Gauge Theories & Particle Physics

Physics 539 | Spring Semester 1997 | Chris Quigg

February 13, 1997

Recommended Reading:

Okun, Chapter 2.
Gauge Theories, Chapter 3.
Howard Georgi, *Weak Interactions and Modern Particle Theory,* (Benjamin, Menlo Park, California, 1984), Chapter 1: "Symmetries in Field Theory."
On the continuation to imaginary time: J. Schwinger, *Proc. Nat. Acad. Sci.* **44**, 956 (1958).

Cultural Reading:

Akira Tonomura, "Electron Holography: A New View of the Microscopic," *Phys. Today* **43**, 22 (April, 1990).
Robert Mills, "Gauge Fields," *Am. J. Phys.* **57**, 493 (1989).
On geometrical phases: M. V. Berry, "Anticipations of the Geometric Phase," *Phys. Today* **43**, 34 (December, 1990).

Deep Background:

T. P. Cheng and L.-F. Li, "Resource Letter: GI-1 Gauge Invariance," *Am. J. Phys.* **56**, 586 (1988).
Murray Peshkin and Akira Tonomura, *The Aharonov-Bohm Effect,* (Springer-Verlag, New York, 1989).
Quantum Coherence, Proceedings of the International Conference on Fundamental Aspects of Quantum Theory to Celebrate Thirty Years of the Aharonov–Bohm Effect, edited by J. S. Anandan (World Scientific, Singapore, 1990).
J. W. Zwanzinger, M. Koenig, and A. Pines, "Berry's Phase," *Ann. Rev. Phys. Chem.* **41**, 601 (1990).
Alfred Shapere and Frank Wilczek, *Geometrical Phases in Physics,* (World Scientific, Singapore, 1989).

Problems (due February 20, 1997)

7. Use the requirement that the Lagrangian be invariant under a continuous symmetry to deduce the conserved quantity that corresponds to a particular transformation. Show that invariance under (i) translations in space, (ii) translations in time, (iii) spatial rotations implies conservation of (i) momentum, (ii) energy, (iii) angular momentum.

8. Making use of the Dirac equation, show that the most general parity-conserving form for the electromagnetic current of the proton is

$$J_\mu \sim \bar{u}(p')[\Gamma_1(q^2)\gamma_\mu + \Gamma_2(q^2)i\sigma_{\mu\nu}q^\nu + \Gamma_3(q^2)q_\mu]u(p),$$

where $\sigma_{\mu\nu} \equiv (i/2)\,[\gamma_\mu, \gamma_\nu]$ and $q \equiv p - p'$. What are the consequences of current conservation, $\partial^\mu J_\mu = 0$?

9. Calculate the differential cross section in the laboratory frame for elastic electron-proton scattering (a) for a structureless proton (*i.e.*, a Dirac particle); (b) for a real proton (using the results of Problem 8).

10. (a) Consider a nonrelativistic particle with charge q moving along the axis of a cylindrical Faraday cage connected to an external generator, which causes the potential $V(t)$ on the cage to vary with time only when the particle is well within the cage. Show that, if the wave

function $\psi^0(\mathbf{x},t)$ is a solution of the Schrödinger equation for $V(t) \equiv 0$, the solution when the generator is operating will be $\psi(\mathbf{x},t) = \psi^0(\mathbf{x},t)e^{iS/\hbar}$, where $S = -\int^t dt'\, qV(t')$.

(b) Now suppose that a single coherent beam of charged particles is split into two parts, each of which is allowed to pass through its own long cylindrical cage of the kind just described. On emerging from the Faraday cages the beams are recombined and the resulting interference pattern is observed. The beam is chopped into bunches that are long compared with the wavelength of an individual particle but short compared with the Faraday cages. The potentials on the two cages vary independently, but are nonzero only when a bunch is well within the tubes. This ensures that the beam traverses a time-varying potential without experiencing electric or magnetic forces. Describe how the interference pattern depends upon the applied voltages. [Reference: Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485 (1959).]

11. If baryon number is absolutely conserved, the conservation law may be a consequence of a global phase symmetry like that of electromagnetism, with the electric charge replaced by baryon number. (a) How would Newton's law of gravitation be modified if the baryonic phase symmetry were a *local* gauge invariance?

    (b) In view of the close equality of inertial and gravitational masses imposed by the Eötvös experiment [P. G. Roll, R. Krotkov, and R. H. Dicke, *Ann. Phys. (NY)* **26**, 442 (1967)], what can be said about the strength of a hypothetical gauge interaction coupled to the baryon current? [Reference: T. D. Lee and C. N. Yang, *Phys. Rev.* **98**, 150 (1955).]

    (c) How would a gauge boson γ_B coupled to baryon number affect the properties of the $b\bar{b}$ resonances? For part (c) only, allow for the possibility that γ_B has a nonzero mass. [Reference: D. Bailey and S. Davidson, *Phys. Lett.* **B348**, 185 (1995).]

    For another recent examination of the possibility that a light $U(1)$ gauge boson is coupled to baryon number, see C. D. Carone and H. Murayama, *Phys. Rev. Lett.* **74**, 3122 (1995). On the possibility of a massless gauge boson coupled to lepton number, see L. B. Okun, *Yad. Fiz.* **10**, 358 (1969) [English translation: *Sov. J. Nucl. Phys.* **10**, 206 (1969)]. For a critical reëxamination of the limits, see S. I. Blinnikov, A. D. Dolgov, L. B. Okun, and M. B. Voloshin, *Nucl. Phys.* **B458**, 52 (1996).

12. Outline a "three-neutrino experiment" to establish that a neutral, penetrating beam of ν_τ materializes into τ upon interacting in matter. [For background, look at the first two-neutrino experiment, J. Danby, *et al.*, *Phys. Rev. Lett.* **9**, 36 (1962). See also the Nobel Lectures of "Murder, Inc.," (as they called themselves), Mel Schwartz, J. Steinberger, and Leon M. Lederman, reprinted in *Rev. Mod. Phys.* **61**, 527, 533, 547 (1989).] What would provide a copious source of ν_τ? What energy would be advantageous for the detection of the produced τ? What characteristics would be required of the detector? What are the important backgrounds, and how would you handle them? Some information about a three-neutrino experiment in preparation at Fermilab can be found at `http://fn872.fnal.gov`.

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Course information, including class schedules and assignments, is available on World Wide Web. Point your web client to `http://lutece.fnal.gov/Princeton/`.