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Addendum to the report on $K^+ \to \pi^+\gamma\gamma$

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1 ChPT at $O(p^6)$

Tamaki used the expression "the ChPT at $O(p^6)$" in the report. Exactly speaking that was incorrect.

The unitarity corrections (UC) plus the vector meson contribution (if exists) in the next-to-leading order at $O(p^6)$ are not a full calculation of the ChPT at $O(p^6)$. In the real case, there is no complete theoretical study at $O(p^6)$ yet [2]. While $K \to \pi\gamma\gamma$ in the leading order: the ChPT at $O(p^4)$ is calculated exactly [3], the ChPT at $O(p^6)$ contains many parameters in the theory and cannot be fully calculated from scratch. What theorists do is to pick up contributions that are supposed to be dominant at $O(p^6)$ and do calculations in a more or less phenomenological way. That was the reason we chose the expressions "with and without unitarity corrections" and "including UC and and without UC", not "the ChPT at $O(p^6)$", in the publication.

2 Final Plot

The final plot for publication (reproduced as Figure 1 in this note) is somehow different from the one in the report (Figure 5:1 in Section 5.1). The Figure 1 is correct; it was turned out that he had made the plot in the report with coarse binning.

There is one event ($P = 215.7352$, $R = 33.39188$, and $E = 115.7036$) located at "just outside the R-E box". This event appeared at the stage of "two-cut failure" study with the 1/3 sample and was understood to be

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[2]See 4.2.2 of G. D'Ambrosio and J. Portolés, Nucl. Phys. B 533 (1998) 494.

[3]The undetermined coupling-constant $\hat{c}$ is needed for $K^+ \to \pi^+\gamma\gamma$, while the $K_L^0 \to \pi^0\gamma\gamma$ amplitude is determined unambiguously without any undetermined coupling constant.

a $K_{\pi 2}$ background (with `itgqualt`$= 1$: the kaon stopped in the I-counter); the event therefore did not appear in the outside-the-box study based on the 2/3 sample. Even if we take this event as a candidate, the S/N value is poor and the likelihood method by T.Junk should provide us with a similar upper limit to the branching ratio.

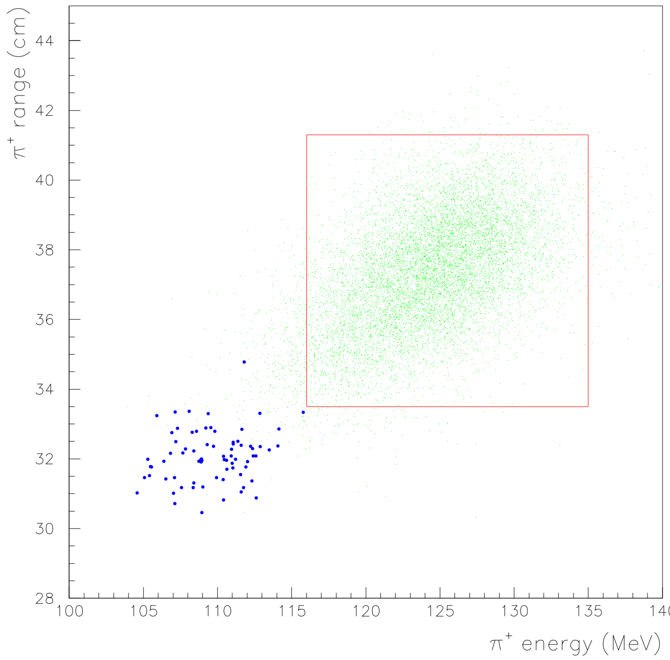

Figure 1: The final plot, used in Figure.3 of the publication.

3 Trigger Acceptance

In the analysis, the lower limit of the momentum box cut is 213 MeV/c. To obtain the partial branching ratio for $P > 213$ MeV/c correctly and avoid the effects due to kinematic resolutions, the following method was used; see Section 5.5 of the report.

1. The Monte Carlo simulation (UMC) generated the $K^+ \to \pi^+\gamma\gamma$ decay in $P > 200$ MeV/c, which started at much lower momentum than the signal region; the "partial" acceptance and single event sensitivity for $P > 200$ MeV/c to each of the assumptions (ChPT including UC, ChPT w/o UC, and Phase Space) were calculated, and then

2. the correction factor to convert them to the ones in $P > 213$ MeV/c, by assuming the π^+ spectrum shape in each assumption, was multiplied. The factors are 1.929 ($\equiv 1/0.5185$) for ChPT including UC, 7.148 ($\equiv 1/0.1399$) for ChPT w/o UC, and 1.811 ($\equiv 1/0.5523$) for Phase Space, respectively.

In the acceptance table for publication (reproduced as Table 1 in this note), the values of the "Trigger" acceptance, in the second row, are the ones after the correction factor was multiplied to the value written in the report.

Acceptance factors	UC	w/o UC	samples
Trigger	0.0666	0.0435	MC
Trigger-counter efficiency	0.936	0.936	K_B
charged-track reconstruction	0.996	0.996	$K_{\mu 2}$
π^+ fiducial cuts	0.984	0.938	MC
π^+ accepted region (P,R,E)	0.912	0.667	MC
π^+ stop without nuclear interaction or decay-in-flight	0.492	0.524	MC
dE/dx and kinematic cuts	0.537	0.537	$K_{\pi 2}$, π_{scat}
$\pi^+ \to \mu^+ \to e^+$ cuts	0.349	0.349	π_{scat}
γ reconstruction and fiducial cuts	0.530	0.492	MC, $K_{\pi 2}$
γ selection cuts	0.216	0.177	MC, $K_{\pi 2}$
Other cuts on beam and target	0.507	0.507	$K_{\mu 2}$
Total acceptance	2.99×10^{-4}	1.10×10^{-4}	

Table 1: Acceptance factors for the $K^+ \to \pi^+\gamma\gamma$ decay in the kinematic region $P > 213$ MeV/c, for $\hat{c} = 1.8$ including unitarity corrections ("UC") and for $\hat{c} = 1.6$ without the corrections ("w/o UC"), and the samples used to determine them.

4 Erratum to the acceptance tables for "ChPT w/o UC" and "Phase Space"

Tamaki found [4] an unfortunate mistake in his forpaw-scripts for acceptance calculations: the π^+ total energy (`etot`) of the UMC events for "ChPT w/o

[4] The motivation was to answer the question raised by S. Kettell: "Is it really true that the π^+ nucint changes from 0.492 to 0.578 without unitarity corrections ? Such a

UC" and for "Phase Space" when π^+ nuclear interaction and decay in flight ("NIDIF") turned on was incorrect. The Monte-Carlo based acceptance factors $A_{fidCT}(NIDIFon)$ and A_{nidif} (exactly speaking, the acceptance of the `PGGBOX` cut in the calculation of $A_{fidCT}(NIDIFon)$) were wrong, and the MC based acceptance factors for photons, A_{fidBV} and A_{PV-MC} should also be changed accordingly because the MC events which survived the PGGBOX cut were used in the calculation. The problem in the scripts was, for these cases the stopping-counter muon energy (`QSUBMU`, 3.04MeV) was added to `etot` by mistake.

Numbers in the following sections should be corrected for.

- 5.2 Acceptance Factors
  (and Tables 5.3, 5.4)
- 5.5 Branching ratio for $K^+ \to \pi^+\gamma\gamma$
- C.1 Monte Carlo Based Acceptance
  (and Tables C.5, C.6, C.8, C.9, C.11, C.12)
- C.5 Categorization of the Acceptances
  (and Tables C.22, C.23)

Concerning the numbers in the publication, there were changes from some old versions of the draft as:

- **ChPT w/o UC**
  - Acceptance: $(1.18 \pm 0.04) \times 10^{-4} \Longrightarrow \underline{(1.10 \pm 0.04) \times 10^{-4}}$
  - SES: $(9.40 \pm 0.45) \times 10^{-9} \Longrightarrow \underline{(1.01 \pm 0.05) \times 10^{-8}}$
  - Upper Limit: $2.1 \times 10^{-9} \Longrightarrow \underline{2.3 \times 10^{-9}}$
- **Phase Space**
  No number was changed.

The tables 5.2 (which did not have mistakes), 5.3, and 5.4 after the corrections are given in this note as Tables 2, 3, and 4, respectively; the Appendix C "Details of Acceptance Measurements" after the corrections is provided as a separate note.

large change over the region from p=213 to p=227 ?" After fixing the problem, the π^+ nucint now changes from 0.492 to $\underline{0.524}$ (see Table 1) without unitarity corrections, which is reasonable.

5 Categorization of the Acceptances

In order to produce the acceptance table for publication,

- "π^+ Fiducial Cuts" in the report (Tables C.4, C.5, and C.6, from the `LAY1617` cut to the `PGGBOX` cut) was further categorized into:
  - new "π^+ fiducial cuts": from the `LAY1617` to `ZUTOUT` cuts,
    - ChPT including UC:
      $0.97412 \pm 0.00054_{(NIDIFon)}$ and $0.98439 \pm 0.00030_{(NIDIFoff)}$
    - ChPT w/o UC:
      $0.91443 \pm 0.00215_{(NIDIFon)}$ and $0.93803 \pm 0.00139_{(NIDIFoff)}$
    - Phase Space:
      $0.97452 \pm 0.00066_{(NIDIFon)}$ and $0.98387 \pm 0.00038_{(NIDIFoff)}$

    and
  - "π^+ accepted region (P,R,E)": the `PGGBOX` cut,
    - ChPT including UC:
      $0.88559 \pm 0.00109_{(NIDIFon)}$ and $0.91225 \pm 0.00069_{(NIDIFoff)}$
    - ChPT w/o UC:
      $0.63914 \pm 0.00386_{(NIDIFon)}$ and $0.66742 \pm 0.00280_{(NIDIFoff)}$
    - Phase Space:
      $0.88464 \pm 0.00136_{(NIDIFon)}$ and $0.91172 \pm 0.00087_{(NIDIFoff)}$

  to isolate and present the acceptance factor of the π^+ signal region.
  Reminder: the factors with NIDIF off are on the acceptance table.

- "Extra Photon Veto cuts", which was a name of more or less misleading in the report (Tables C.10, C.11, and C.12, from the `HEX` cut to the `TGPVCUT` cut) was renamed as "γ selection cuts".

Definition	Symbol	Acceptance	Data
Trigger	$A^{\pi^+\gamma\gamma}_{\mathrm{Trigger}}$	0.03455 ± 0.00008	Monte Carlo
π^+ Reconstruction Efficiency	$A^{\pi^+\gamma\gamma}_{\mathrm{Reconstruction}}$	0.99556 ± 0.00012	$K_{\mu 2}$
π^+ Fiducial Cuts	$A^{\pi^+\gamma\gamma}_{\mathrm{Fiducial}}$	0.89802 ± 0.00073	Monte Carlo
π^+ Stop Efficiency	$A^{\pi^+\gamma\gamma}_{\pi^+\mathrm{StopEff.}}$	0.49226 ± 0.00215	Monte Carlo
$\pi^+ \to \mu^+ \to e^+$ Decay Chain	$A^{\pi^+\gamma\gamma}_{\pi^+\to\mu^+\to e^+}$	0.34872 ± 0.00548	π_{scat}
γ Reconstruction and Fiducial	$A^{\pi^+\gamma\gamma}_{\gamma\mathrm{Recon.\&Fid.}}$	0.53047 ± 0.00204	Monte Carlo, $K_{\pi 2}$
Extra Photon Veto Cuts	$A^{\pi^+\gamma\gamma}_{\mathrm{ExtraPV}}$	0.21633 ± 0.00185	Monte Carlo, $K_{\mu 2}$
Kinematic Cuts	$A^{\pi^+\gamma\gamma}_{\mathrm{Kinematics}}$	0.53706 ± 0.00613	$K_{\pi 2}$, π_{scat}
Beam and Target	$A^{\pi^+\gamma\gamma}_{\mathrm{Beam\&Target}}$	0.50697 ± 0.00077	$K_{\mu 2}$
$T \bullet 2$ Efficiency($\pi^+\gamma\gamma$)	$A^{\pi^+\gamma\gamma}_{T\bullet 2\mathrm{Eff.}}$	0.93570 ± 0.00110	K_B
Total Acceptance	$A^{\pi^+\gamma\gamma} = 1.550 \pm 0.034 \times 10^{-4}$		

Table 2: Acceptance factors for the $K^+ \to \pi^+\gamma\gamma$ (ChPT including UC), which do not change from Table 5.2 in the report. After multiplying the correction factor 1.929, Trigger acceptance is 0.06663 ± 0.00015 and Total acceptance is $2.990 \pm 0.066 \times 10^{-4}$, as you find in Table 1.

Definition	Symbol	Acceptance	Data
Trigger	$A^{\pi^+\gamma\gamma}_{\text{Trigger}}$	0.00609 $\pm$ 0.00003	Monte Carlo
π^+ Reconstruction Efficiency	$A^{\pi^+\gamma\gamma}_{\text{Reconstruction}}$	0.99556 $\pm$ 0.00012	$K_{\mu 2}$
π^+ Fiducial Cuts	$A^{\pi^+\gamma\gamma}_{\text{Fiducial}}$	0.62606 $\pm$ 0.00279	Monte Carlo
π^+ Stop Efficiency	$A^{\pi^+\gamma\gamma}_{\pi^+\text{StopEff.}}$	0.52350 $\pm$ 0.00647	Monte Carlo
$\pi^+ \to \mu^+ \to e^+$ Decay Chain	$A^{\pi^+\gamma\gamma}_{\pi^+\to\mu^+\to e^+}$	0.34872 $\pm$ 0.00548	π_{scat}
γ Reconstruction and Fiducial	$A^{\pi^+\gamma\gamma}_{\gamma\text{Recon.\&Fid.}}$	0.49229 $\pm$ 0.00493	Monte Carlo, $K_{\pi 2}$
Extra Photon Veto Cuts	$A^{\pi^+\gamma\gamma}_{\text{ExtraPV}}$	0.17665 $\pm$ 0.00459	Monte Carlo, $K_{\mu 2}$
Kinematic Cuts	$A^{\pi^+\gamma\gamma}_{\text{Kinematics}}$	0.53706 $\pm$ 0.00613	$K_{\pi 2}$, π_{scat}
Beam and Target	$A^{\pi^+\gamma\gamma}_{\text{Beam\&Target}}$	0.50697 $\pm$ 0.00077	$K_{\mu 2}$
$T \bullet 2$ Efficiency($\pi^+\gamma\gamma$)	$A^{\pi^+\gamma\gamma}_{T\bullet 2\text{Eff.}}$	0.93570 $\pm$ 0.00110	K_B
Total Acceptance	$A^{\pi^+\gamma\gamma} = 1.536 \pm 0.055 \times 10^{-5}$		

Table 3: Acceptance factors for the $K^+ \to \pi^+\gamma\gamma$ (ChPT w/o UC), after the corrections to Table 5.3 in the report. After multiplying the correction factor 7.148, Trigger acceptance is 0.0435 ± 0.0002 and Total acceptance is $1.098 \pm 0.040 \times 10^{-4}$, as you find in Table 1.

Definition	Symbol	Acceptance	Data
Trigger	$A^{\pi^+\gamma\gamma}_{\mathrm{Trigger}}$	0.03490 ± 0.00010	Monte Carlo
π^+ Reconstruction Efficiency	$A^{\pi^+\gamma\gamma}_{\mathrm{Reconstruction}}$	0.99556 ± 0.00012	$K_{\mu 2}$
π^+ Fiducial Cuts	$A^{\pi^+\gamma\gamma}_{\mathrm{Fiducial}}$	0.89701 ± 0.00092	Monte Carlo
π^+ Stop Efficiency	$A^{\pi^+\gamma\gamma}_{\pi^+\mathrm{StopEff.}}$	0.49943 ± 0.00274	Monte Carlo
$\pi^+ \to \mu^+ \to e^+$ Decay Chain	$A^{\pi^+\gamma\gamma}_{\pi^+\to\mu^+\to e^+}$	0.34872 ± 0.00548	π_{scat}
γ Reconstruction and Fiducial	$A^{\pi^+\gamma\gamma}_{\gamma\mathrm{Recon.\&Fid.}}$	0.53887 ± 0.00242	Monte Carlo, $K_{\pi 2}$
Extra Photon Veto Cuts	$A^{\pi^+\gamma\gamma}_{\mathrm{ExtraPV}}$	0.20661 ± 0.00218	Monte Carlo, $K_{\mu 2}$
Kinematic Cuts	$A^{\pi^+\gamma\gamma}_{\mathrm{Kinematics}}$	0.53706 ± 0.00613	$K_{\pi 2}$, π_{scat}
Beam and Target	$A^{\pi^+\gamma\gamma}_{\mathrm{Beam\&Target}}$	0.50697 ± 0.00077	$K_{\mu 2}$
$T \bullet 2$ Efficiency($\pi^+\gamma\gamma$)	$A^{\pi^+\gamma\gamma}_{T\bullet 2\mathrm{Eff.}}$	0.93570 ± 0.00110	K_B
Total Acceptance	$A^{\pi^+\gamma\gamma} = 1.539 \pm 0.036 \times 10^{-4}$		

Table 4: Acceptance factors for the $K^+ \to \pi^+\gamma\gamma$ (Phase Space), after the corrections to Table 5.4 in the report. After multiplying the correction factor 1.811, Trigger acceptance is 0.06319 ± 0.00018 and Total acceptance is $2.787 \pm 0.066 \times 10^{-4}$.

6 new Acceptance Factors for publication

(note added on 2005.May.22)

Suggested by T. Numao, the acceptance table for publication was re-organized once again (Table 5 in this note).

- "Trigger" and "Trigger-counter efficiency" were multiplied into the new "Trigger" acceptance factor.
  - ChPT including UC:
    $0.062345 \pm 0.000158_{(NIDIFoff)}$
  - ChPT w/o UC:
    $0.040703 \pm 0.000193_{(NIDIFoff)}$
  - Phase Space:
    $0.059127 \pm 0.000182_{(NIDIFoff)}$

- "charged-track reconstruction" and "π^+ fiducial cuts" were multiplied into the new"π^+ reconstruction and fiducial cuts" acceptance factor.
  - ChPT including UC:
    $0.98002 \pm 0.00032_{(NIDIFoff)}$
  - ChPT w/o UC:
    $0.93387 \pm 0.00139_{(NIDIFoff)}$
  - Phase Space:
    $0.97950 \pm 0.00040_{(NIDIFoff)}$

Acceptance factors	UC	w/o UC	samples
Trigger	0.0623	0.0407	MC, K_B
π^+ reconstruction and fiducial cuts	0.980	0.935	MC, $K_{\mu 2}$
π^+ accepted region (P,R,E)	0.912	0.667	MC
π^+ stop without nuclear interaction or decay-in-flight	0.492	0.524	MC
dE/dx and kinematic cuts	0.537	0.537	$K_{\pi 2}$, π_{scat}
$\pi^+ \to \mu^+ \to e^+$ cuts	0.349	0.349	π_{scat}
γ reconstruction and fiducial cuts	0.530	0.492	MC, $K_{\pi 2}$
γ selection cuts	0.216	0.177	MC, $K_{\pi 2}$
Other cuts on beam and target	0.507	0.507	$K_{\mu 2}$
Total acceptance	2.99×10^{-4}	1.10×10^{-4}	

Table 5: Acceptance factors for the $K^+ \to \pi^+\gamma\gamma$ decay in the kinematic region $P > 213$ MeV/c, for $\hat{c} = 1.8$ including unitarity corrections ("UC") and for $\hat{c} = 1.6$ without the corrections ("w/o UC"), and the samples used to determine them.

7 An important missing reference ?

(note added on 2005.August.03)

After the manuscript was submitted and was posted to arXiv as hep-ex/0505069 (v1 24 May 2005), I received an email from Joaquim Prades (Universidad of Granada, Spain), who pointed of an missing reference:

C. Bruno and J. Prades, "Rare kaon decays in the $1/N_c$-expansion" Z. Phys. C57, 585-593 (1993) and hep-ph/9209231.

In this paper [5] they predicted $0.05 < \hat{c} < 0.85$ and the total branching ratio to be $(7 \pm 1) \times 10^{-7}$.

8 revised manuscript for the publication

(note added on 2005.August.03)

Upon requests in the referee's report [6], the following modifications were added to the revised manuscript, which was finally accepted for publication in Physics Letters B.

- kaon flux:
  One sentence was added to describe how one derives the kaon flux (K_{Blive}) in the experiment.

  > The exposure of kaons in each spill was recorded by a scaler which counted the number of kaons entering the target during the time the data acquisition system was ready to accept data.

- systematic uncertainty:
  The paragraph on the acceptance and sensitivity was revised, in order to clarify that:

  1. in using the K^+-stopping fraction from the $K^+ \to \pi^+\pi^0$ ($K_{\pi 2}$) measurement, the sensitivities for $K^+ \to \pi^+\gamma\gamma$ and for $K^+ \to$

[5] They should have let us know when the E787 $\pi^+\gamma\gamma$ paper was published in 1997 !

[6] The referee's report pointed out correctly the two shortcomings of the $\pi^+\gamma\gamma$ paper: (1) it lacks describing the systematic uncertainty in the sensitivity and (2) the result depends on a very specific prescription (ChPT with unitarity corrections) and is not quoted in a model-independent way, although the limit assuming Phase Space is given. The referee also requested to give the description on how one derives the kaon flux.

$\mu^+\nu$ were normalized to $K_{\pi 2}$; many systematic uncertainties in the measurement of the acceptance factors (in particular in the small trigger acceptance) canceled, and

2. the systematic uncertainty in the sensitivity was estimated to be less than 10% [7].

> The efficiency was measured to be 0.754 ± 0.024 with the $K_{\pi 2}$ events collected with the $K^+ \to \pi^+\gamma\gamma$ trigger and selected using all the analysis cuts except those designed to remove $K_{\pi 2}$ events. Thus the sensitivity for $K^+ \to \pi^+\gamma\gamma$ was normalized to $K_{\pi 2}$ and many systematic uncertainties in the measurement of the acceptance factors, in particular in the trigger acceptance, canceled.
> ...
> The branching ratio of 0.628 ± 0.020 measured with the same K^+-stopping efficiency was consistent with the Particle Data Group value [7]. The systematic uncertainty in the sensitivity was estimated to be less than 10%.

The systematic uncertainty, which is small and does not change the upper limits (using the formula by R.D. Cousins and V.L. Highland: "Incorporating systematic uncertainties into an upper limit" in Nucl. Instr. and Meth. A320 (1992) 331, for example:

$$\text{Upper Limit} = \boxed{2.30\ (\ 1 + 2.30\ \sigma_r^2\ /\ 2\ )} \times \text{S.E.S}$$
$$(\sigma_r\text{: relative systematic-uncertainty})$$

the 10% uncertainty changes the upper limits by 1.2% [8]), is not taken into consideration in deriving the limits.

- Reference [26]
  The PLB editor agreed to update the reference [26] as follows.

  B. Melic, private communication.

[7] The errors in the single event sensitivity are 4–5%. The estimated systematic uncertainty in the acceptance of the $\pi^+\nu\bar{\nu}1$ analysis was about 10% and was due mostly to the uncertainty in pion-nucleus interactions; Since the $\pi^+\gamma\gamma 1$ analysis was normalized to the $K_{\pi 2}$ branching ratio and the uncertainty in pion-nucleus interactions is expected to be reduced, it is reasonable to estimate the systematic uncertainty to be less than 10%. For reference, the estimated systematic uncertainty in the E787 $\pi^+\gamma$ analysis was 8.5%.

[8] Replacing 2.30 by 2.24, "1.2%" gets to "1.1%".

$\longrightarrow$

B. Melić, K. Passek-Kumerički, and J. Trampetić, hep-ph/0507231.