16

Neutrino Masses and Oscillations

The Old Enigma

Neutrinos are the most enigmatic of elementary particles. Postulated in 1930, they were not observed until a quarter of a century later. It has taken another forty years to determine whether they are massless.

As explained in Chapter 6, if the electron neutrino were sufficiently massive the electron spectrum in tritium beta decay would be influenced near the end point. This prompted many measurements of this spectrum over the past thirty years. The expression for the spectrum actually depends on the square of the neutrino mass and for some time the best fits returned unphysical, negative values for this. More recent measurements give an upper limit near 3 eV.

The best measurement of the mass of ν_μ is obtained from $\pi^+ \to \mu^+\nu_\mu$, which gives a 90% CL upper limit of 170 keV.

The mass of ν_τ can be sought by studying τ decays of the sort $\tau^- \to 2\pi^-\pi^+\nu_\tau$ and $\tau^- \to 3\pi^-2\pi^+\nu_\tau$. If ν_τ is massive, the invariant mass spectrum of the charged pions will terminate below the mass of the τ. The best limit obtained to date is $m_{\nu_\tau} < 18.2$ MeV.

It would seem natural the neutrino masses would be just like the masses of other fermions like electrons and quarks. However, neutrinos can have a character that is quite unlike that of charged fermions. The electron-positron system has four degrees of freedom, which we can represent by e_L, e_R, e^c_L, and e^c_R, where we have chosen to write e^c for e^+. For the neutrino we can write similarly ν_L, ν_R, ν^c_L and ν^c_R.

To make a massive particle, we need both "left-handed" and "right-handed" pieces. For neutrinos we can suppose that the massive particle is combination of the left-handed neutrino and the right-handed antineutrino:

$$N = \nu_L + \nu^c_R \tag{16.1}$$

This provides all the degrees of freedom required.

The mass of the electron is described in terms of the electron field by the expression $m_e\overline{e}e = m_e(\overline{e}_L e_R + \overline{e}_R e_L)$. The mass term changes a left-handed electron into a right-handed electron, with amplitude m_e. Of course this is a colloquialism since the freely propagating electron doesn't spontatneously change its angular momentum! The imprecision arises because $e_L = \frac{1}{2}(1-\gamma_5)e$ describes a left-handed electron only in the ultra-relativistic limit. Thus an electron emitted in beta decay has polarization, on average, $-v/c$.

While N has the degrees of freedom required for a massive fermion, by combining a lepton with an antilepton, we have broken lepton conservation. If we tried the same thing with an electron, joining the left-handed electron with the right-handed positron, we would have broken charge conservation, something that is certainly impermissible. Whether lepton number is truly conserved is an experimental question.

There are a number of nuclides that are stable against both β^- and β^+ decay, but which can decay by double beta decay. An example is Ge_{32}^{76}. Energy conservation forbids $\mathrm{Ge}_{32}^{76} \to \mathrm{Ga}_{31}^{76}\ e^+\nu_e$ and $\mathrm{Ge}_{32}^{76} \to \mathrm{As}_{33}^{76}\ e^-\overline{\nu}_e$, but $\mathrm{Ge}_{32}^{76} \to \mathrm{Se}_{34}^{76}\ e^-\nu_e e^-\nu_e$ occurs with a half-life of about 1.8×10^{21}y. The neutrinoless double beta decay $\mathrm{Ge}_{32}^{76} \to \mathrm{Se}_{34}^{76} e^- e^-$ would violate lepton number. If ν_e is a Majorana particle, such a process is allowed.

Imagine this decay occurs through the intermediate virtual process $\mathrm{Ge}_{32}^{76} \to \mathrm{Se}_{34}^{76}\ W^-W^-$. One W decays to $e^-\overline{\nu}_{eR}$, where the antineutrino is virtual. If the neutrino is a Majorana particle, the $\overline{\nu}_{eR}$ can become ν_{eL}. This latter combines with the W^- to make the second e^-. The amplitude for this process is proportional to m_{ν_e}, so observing it would establish a non-zero neutrino mass, as well as showing that lepton number is violated. The experimental lower limit on the half-life of Ge_{32}^{76} against neutrinoless double beta decay is about 10^{25}y. Using calculated nuclear matrix elements, this gives an upper limit of about 0.5 eV for the mass of a Majorana electron-neutrino.

The Standard Model together with Majorana neutrinos can provide a potential explanation of the smallness of neutrino masses (if they are indeed not massless). The mass term changes a left-handed electron into a right-handed electron, with amplitude m_e, changing the weak isospin from $I_z = -1/2$ to $I_z = 0$. This is permissible because the electron interacts with the ubiquitous Higgs field, which has $I_z = \pm 1/2$ and which is non-zero everywhere.

Our Majorana neutrino N behaves differently. To change ν_{eL} to ν_{eR}^c requires $\Delta I_z = 1$, more than the Higgs field supplies. Thus we expect this amplitude to be zero or very, very small. Suppose, however, that in addition there is a right-handed neutrino, together with its conjugate, a left-handed antineutrino. Together they can form a second Majorana neutrino,

$$N' = \nu_R + \nu_L^c \tag{16.2}$$

To change from the left-handed piece of N' to the right-handed piece doesn't change I_z at all, since both pieces are neutral under weak isospin. There is no reason for this not to be very large since it does not depend on the "low" scale at which electroweak symmetry is broken. Its value M might even be as large as 10^{15} GeV, the scale at which the strong and electroweak forces may be unified.

It is also possible for N to become N' through a mass. In particular, the ν_{eL} in N can become ν_{eR} in N', just as e_L becomes e_R. Indeed, we might anticipate an amplitude of the same scale, m. The same is true for the transition of N' to N. These results can be summarized in mass matrix connecting N and N':

$$\mathbf{M} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \tag{16.3}$$

For $m << M$, the eigenvalues of the matrix are nearly M and $-m^2/M$. The negative sign has no physical significance. It corresponds to a mass m^2/M. The lighter eigenstate is mostly the weakly interacting Majorana neutrino, while the heavier one is mostly the non-interacting Majorana neutrino:

$$\begin{aligned} |N_1\rangle &\approx |N\rangle & -\left(\frac{m}{M}\right)|N'\rangle \\ |N_2\rangle &\approx |N'\rangle & +\left(\frac{m}{M}\right)|N\rangle \end{aligned}$$

If we guess that $m = m_e$ and $M = 10^{15}$ GeV, we get a neutrino mass of less than 10^{-12} eV, very small indeed. This is known as the seesaw mechanism.

If neutrinos have mass, the leptonic system is quite analogous to the quark system. We thus expect that the weak eigenstates may not correspond to the mass eigenstates: there is a leptonic version of the Kobayashi-Maskawa matrix connecting the two. To see this, consider just two species of neutrinos, with weak eigenstates ν_e and ν_μ and mass eigenstates

$$\begin{aligned} |\nu_1\rangle &= \cos\theta_0|\nu_e\rangle - \sin\theta_0|\nu_\mu\rangle \\ |\nu_2\rangle &= \sin\theta_0|\nu_e\rangle + \cos\theta_0|\nu_\mu\rangle \end{aligned}$$

Suppose a beta decay produces a ν_e. It will undergo oscillations described by

$$|\nu_e(t)\rangle = e^{-iE_1t}\cos\theta_0|\nu_1\rangle + e^{-iE_2t}\sin\theta_0|\nu_2\rangle \tag{16.4}$$

If the state has well defined momentum $p = E >> m_\nu$, then

$$E_1 \approx E + \frac{m_1^2}{2E}; \qquad E_2 \approx E + \frac{m_2^2}{2E} \tag{16.5}$$

After traveling a distance $L = t$, the two pieces will have a relative phase $(m_2^2 - m_1^2)L/(2E)$. The probability that the ν_e will have become a ν_μ is easily determined to be

$$P_{\nu_e\to\nu_\mu}(t) = \sin^2 2\theta_0 \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \tag{16.6}$$

With E measured in GeV, L in km, and Δm^2 in eV^2, the last factor is

$$\sin^2\left(1.27\frac{\Delta m^2 L}{E}\right) \tag{16.7}$$

These oscillations are similar to those in the $K^0-\overline{K}^0$ and $B^0-\overline{B}^0$ systems. There the oscillation is manifested in the variation in the sign of charged leptons emitted in semileptonic decays. Here it is the type of lepton itself that varies. The specific phenomenon observed depends on the energy of the neutrino that is oscillating. Antineutrinos generated by beta decays in nuclear reactors have energies in the MeV range. If these antineutrinos oscillate from electron-type to muon- or tau-type, their energies will be too low to produce the associated charged leptons. What would be measurable would be simply a drop in the number of charged current reactions. The neutrinos would seem to disappear.

A neutrino beam generated by decaying pions will be dominantly ν_μ or $\overline{\nu}_\mu$ depending on the sign of the pions. Its charged-current interactions will regenerate muons. If, however, the beam oscillates to electron- or tau-type neutrinos, the corresponding charged leptons could be produced. Such an experiment would establish oscillations by appearance.

Neutrino oscillation experiments are remarkable for the diversity of neutrino sources that have been employed. These sources include besides both low-energy and high-energy particle beams, the sun, nuclear reactors, and cosmic rays. The history of neutrino experiments is replete with contradictory results. The complexity of many of the sources for neutrino oscillation measurements further obscures the findings.

The earliest indications of neutrino oscillations came in solar neutrino experiments. The initial step in the fusion cycle that powers the Sun is the weak process $pp\to de^+\nu_e$. Because the total rate of energy production is proportional to the rate at which this reaction occurs, there is little uncertainty about the neutrino flux it generates. This turns out to be about $6\times10^{10}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$. These neutrinos have energies below 0.5 MeV and are thus below threshold for charged current interactions except with a few nuclides. The next most copious source of solar neutrinos is electron capture on Be^7: $\mathrm{Be}^7e^-\to\mathrm{Li}^7\nu_e$, with discrete neutrino energies near 0.4 and 0.9 MeV. The third significant source of solar neutrinos is the decay $\mathrm{B}^8\to\mathrm{Be}^{8*}e^+\nu_e$, which produces a continuum of neutino energies up to nearly 18 MeV. Even though the flux of these neutrinos is about 10^{-4} of those from the pp reaction their high energy makes them very important in solar neutrino experiments.

The solar neutrinos can be detected if they are captured by isotopes like Cl^{37} and Ga^{71}, which become radioactive and whose subsequent decays can then be observed. The threshold for the former capture is 814 keV, while for the latter it is 233 keV. As a result, chlorine experiments are blind to the pp reaction, while gallium experiments can detect it. The chlorine experiments are dominated by neutrinos from B^8 and Be^7. The chlorine experiments were pioneered by R. Davis at the Homestake Mine in South Dakota, starting back in the 1960's. Gallium experiments are pursued by the GALLEX collaboration at the Gran Sasso in Italy and by the SAGE collaboration at Baksan, in Russia.

An alternative detection scheme relies on Cerenkov light from charged-current reactions induced by the neutrinos. Because an enormous target is required to obtain sufficient rate, the natural medium is water. Kamiokande and its successor, Super-Kamiokande have been the leading experiments using this approach. The threshold for observability is several MeV and these experiments are thus dominated by neutrinos from B^8 decay.

Every one of these techniques is extremely challenging because of the small rates and large detectors employed. What is striking is that the results of all these experiments tell about the same story: about half the expected rate of neutrino interactions is actually observed.

The abundances of elements like beryllium and boron must be deduced from solar models and this adds some doubt to the predictions for these contributions to the solar neutrino flux. However, there is good agreement between the various calculations that have been done to estimate these abundances. This makes it hard to dismiss the results from the Cerenkov and chlorine experiments. Moreover, fully half of the reaction rate expected in the gallium experiments is due to the *pp* reaction, about whose rate there can be little doubt.

The discrepancy between the expected and observed rates for solar neutrino experiments is consistent and persistent. Attempts to blame the problem on solar models are weakened by the GALLEX and SAGE results. What remains suggests strongly that there are neutrino oscillations involving electron neutrinos.

If there is mixing between ν_e and ν_μ, the combinations that are eigenstates in free space will not remain eigenstates when passing through matter. This is completely analogous to the phenomenon of regeneration in the neutral K system. There regeneration occurs because K^0 and $\overline{K}^0$ have different forward scattering amplitudes on nuclei. In the neutrino system the corresponding difference is between the elastic scattering of ν_e on electrons and ν_μ on electrons. While $\nu_\mu e$ elastic scattering occurs only through the neutral current, $\nu_e e$ elastic scattering has a contribution from the charged current process. For neutrinos, where the mass is apparent in the relation $E \approx p + \frac{1}{2}m^2/p$, the mass-squared matrix is of interest. The effect of the extra scattering of ν_e is to add to its diagonal element in this matrix the quantity

$$A = 2\sqrt{2}G_F N_e E = 0.76 \times 10^{-7}\mathrm{eV}^2 \times \rho[\mathrm{g/cm}^3] \times E[\mathrm{MeV}] \times [2Y_e] \qquad (16.8)$$

where N_e is the electron density in the matter and E is the neutrino energy. The mass density is ρ and the number of electrons per nucleon is Y_e. No other matrix element is affected. The ν_e component of a mixed electron picks up an extra phase $\frac{1}{2}AL/E$ in traversing a distance L. If the material is hydrogen with a density of 1 g cm^{-3}, a full cycle is accumulated in a distance of 1.6×10^4 km, a bit more than the diameter of the Earth.

The mixing that results in the eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ with masses squared M_1^2 and M_2^2 is described by

$$M^2 = \frac{M_2^2 - M_1^2}{2} \begin{pmatrix} -\cos 2\theta_0 & \sin 2\theta_0 \\ \sin 2\theta_0 & \cos 2\theta_0 \end{pmatrix} \tag{16.9}$$

where we dropped the common diagonal term with the average mass squared. Because the energy of a neutrino with momentum p is $\approx p + \frac{1}{2}m^2/E$ we can write a Schrodinger equation for the system

$$i\frac{d}{dt}\begin{pmatrix} C_e \\ C_\mu \end{pmatrix} = \frac{1}{2E}M^2 \begin{pmatrix} C_e \\ C_\mu \end{pmatrix} \tag{16.10}$$

We see that this system is analogous to a spin-1/2 particle in a magnetic field with $\vec{B} \propto -\cos 2\theta_0 \hat{z} + \sin 2\theta_0 \hat{x}$. The eigenstate $|\nu_1\rangle$ is the up state rotated by $-2\theta_0$ about the y axis. The effect of the extra elastic scattering of ν_e on electrons with density N changes the mass squared matrix to

$$M_{eff}^2 = \frac{M_2^2 - M_1^2}{2} \begin{pmatrix} -\cos 2\theta_0 + \frac{2\sqrt{2}G_F NE}{M_2^2 - M_1^2} & \sin 2\theta_0 \\ \sin 2\theta_0 & \cos 2\theta_0 - \frac{2\sqrt{2}G_F NE}{M_2^2 - M_1^2} \end{pmatrix} \tag{16.11}$$

Using the magnetic analogy, it is clear that the new eigenstates will be

$$\begin{aligned} |\nu_1, N\rangle &= \cos\theta_N |\nu_e\rangle - \sin\theta_N |\nu_\mu\rangle \\ |\nu_2, N\rangle &= \sin\theta_N |\nu_e\rangle + \cos\theta_N |\nu_\mu\rangle \end{aligned}$$

where the geometry shows that

$$\cot 2\theta_N = \cot 2\theta_0 - \frac{2\sqrt{2}G_F NE}{\sin 2\theta_0 (M_2^2 - M_1^2)} \tag{16.12}$$

If a neutrino begins at t_0 where the electron density is N(t_0) in the eigenstate $|\nu_1, N(t_0)\rangle$ and proceeds through matter in which the electron density changes only gradually, we might expect the state to evolve at time t to $|\nu_1, N(t)\rangle$. This adiabatic evolution is analogous to the magnetic moment of the spin-1/2 particle following the gradual change in $\vec{B}$. As long as the z component of the magnetic field remains large compared to the x component, this is a compelling argument. However, in the region where the electron density makes the z component vanish,

$$\cos 2\theta_0 = \frac{2\sqrt{2}G_F NE}{M_2^2 - M_1^2} \tag{16.13}$$

the spin may longer follow the field and transitions from $|\nu_1\rangle$ to $|\nu_2\rangle$ become much more likely.

In the sun, neutrinos are produced near the core, where the density is of order 130 g/cm^3. For $M_2^2 - M_1^2$ around 10^{-5} eV^2, and neutrino energy E of order an MeV, the right hand side is about 0.9. Thus, for this value of Δm^2, electron-type neutrinos with energies of a few MeV are produced in a region where the "magnetic field" has the opposite sign to what is has outside the sun, where $N = 0$. If they evolve adiabatically, they will emerge as muon-type neutrinos. Much lower energy neutrinos will be produced in a region where the "magnetic field" is just about what it outside the sun, so these neutrinos will not be "flipped." The emerge as electron-type neutrinos.

The apparent solar neutrino deficit can be explained either by vacuum oscillations, with a large value of $\sin^2 2\theta$, or using the MSW effect, with a small value of $\sin^2 2\theta$.

Reactor experiments produce antineutrinos, which accompany the beta particles emitted by fission products. Since the energies here are at most a few MeV, there is no possibility of observing the oscillation of $\overline{\nu}_e$ to $\overline{\nu}_\mu$ in a charged current interaction: these neutrinos are below threshold for muon production. However, these oscillations would lead to a reduction in the number of charged current events producing electrons. For sufficiently large mixing angles, such an effect would be observable by comparing the event rate with the reactor on and off, and comparing with the expected rate, based on calculations. Such calculations are believed to be accurate at the few percent level.

A nuclear power station located near Chooz in the Ardennes region of France served as the antineutrino source for one experiment. The antineutrinos were detected through inverse beta decay: $\overline{\nu}_e p \to e^+ n$. The positron was observed through its two-gamma annihilation with an electron. The neutron was observed by incorporating gadolinium in liquid scintillator. Gadolinium has a large cross section for neutron absorption, which is signaled by the emission of a gamma ray of 8 MeV. The neutrons could also be observed by their absorption by protons, producing a deuteron and a 2.2 MeV gamma. The delay of 2 to 100 μs between the positron annihilation and the neutron absorption provided a signature for the events. The technique used was very much the same as the one used by Reines and Cowan in their original detection of antineutrinos. At Chooz, the signal event rate was found to be proportional to the instantaneous power of the reactor, as it should have been. The value of about 25 neutrino events per day at full power was much larger than the background of about 1 event per day.

The anticipated rate in the absence of neutrino oscillations depended on the intensity and energy spectrum of the neutrinos emitted by the reactor. Including this uncertainty and others associated with the detector, the ratio of the measured to the expected rate was found to be $0.98 \pm 0.04 \pm 0.04$, where the first error is statistical and the second systematic. At 90% CL, this is greater than 0.91, so at the same confidence level, for large Δm^2, $\sin^2 2\theta < 0.18$. Using a mean neutrino energy of 3 MeV, in the limit $\sin^2 2\theta = 1$, $\Delta m^2 < 0.9 \times 10^{-3}\text{eV}^{-2}$.

Accelerators produce primarily $\overline{\nu}_\mu$, which result from the decays $\pi^+ \to \mu^+\overline{\nu}_\mu$ and $K^+ \to \mu^+\overline{\nu}_\mu$, and ν_μ from the analogous decays of negative particles. The semileptonic decay $K^+ \to \pi^0 e^+\overline{\nu}_e$ has a 4% branching ratio and serves either as a welcome source of $\overline{\nu}_e$ or as a contaminant.

By working with a low energy primary proton beam, K production can be excluded. The decay $\pi^+ \to \mu^+\nu_\mu$ guarantees a pure ν_μ beam with little $\overline{\nu}_\mu$ contamination since the muon is so long-lived. On the other hand, a pure μ^+ beam that is stopped in matter will produce a pure $\overline{\nu}_\mu$ source without a ν_μ component. This provides the means to search for both $\overline{\nu}_\mu \to \overline{\nu}_e$ and $\nu_\mu \to \nu_e$ oscillations. The Liquid Scintillator Neutrino Detector (LSND) at Los Alamos found evidence for both kinds of oscillations, with similar allowed regions of $\sin^2 2\theta$ and Δm^2.

A contrary result was obtained by the KARMEN Collaboration working at the Rutherford Appleton Laboratory in England. An 800-MeV proton beam struck a thick target, which stopped the produced particles. The negative pions were absorbed by nuclei, while the positive pions decayed at rest. The resulting μ^+s also decayed at rest. Altogether, these decays produce equal numbers of ν_μ, $\overline{\nu}_\mu$, and ν_e, but no $\overline{\nu}_e$s. The background from the decay in flight of μ^- was small. The signal for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations would be the reaction $\overline{\nu}_e p \to e^+ n$. The positron would be apparent in the 56 t liquid-scintillator target. The neutron would be apparent after slowing to thermal velocities, when it would be absorbed by $np \to d\gamma$ or in the gadolinium added to the detector to increase neutron sensitivity. While nearly three events were expected from background, none was observed. The limit at 90% CL was set at $N < 1.07$, while for maximal mixing and large Δm^2, $N = 811 \pm 89$ would have been expected. Thus at 90% CL, $\sin^2 2\theta < 1.3^{\times}10^{-3}$, which conflicts somewhat with the LSND experiment.

Neutrinos are a ubiquitous if imperceptible part of our environment. Extraordinarily low energy neutrinos created in the Big Bang, like the cosmic background radiation pervade the entire Universe. The Sun is a poweful source of MeV neutrinos. Neutrinos in the GeV range are created when cosmic rays strike the atmosphere, 15 km or so above the Earth's surface. In the hadronic showers, pions are created that decay to $\mu\nu$, and the muons subsequently decay to $e\nu\nu$. More precisely, two ν_μs and one nu_e are generated for each pion created, ignoring the difference between neutrinos and antineutrinos.

The actual flux of particles created by the collisions high in the atmosphere is not so well known, so there is an advantage in comparing the ratio of events producing a μ to those producing an electron to the ratio expected from Monte Carlo simulations: $R = (\mu/e)_{DATA}/(\mu/e)_{MC}$. The absolute strength of the flux cancels in the ratio of the simulations. A number of experiments using water cerenkov counters, including one at Kamiokande in Japan, IMB near Cleveland, Ohio, and Super-Kamiokande, a larger Japanese detector, observed values of R

less than one, indicating that the μ_μ were somehow disappearing.

With its large mass of water, 50 kilotons, Super-Kamiokande obtained the most impressive evidence for neutrino oscillations. The ring of cerenkov light produced by a muon has a sharper definition than that produced by the shower of an electron and the two categories can be reliably separated. More than 11 thousand photomultiplier tubes viewed the central 22.5 kilotons of detector, in which events were required to begin. The Super Kamiokande collaboration recorded more than 4000 events that were fully contained within the inner fiducial voluem. The ratio R thus found differed substantially from unity, both for lower energy events (visible energy below 1.33 GeV), with $R = 0.63 \pm 0.03 \pm 0.05$ and higher energy events, with $R = 0.65 \pm 0.05 \pm 0.08$.

From the cerenkov light, it was possible to determine the direction of the incoming neutrino. Those that came from below must have been created in the atmosphere on the other side of the Earth, thousands of kilometers away. Those that came from above, were created realitively nearby. While the e=like events showed no particular direcctional dependence, the μ-like events that came from below were substantially depleted. The simplest interpretation is that the ν_μ oscillate to ν_τ with an oscillation wavelength comparable to the Earth's radius. Alternatively, the ν_μ might oscillate to some previoiusly unknown neutrino type, a sterile neutrino that lacks interactions. For such a depletion to be observable, the mixing would have to be substantial. A fit to the data for $\nu_\mu - \nu_\tau$ oscillations gave $\sin^2 2\theta > 0.82$ and $5 \times 10^{-4}\text{eV}^2 < \Delta m^2 < 6 \times 10^{-3}\text{eV}^2$ with 90% confidence.

EXERCISES

16.1 Verify the claim that the MSW effect would accumulate a phase of 2π traversing 1.6×10^4 km of hydrogen with a density of 1 g/cm^{-3}.

BIBLIOGRAPHY

A convenient review of neutrino oscillations up to 1999 is given by P. Fisher, B. Kayser, and K. MacFarl, *Ann. Rev. Nucl. Part. Sci.*, **49**, 481 (1999).

REFERENCES

16.1 SuperKamiokande Collaboration, "Evidence for Oscillation of Atmospheric Neutrinos." *Phys. Rev. Lett.*, **81**, 1562 (1998).