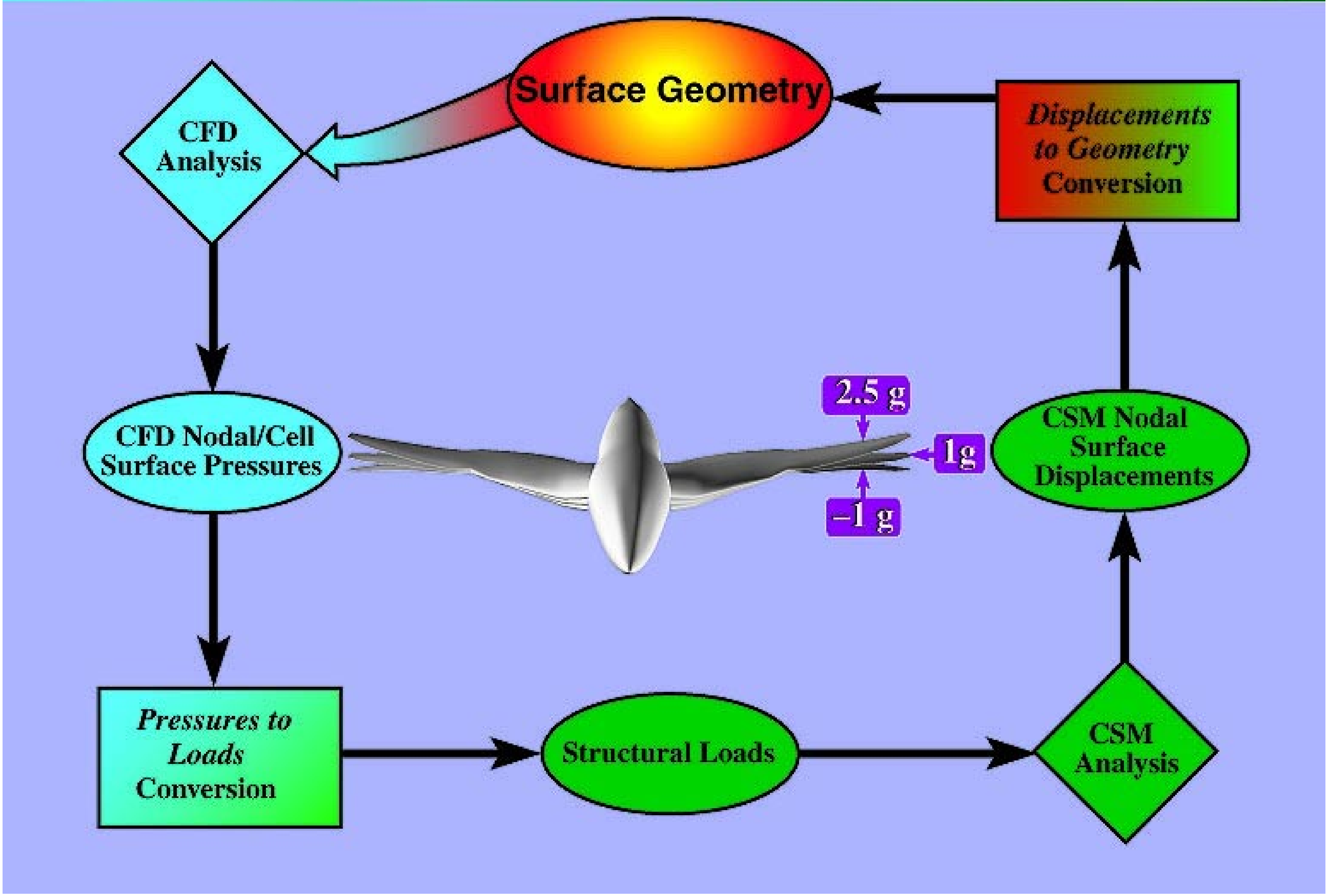
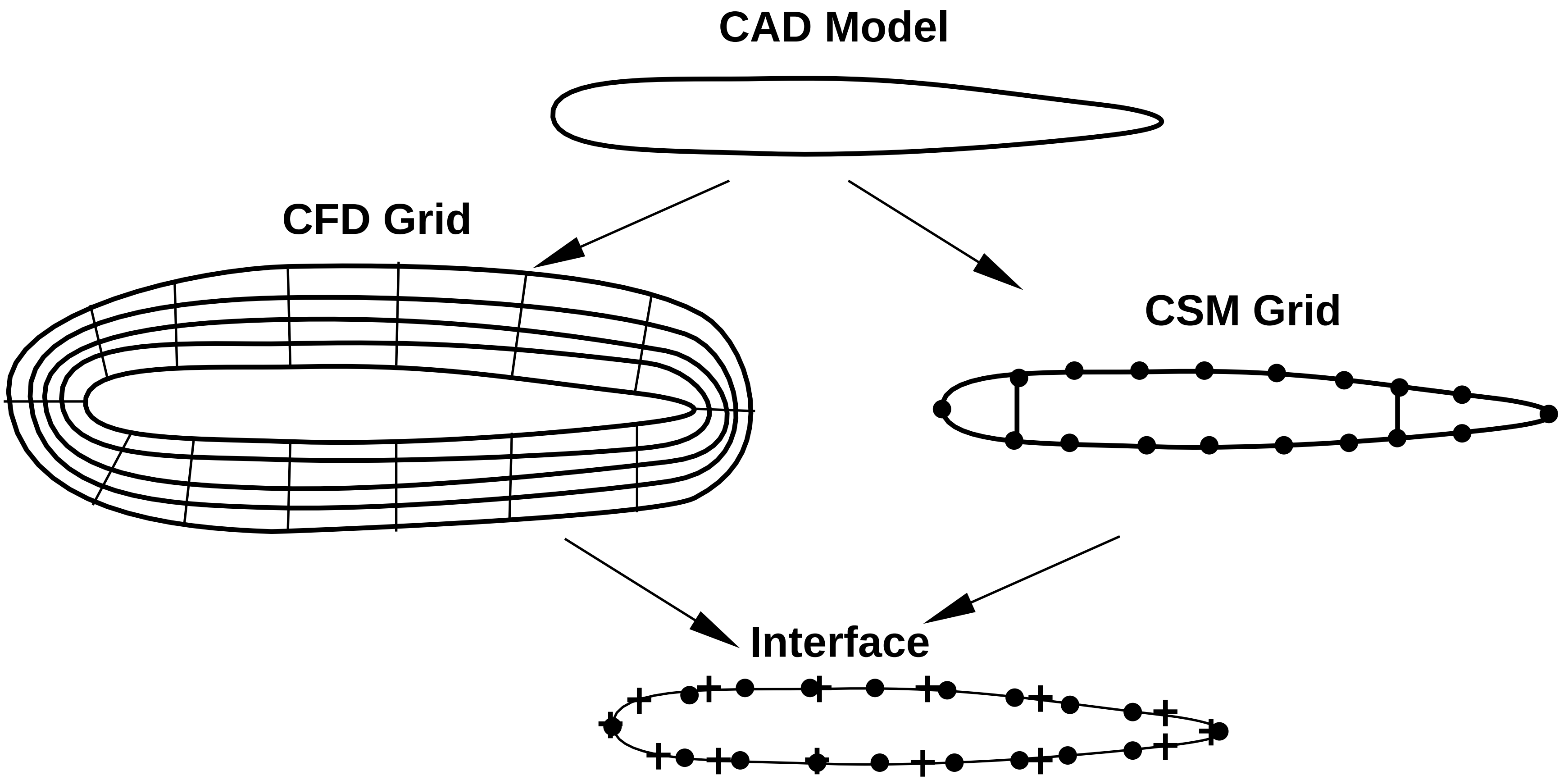
Overview

Integration of CAD systems in an aircraft design process requires manipulation of CAD−based geometry. One such manipulation is the aeroelastic coupling between structural and aerodynamics analyses.

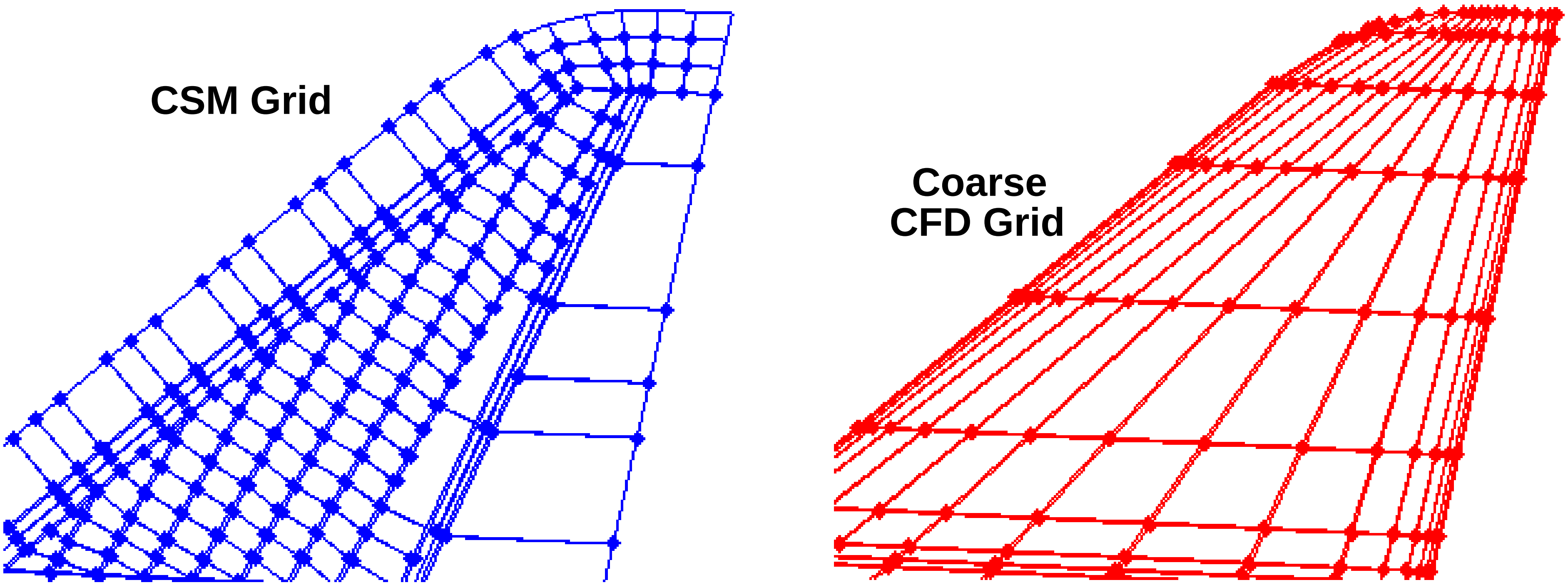
Consistent Aero/Structural/Geometry Coupling



Grid−Based Aeroelastic Coupling

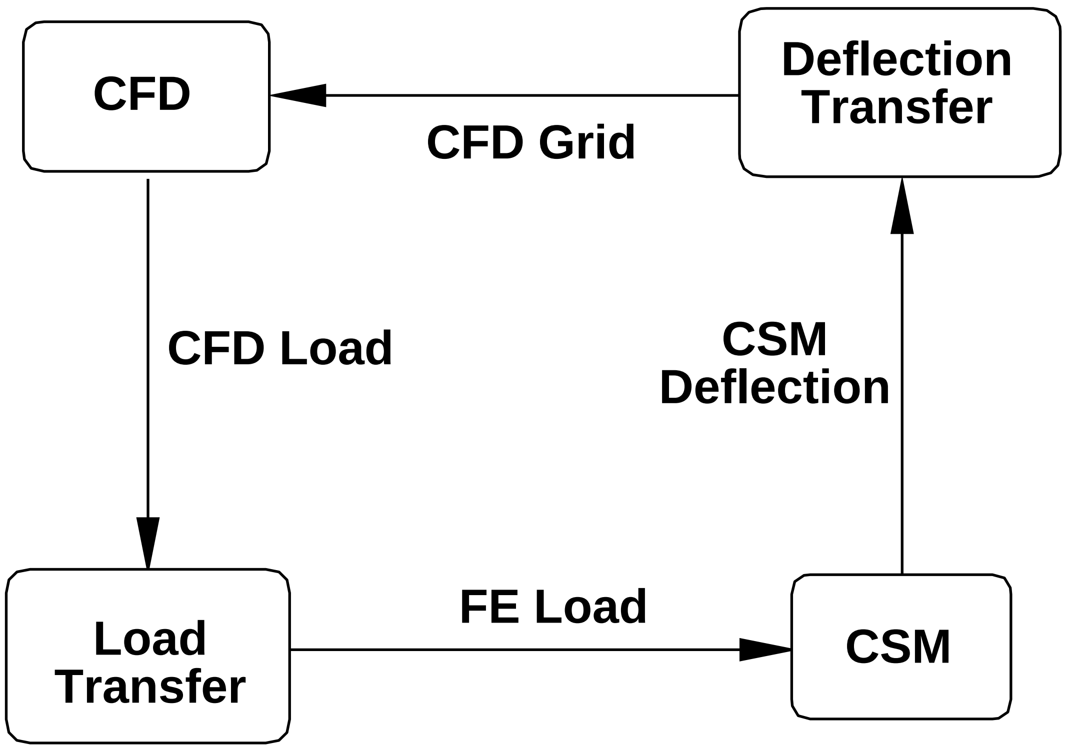


Grid−Based Aeroelastic Coupling



Grid−Based Aeroelastic Coupling

- Ignore x− and y−components of deflection vector
- Assume z−component of deflection to be a function of x and y , D = D(x, y)
- Interpolate z−component of deflection directly from CSM grid to CFD grid



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Grid−Based Interpolation Techniques

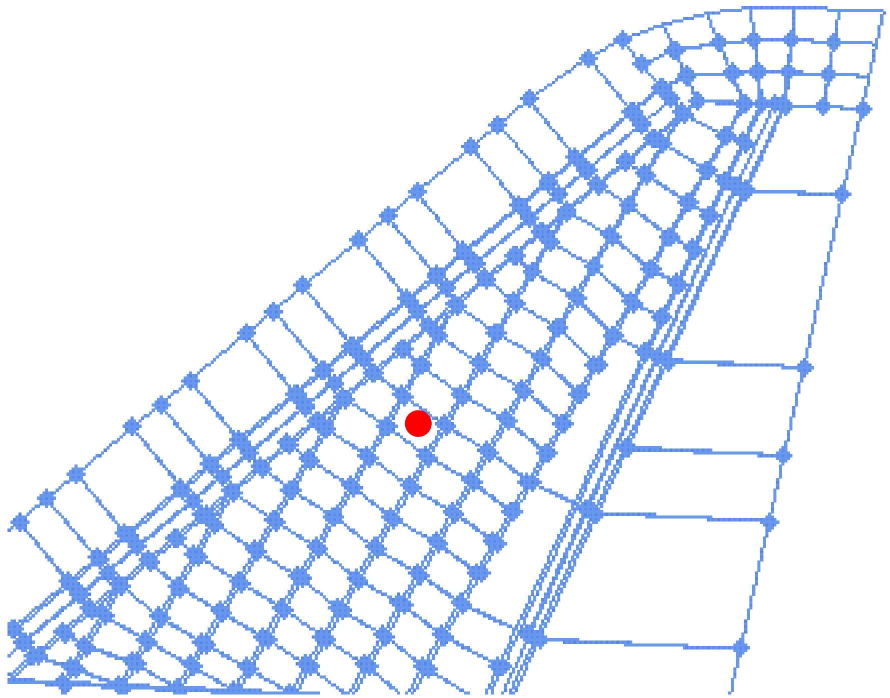
- o Global Interpolation Techniques
  - o Explicit Scatter Data Interpolation (Shepard)
  - o Implicit Scatter Data Interpolation
    - Infinite−Plate Spline (NASTRAN, ASTROS)
    - Thin−Plate Spline
    - Multiquadric−Biharmonic
  - o Finite−Plate Spline (ENSAERO)
- o Local Interpolation Technique
  - o Inverse Isoparametric Mapping
- o Piecewise Interpolation Technique
  - o NUBS as Implemented by Smith et al.
- o Piecewise Least−Squares Using NURBS

Explicit Scatter Data Interpolation
(Originally Proposed by Shepard)

$$D(x, y) = \frac{\sum d_i W_i(x, y)}{\sum W_i(x, y)}, \; \Sigma \text{ is over all CSM Grid Points}$$

$$W_i(x, y) = \left(|R - R_i|^p + \varepsilon \right)^{1/p}$$

If $p = 2$, $W_i(x, y)$ = distance



Implicit Scatter Data Interpolation

$D(x, y) = \sum F_i \, W_i(x, y)$, $\sum$ is over all CSM Grid Points

$[D_j] = [f(W_{i,j})][F_i]$

Infinite−Plate Spline $W_{i,j} = |R_j - R_i|^2 \mathrm{Ln}\ |R_j - R_i|^2$

Thin−Plate Spline $W_{i,j} = |R_j - R_i|^2 \,\mathrm{Log}\, |R_j - R_i|^2$

Multiquadric−Biharmonic $W_{i,j} = (|R_j - R_i|^2 + c^2)^{1/2}$

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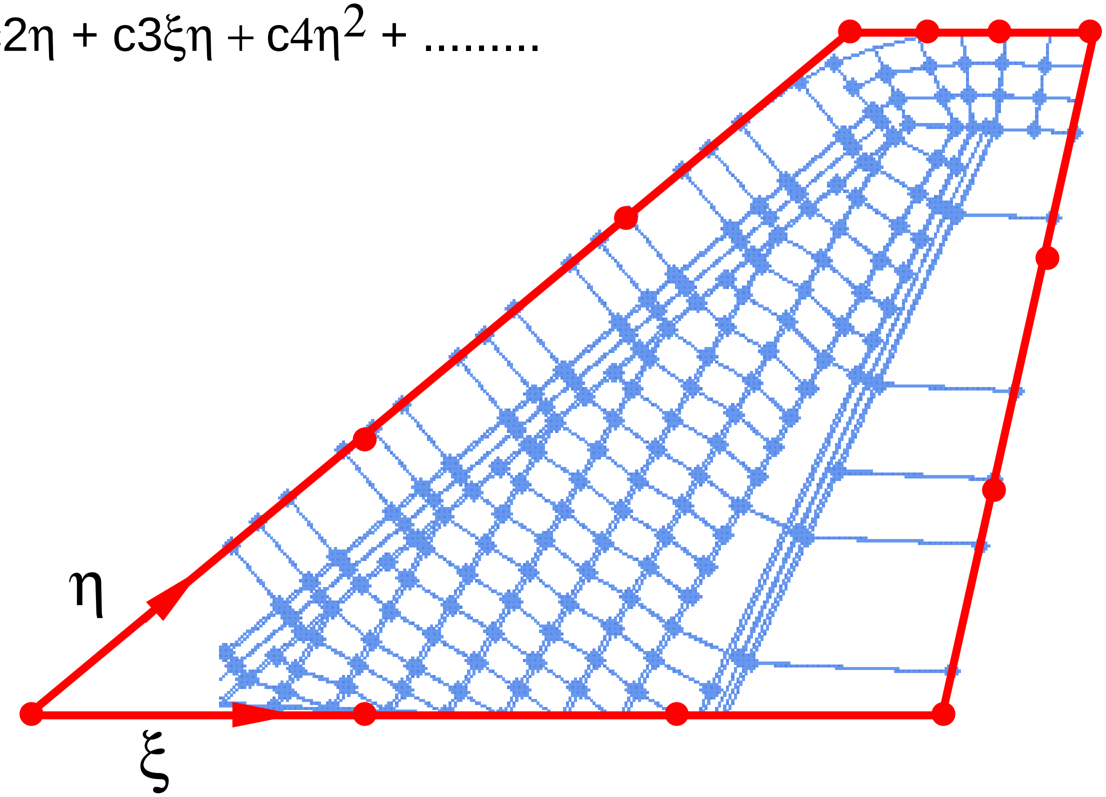
Finite−Plate Spline
(Cubic Serendipity Element)

$D(\xi, \eta) = \sum F_i \, W_i(\xi, \eta)$, $\sum$ is over 12−points of cubic element

$[D_j] = [W_{i,j}][F_i]$

$W_{i,j} = c0 + c1\xi + c2\eta + c3\xi\eta + c4\eta^2 + \ldots\ldots\ldots$



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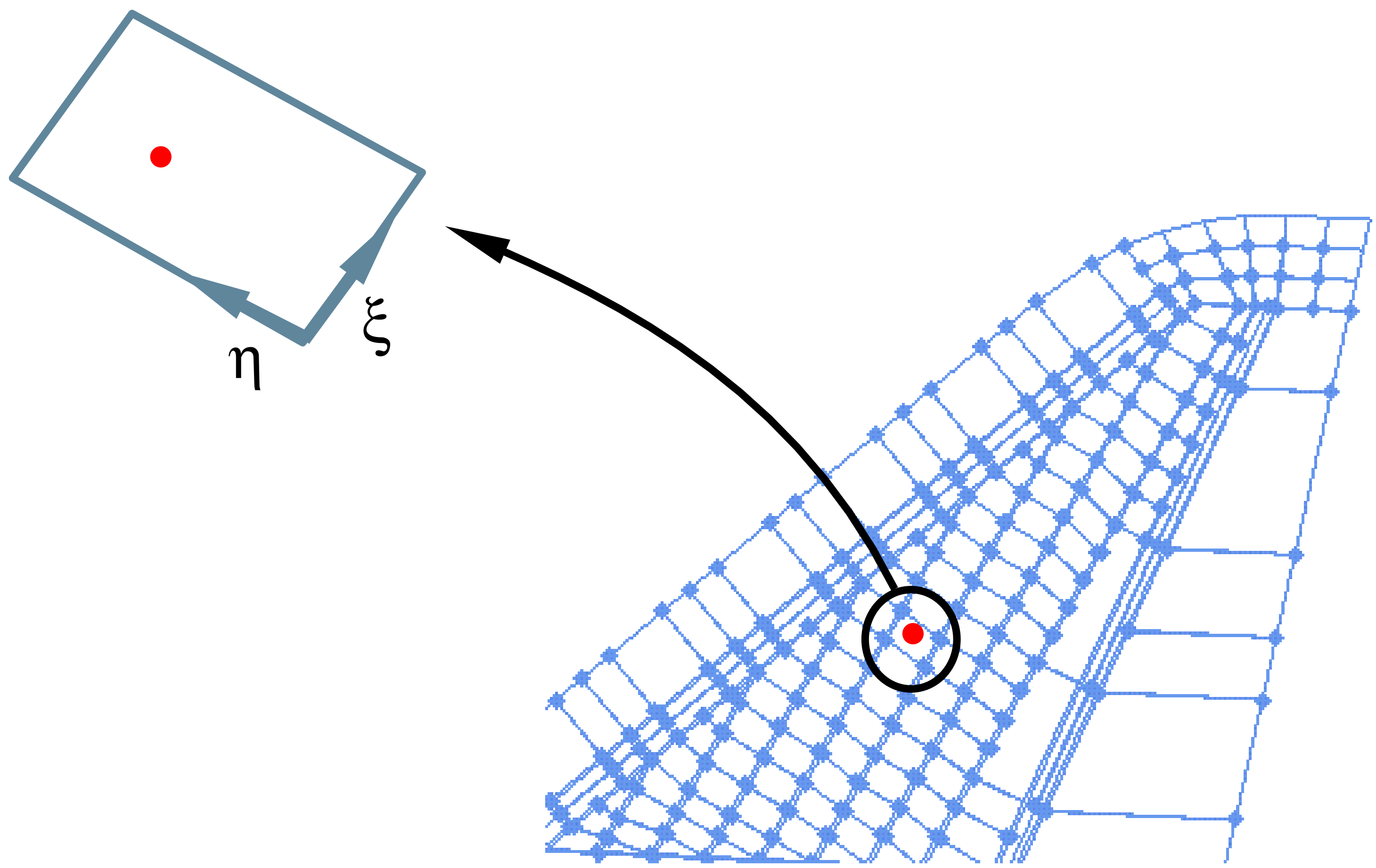
Inverse Isoparametric Mapping

$D(\xi, \eta) = \sum F_i W_i(\xi, \eta)$, $\sum$ is over points forming the CSM element



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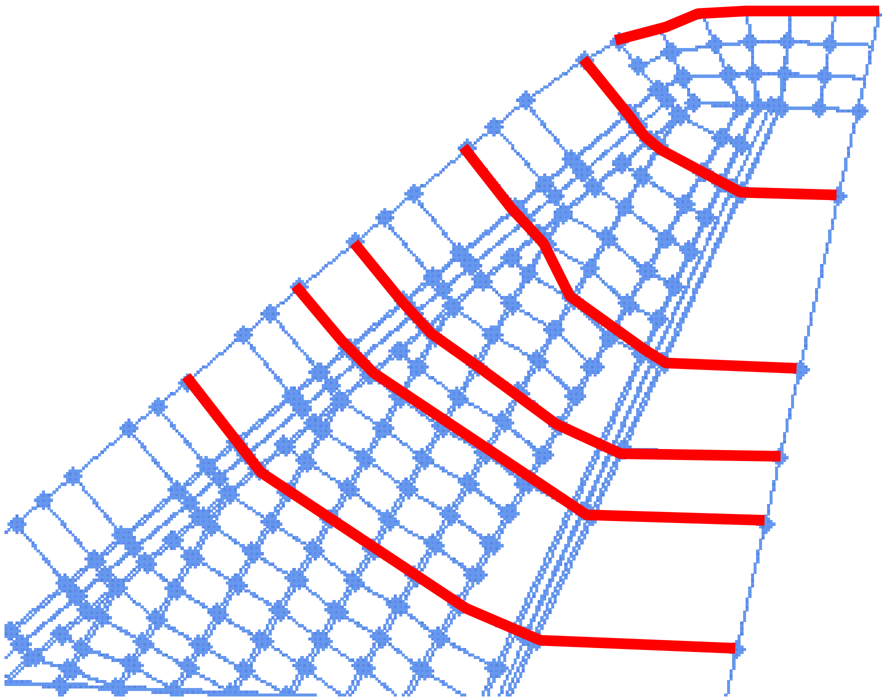
NUBS as implemented by Smith et. al.

- **Select a set of points that forms a complete cross−section**
- **Create a structured grid from the cross−sections**
- **Interpolate a NUBS surface through the resulting structured grid**



Representing Deflection with a NURBS Surface

Steps:

1. **Extract leading and trailing edges from CAD definition (i. e., NURBS curves)**
2. **Find normalized local chord (ξ) and span location (η) for all CSM grid points**
3. **Fit a NURBS surface through** $D_n = D(\xi_n, \eta_n)$, $n = 1$**, number of CSM points**
4. **Find normalized local chord (ξ) and span location (η) for all CFD grid points**
5. **Evaluate deflection for all CFD grid points**

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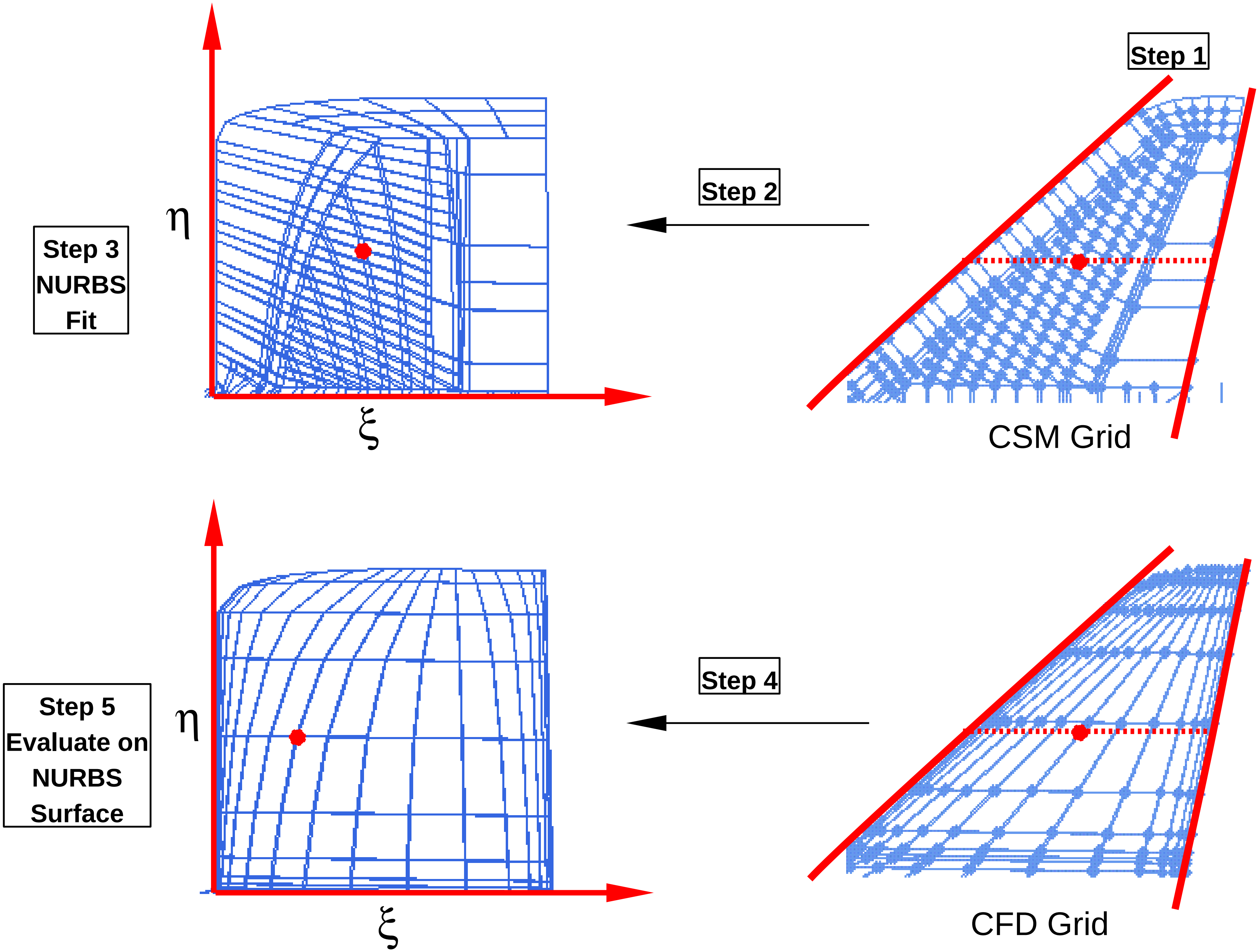

Step 1
Step 2
Step 3
NURBS
Fit
η
ξ
CSM Grid
Step 4
Step 5
Evaluate on
NURBS
Surface
CFD Grid

Step 3: Fit a NURBS Surface, $D(\xi, \eta)$, Through Deflection

$$D(\xi, \eta) = \frac{\sum\sum B_{i,p}(\xi)\, B_{j,q}(\eta)\, W_{i,j}\, R_{i,j}}{\sum\sum B_{i,p}(\xi)\, B_{j,q}(\eta)\, W_{i,j}}$$

NURBS in homogeneous coordinates,

$$d(\xi, \eta) = \sum\sum B_{i,p}(\xi)\, B_{j,q}(\eta)\, r_{i,j}$$

Where

$$d(\xi, \eta) = D(\xi, \eta) \sum\sum B_{i,p}(\xi)\, B_{j,q}(\eta)\, W_{i,j}$$

$$r_{i,j} = W_{i,j}\, R_{i,j}$$

$B_{i,p}(\xi)$	i−th B−Spline basis function of order p
$W_{i,j}$	Weights
$R_{i,j}$	control points

Cont. Step 3: Fit a NURBS, $D(\xi, \eta)$, surface through deflection

$$d(\xi, \eta) = \sum\sum B_{i,p}(\xi)\, B_{j,q}(\eta)\, r_{i,j}$$

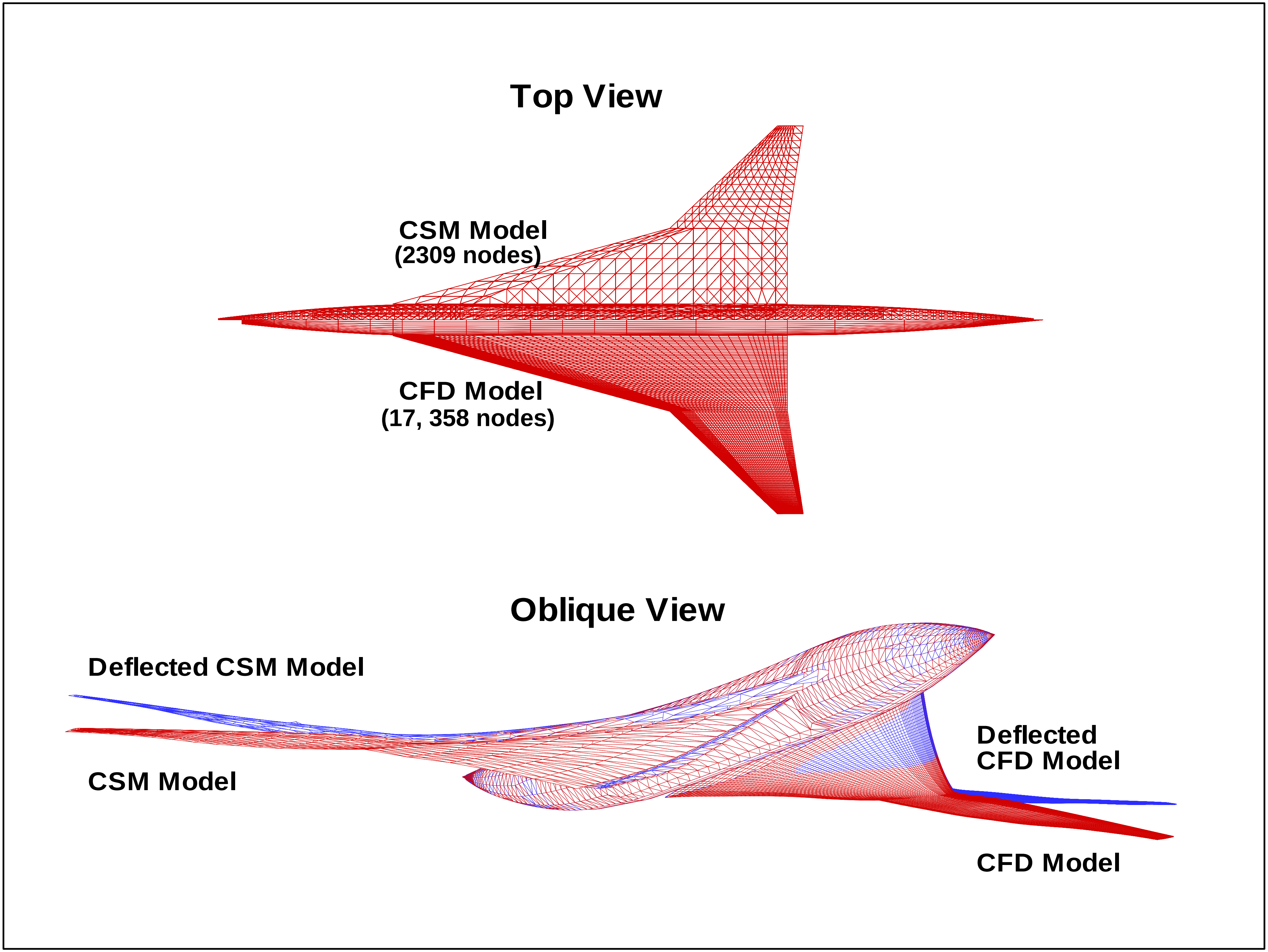
$$d_n = d(\xi_n, \eta_n) = \sum\sum B_{i,p}(\xi_n)\, B_{j,q}(\eta_n)\, r_{i,j}, \quad \underset{N\mathrm{x}1}{[d]} = \underset{N\mathrm{x}(I\mathrm{x}J)}{[B]}\ \underset{(I\mathrm{x}J)\mathrm{x}1}{[r]}$$

If $N = I\mathrm{x}J$, $[r] = [B]^{-1}[d]$

If $N > I\mathrm{x}J$, $[r] = \left[[B]^T[B]\right]^{-1}[B]^T\,[d]$

Band Width of $[B] = p \times J + q + 1$

[B] may be rank deficient



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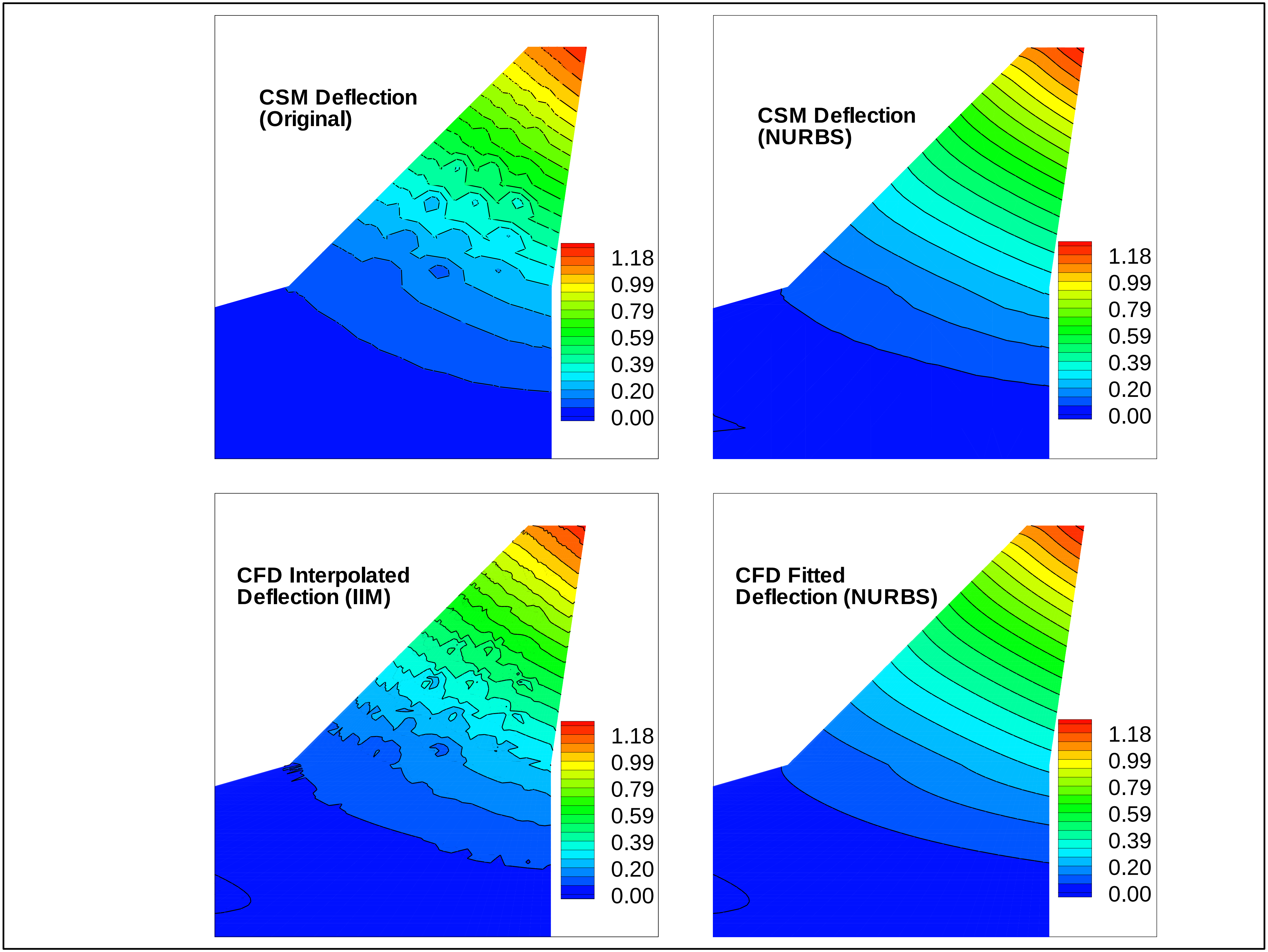

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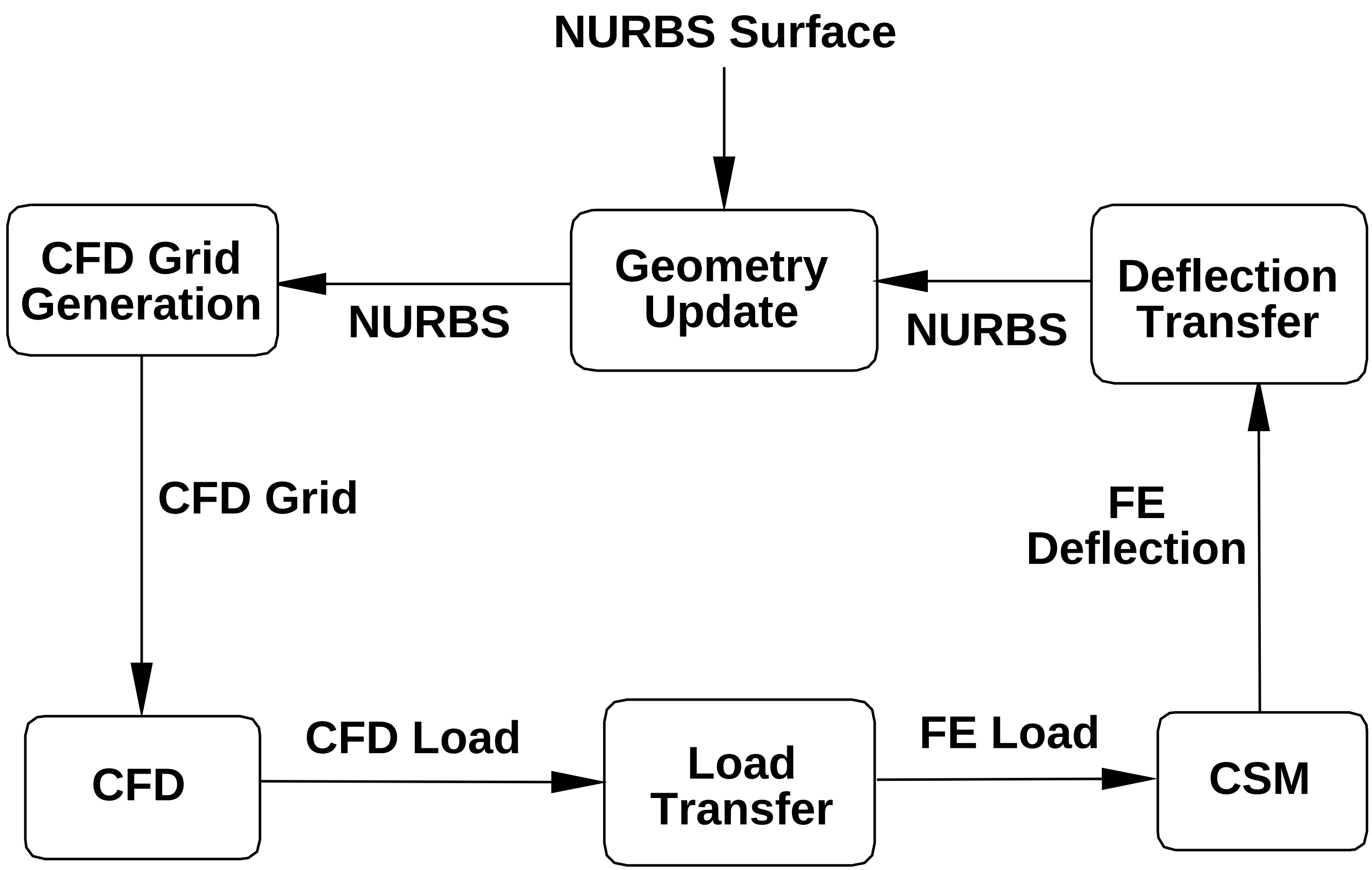
CAD−Based Aeroelastic Coupling



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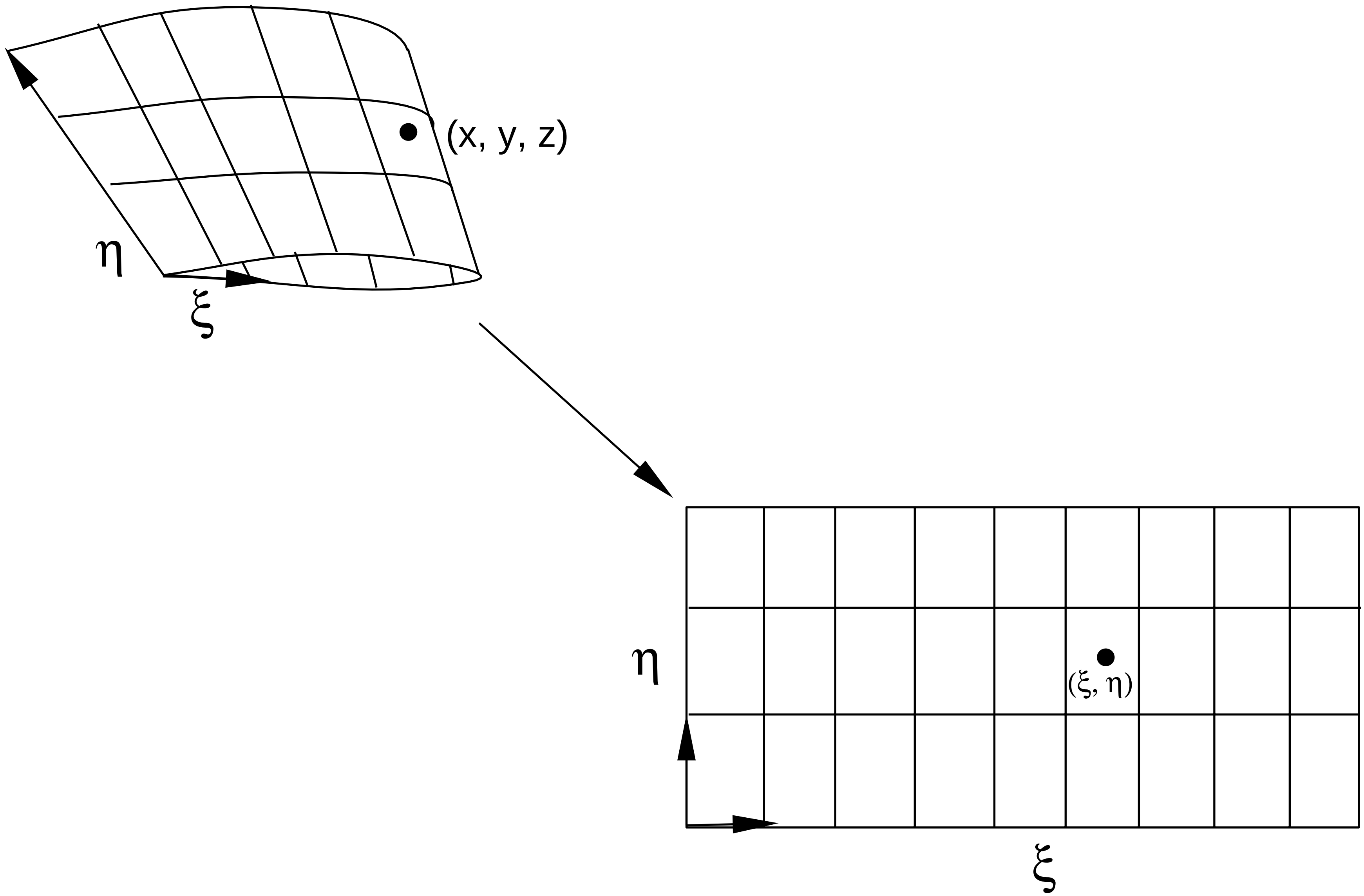
Projecting Point to NURBS Surface



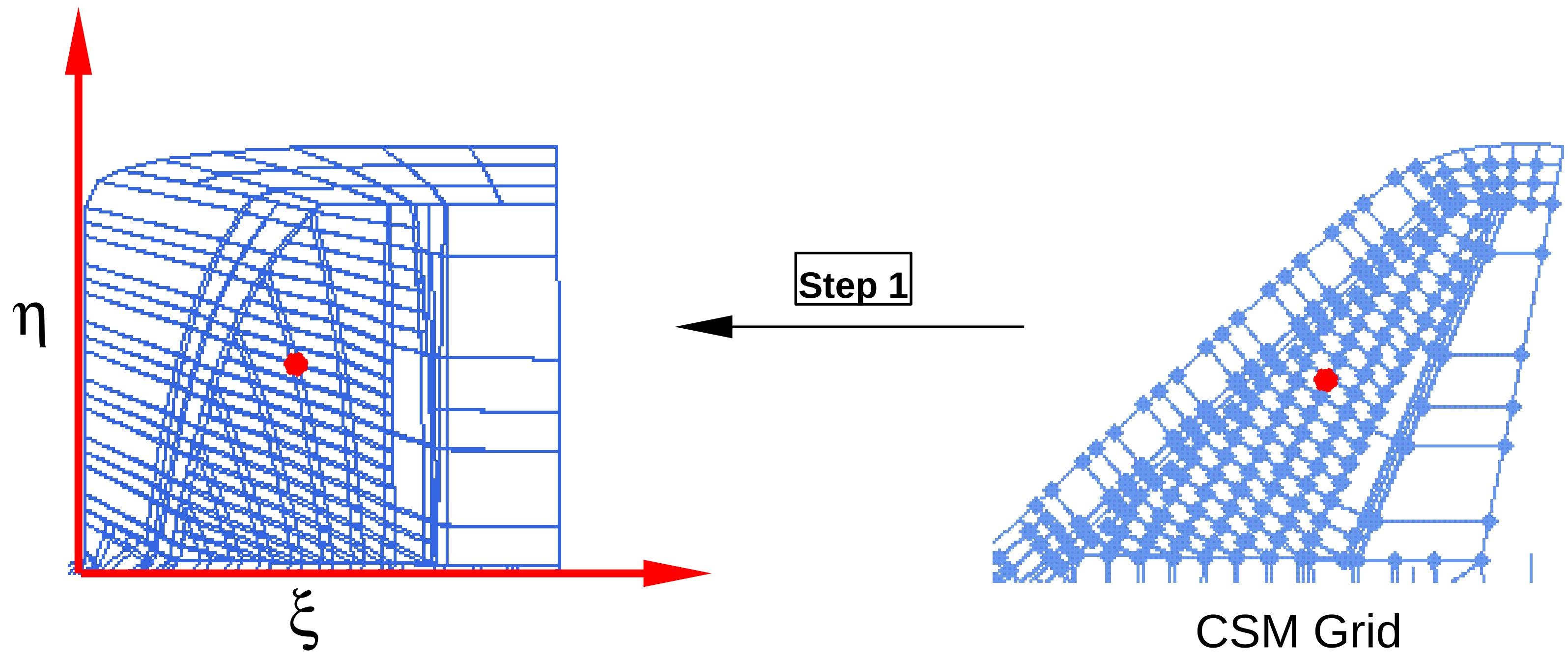
CAD−Based Aeroelastic Coupling

1. **Project each CSM grid point to the CAD geometry, and compute the parametric coordinates (**ξ, η**) on the NURBS surface**

2. **Fit a NURBS surface through** $D_n = D(\xi_n, \eta_n)$

3. **Resolve the knot vector distribution between** $D(\xi, \eta)$ **and CAD geometry**

4. **Add deflection control points to original NURBS surface,** $R_{new} = R(\xi, \eta) + D(\xi, \eta)$



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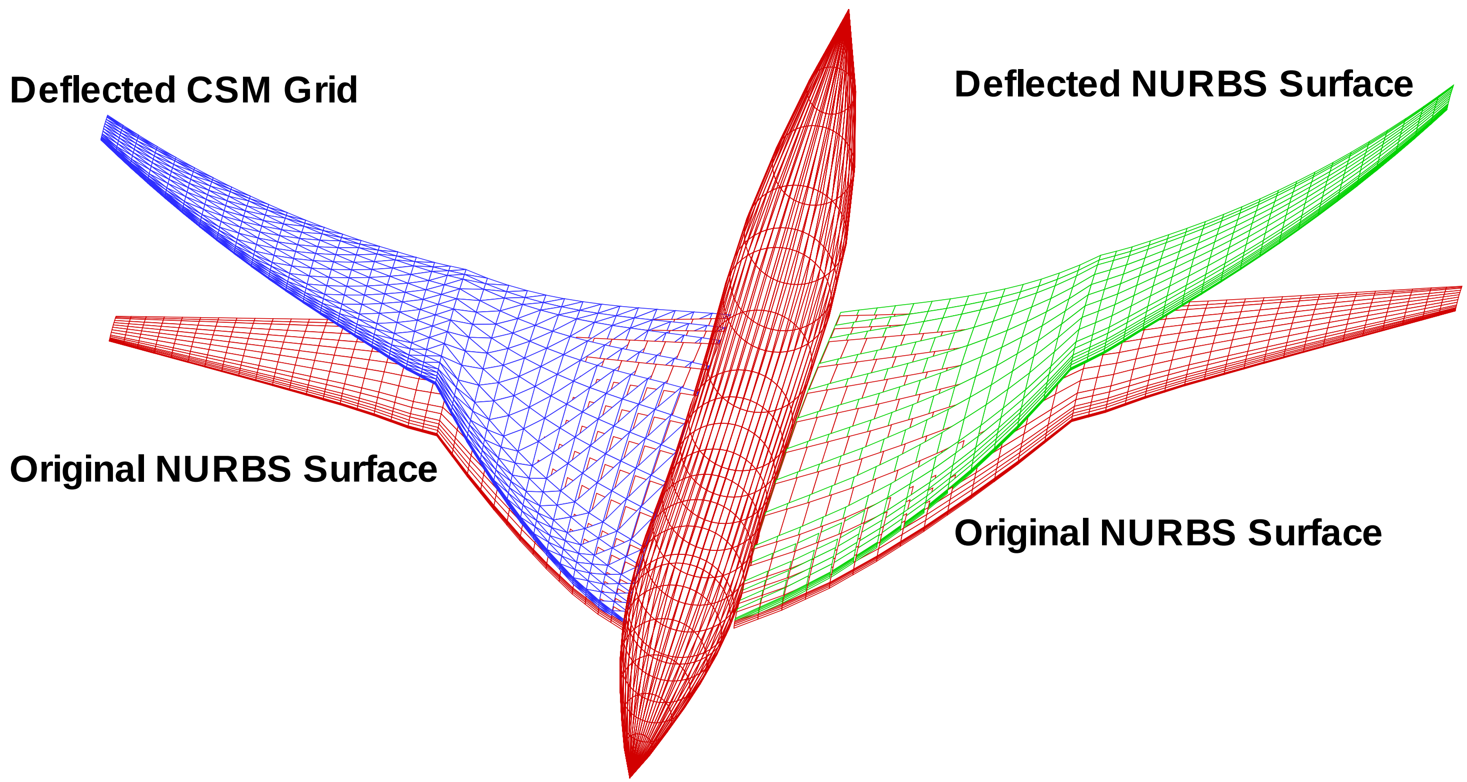
CAD−Based Aeroelastic Coupling



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Summary

- Reviewed existing aeroelastic coupling techniques
  - Simple & 2D
  - No connection to CAD

- Presented a grid−based aeroelastic coupling using NURBS
  - Ability to use the entire data set
  - Ability to control the trade−off between accuracy and smoothness
  - Piecewise representation with guaranteed continuity

- Presented a CAD−Based aeroelastic coupling
  - Can be incorporated in a CAD−based environment

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