Default Risk on Derivatives Exchanges:

Evidence from Clearing-House Data

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Abstract

In this paper, we analyze empirically the clearing house exposure to the risk of default by a clearing house member. Using actual daily data on margins and variation margins for all clearing members of the Chicago Mercantile Exchange's clearing house, we identify many occurrences when the member's daily loss exceeds his posted margin. Furthermore, we find that the major source of default risk for a clearing member is proprietary trading and not trading by customers. In order to quantify the default risk exposure, we provide a characterization of the tail risk of the clearing house using Extreme Value Theory. We then design and price a realistic insurance contract covering the loss to the clearing house from default by one or several clearing members. We investigate the impact on the insurance premium of including data from the Black Monday of 1987 in our sample. Our empirical analysis also allows us to put a dollar amount on the service provided by the central bank, which is the implicit insurer of the clearing house.

JEL classification: G13, G18

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1 Introduction

Derivatives contracts, as they call for future delivery or payments, are clearly exposed to the risk of counterparty default. On organized derivatives exchanges, the central counterparty clearing house (hereafter CCP) greatly reduces this concern by becoming counterparty to every transaction undertaken by a clearing member (hereafter CM).[1] As an intermediary, the CCP does not incur market risk but it does bear default risk (also called counterparty risk, or nonperformance risk) vis-a-vis their CMs. Indeed, a payment default occurs if the daily loss of a CM's total position exceeds his margin and he does not pay the deficiency on time.[2]

In this paper, we analyze empirically the default risk of CM using actual data of daily gains and losses of all the CMs of the Chicago Mercantile Exchange's CCP. Characterizing the probability of a default by a CM, as well as the magnitude of the resulting loss, is of great interest to many financial market participants. First in line is the CCP since, in the event of a CM default, it will need to utilize its own resources to compensate the winning CMs, and once resources are exhausted, it may also default. Other concerned market participants include the non-defaulting CMs because of the mutualization of losses through loss-sharing rules. Individual investors trading through the defaulting CM can also be at risk since, as made clear by Jordan and Morgan (1990), they are not contractually protected by the CCP – especially if the default of the CM is due to the loss of another customer trading through the defaulting CM. The parent companies of the CM are also directly concerned since they provide the financial guaranty necessary to the CM to operate. Furthermore, as implicit and explicit insurers of the CCP against default, central banks (Bernanke, 1990) and insurance companies may find the present analysis informative.

Although actual defaults by a CM have remained infrequent (Examples of CCP having experienced a CM default are US Options Clearing Corporation in 1973, New York Mercantile Exchange, Inc. in 1976, Commodities Exchange Inc. in 1985, and US Board of

[1] The clearing activity consists in confirming, matching, and settling all trades on an exchange. On most derivatives exchanges, only a subset of market participants (i.e., the CMs) can directly trade with the CCP whereas all non-CM participants have to route their trades through a designated CM. Some end-customers deal directly with a CM (e.g. non-CM institutional investors, hedge funds) while others trade through a broker who still needs to route the trades through a CM.

[2] Our definition of default does not include all occurrences when a CM does not meet a margin call. As long as the balance of the margin account is non-negative, the position of the CM can be unwinded at no cost to the CCP and other CMs. It is only when the balance is negative - the money has already been lost and needs to be transferred immediately to the winning party - that failing to meet a margin call is perceived as a default.

Trade Clearing Corporation in 1992), concerns about default risk in the clearing process have recently increased for a number of reasons. Indeed, recent years have witnessed an extraordinary expansion of the derivatives markets, which was fueled in part by the rise of the hedge fund industry. In parallel to this increase, the emergence of mega-exchanges resulting from mergers (e.g. Chicago Mercantile Exchange & Chicago Board of Trade, EURONEXT & LIFFE) leads to fewer and bigger clearing facilities. As a result, CCPs concentrate an increasing amount of risk which raises substantial systemic risk concerns. As recently noticed by Federal Reserve Governor Kroszner (2006), CCPs are now clearing new products, some of which being illiquid or very complex, and are frequently involved in cross-border clearing activities. In response to this growing concern, a number of CCPs have purchased default protections from insurance companies to further strengthen their financial safeguard package. Recent examples include the CCP of the New York Mercantile Exchange, Inc., the CCP of the Sydney Futures Exchange, and the Norwegian Futures and Options Clearinghouse.[3]

Over the years, CCPs have assembled a battery of safeguards to control their default risk exposure. First, CMs are subject to financial and capital adequacy requirements to guaranty their creditworthiness. Second, CCPs impose a system of margining which ensure that obligations of both CM and their customers are collateralized. Both house margin accounts and customer margin accounts are adjusted daily to reflect changes in the mark-to-market value of positions. These daily aggregate gains or losses are called variation margins. If the margin account balance falls below the maintenance margin then additional margin must be posted to bring the balance up to the required level.[4] In most cases, CMs are permitted to post a margin on the aggregate *net* position for their own house positions but they are required to deposit margins with the CCP sufficient to cover the *gross* positions of their customers. Furthermore, positive balances in the house account are typically pledged to offset negative balances in the customer account, but not the reverse.

The academic literature on default risk on derivatives exchanges has developed concurrently with derivatives markets. A first strand of the literature focuses on the probability of facing a daily loss greater than the actual margin and, in turn, that additional funds must be raised (Figlewski, 1984; Gay, Hunter and Kolb, 1986; Knott and Polenghi, 2006) and on

[3]In their 10K form filed in September 2002 to the Security Exchange Commission, the Chicago Mercantile Exchange stated that they were in the process of obtaining default insurance. However, to our knowledge, this has not materialized yet.

[4]Intraday margin calls are possible in particularly volatile market conditions. Extraordinary intraday margin calls occurred three times on October 19th, 1987, and ten more times in the remainder of October (Fenn and Kupiec, 1993).

the expected value of additional funds that need to be raised conditional or not on additional funds being needed (Bates and Craine, 1999). Other authors have proposed models of optimal margin that lead to a given target probability of a loss in excess of the margin (Booth et al., 1997 and Cotter, 2001), that minimize the total cost of margin, settlement, and default costs (Fenn and Kupiec, 1993), that reflect the option to default on a futures position (Day and Lewis, 2004), or that take account of the CCP risk aversion (Cotter and Dowd, 2006). Recently, Shanker and Balakrishnan (2005) have used the framework of Brennan (1986) to set the optimal margin, capital, and price limits that will minimize the cost of clearing firms and simultaneously provide protection against default risk to the CCP. Differently, Bailey and Ng (1991) estimate the default premium in the futures prices of precious metals and show that nonperformance risk has significantly affected futures prices during the 1979-1980 "Hunt Brothers" episode.

A common feature of the aforementioned studies is that they all focus on an individual futures position. As a result, their conclusions do not directly apply to CMs since, at any point in time, they maintain a *portfolio* of positions with complex diversification effects. Indeed, a CM's aggregate portfolio includes different derivatives types (e.g., futures and options), offsetting positions (i.e. long and short), different underlying assets, and various maturities.[5] Moreover, some of the positions are proprietary positions of the CM while others come from all the end-customers that access the derivatives exchange through this CM.

Portfolio-based margin requirements are computed in most CCPs (including the Chicago Mercantile Exchange) using the Standard Portfolio Analysis of Risk (hereafter SPAN) margining system.[6] SPAN evaluates the risk of an entire account' futures/options portfolio and computes at the end of each trading day its aggregate margin requirement called performance bond.[7] To arrive at a margin level, SPAN considers a series of scenarios representing potential changes over a one-day horizon in the underlying security's price and volatility.

[5] In an early attempt to analyze the default risk of a CCP, Gemmill (1994) highlights the dramatic diversification benefit from combining contracts on uncorrelated or weakly correlated assets.

[6] The SPAN system was originally developed by the Chicago Mercantile Exchange. A smaller number of CCPs use the Theoretical Intermarket Margining System (TIMS) developed by the US Options Clearing Corporation.

[7] On the Chicago Mercantile Exchange, performance bond can be in cash (ten acceptable currencies), U.S. Treasuries, letters of credit, selected stocks from the S&P 500 index, selected sovereign debt, mortgage backed securities, and several types of notes and bonds from selected financial institutions. As of June 30, 1998, the Chicago Mercantile Exchange's CCP held a total of \$3.771 billion in house margin funds and \$7.949 billion in customer margin funds. U.S. Treasuries account for 89.76% of house margin funds, letters of credit for 4.97%, and cash for 0.17%. Corresponding percentages for customer margin funds are 83.46%, 12.96%, and 0.61%, respectively (source: CFTC, 1998).

For each scenario, the value changes are aggregated across all positions and the CM's margin requirement for the day is set to the maximum aggregate loss across all scenarios.[8] The empirical performance of SPAN has been tested by Kupiec (1994) for selected portfolios of S&P 500 futures and futures-options contracts. He finds that, over the period 1988-1992, the historical margin coverages exceed 99% for most considered portfolios. Furthermore, Kupiec and White (1996) find that SPAN provides similar risk protection than the alternative margining system, "Reg T", but with substantially smaller collateral requirements.[9]

The present study is the first one to analyze the default risk of CMs using actual daily performance bonds and variation margins. Our dataset includes the performance bonds and variation margins for all CMs of the Chicago Mercantile Exchange's CCP over the period January 4th, 1999 - December 31st, 2001. Chicago Mercantile Exchange is the largest and most diverse financial exchange in the world for trading futures and options. We conduct an in-depth statistical analysis of the performance bonds ($B \geqslant 0$), variation margins ($V > 0$ corresponds to a gain for the CM and $V < 0$ corresponds to a loss), and of their ratio V/B, i.e., the relative profit-and-loss. We are particularly interested in the left tail of the distribution of the V/B ratio since a ratio smaller than -1 indicates a situation in which the CM has an incentive to default. Over our three-year sample period, we identify 68 occurrences when the daily loss in the house account exceeds the posted margin. Almost one third of our sample CMs have experienced a margin-exceeding loss and in one case the loss was as high as 173% of the posted margin. We also find that the value of the V/B ratio seems to be independent of the size of the performance bond. Furthermore, margin-exceeding losses are much less frequent on the customer side, with only four occurrences in total. As a result, the major source of default risk for a CM is proprietary trading.

In this paper, not only we quantify the default risk but we also show how this risk can be hedged through the purchase of a default insurance. We design and price a realistic insurance contract covering the loss to the CCP from default by one or several CMs. The insurance policy is based on the accumulated default losses over a T-year insured period, subject to both a policy deductible and an overall payout limit. Valuation proceeds in two steps. First, we empirically model the left tail of the V/B ratio using a generalized Pareto distribution. Second, the actuarial insurance premium is determined by Monte

[8] The SPAN lets each CCP determine the minimum margin for each contract, the volatility range of each underlying asset, the definition of an extreme move in the value of the underlying asset, as well as the diversification effect between futures with different maturities or between futures written on different underlying assets (see CFTC, 2001 for more details).

[9] Kupiec (1994) and Kupiec and White (1994) do not use actual historical SPAN margins but hypothetical margins generated using a model that mimics the SPAN system.

Carlo simulations of the joint default process and payout under the policy. Furthermore, we investigate the effect on the fair value of the premium of including data from October 19th, 1987 in our sample. We find that including the Black Monday dramatically impacts the fair value of the default insurance premium.

Our approach drastically differs from previous attempts to compute the fair value of an insurance against losses from futures trading. Bates and Craine (1999) compute the daily premium on an insurance policy that would cover the loss induced by a futures price change exceeding the margin. Using an option pricing approach, they estimate the insurance premium around the October 1987 crash. A key difference between the Bates and Craine's analysis and ours is that they consider a hypothetical insurance on a single futures contract on the S&P 500 index. Gemmill (1994) assesses the cost of default of a stylized CCP assumed to clear three generic futures contracts – soft commodity, metal, and financial – and to collect £50 million of margin from each market. He considers different levels of default cost and, as acknowledged by the author (on page 991), assigns "highly subjective" default probabilities. Under the assumed default scheduled, Gemmill estimates a rough estimate for the premium of an insurance covering against a default by the CCP. Contrary to previous research, we use *actual* data on CM's *portfolios* to price a realistic insurance policy covering the loss due to the default of one or several CMs.

We claim that there is a clear analogy between the premium of the default insurance and the fair cost of the guarantee provided by the central bank. Indeed, just like a standard insurance company, the Federal Reserve (or U.S. Federal Treasury) may have to compensate the CCP in the event of a default by one or several CMs in order to prevent a breakdown of the financial system. As Bernanke (1990) puts it "the Fed became the insurer of last resort" during past episodes of extreme volatility, such as October 1987. For the Federal Reserve, the deductible corresponds to any guarantee fund held by the CCP. As a result, our study permits to put a dollar amount on the service provided by the central bank, which is the *implicit* insurer of the clearing house. This is ultimately the cost to the tax payer of the implicit protection of the clearing house.

Although the focus of the current paper is on organized derivatives exchanges, our analytic tools can be applied to clearing companies on the over-the-counter (hereafter OTC) market. Indeed, a growing fraction of OTC products are cleared through one of the three main clearing processes. First, some OTC products are converted into equivalent exchange-traded contracts and cleared on derivatives exchanges CCPs.[10] Second, SwapClear, a CCP

[10] For instance, Clearing 360, an exchange service offered by the Chicago Mercantile Exchange since

for OTC interest rate swaps, clears approximately 40% of the global inter-dealer market for interest rate swaps (as of December 2006, Bank for International Settlements, 2007). SwapClear exhibits most of the features of a derivatives exchange CCP, e.g., CMs, regulatory capital and minimum credit rating for CMs, and parent firms. Third, OTC derivatives prime brokers operate very much like CCPs since they clear and settle the trades of a small number (around 30) of large buy-side investors, typically hedge funds. For each transaction, the prime broker interposes itself between the hedge fund and the other party. As a result, a prime broker is a de facto CCP which is exposed to the risk of default by a large end-user.

The outline of the paper is as follows. In the next section, we present our data and analyze the risk properties of the performance bonds and margin variations for house and customer accounts. We pay special attention to the (left) tail behavior of the variation margin - performance bond ratio. In Section 3, we describe the default insurance contract and implement our valuation strategy. We summarize and conclude our study in Section 4.

2 Risk Analysis

2.1 Data and Preliminary Results

Our empirical analysis is based on daily performance bond requirements (B) and daily variation margins (V) for all the CMs of the Chicago Mercantile Exchange's CCP. The performance bonds are computed at the end of each trading day by the SPAN margining system and the variation margins simply reflect daily mark-to-market gains or losses. The sample period covers the period January 4th, 1999 - December 31st, 2001, which includes several prominent episodes such as the burst of the internet bubble and the terrorist attack of September 11th, 2001. For each of our 71 sample CMs, amounts are separated between house accounts, B_H and V_H, and customer accounts, B_C and V_C. This partition of the dataset between proprietary and customer trading reflects the principle of strict segregation of customer funds in place in most CCPs. In our dataset, 60 CMs have both a house and a customer accounts, nine only have a house account, and two manage a customer account

April 2006, takes a bilaterally negotiated OTC swap trade and converts it into a strip of futures contracts, which are then submitted to Chicago Mercantile Exchange for clearing. Similar OTC clearing services are provided by Bclear (Euronext.Liffe), OTC Trade Entry Facility (Eurex), Converge (Canadian Derivatives Clearing Corporation), and Clearport (New York Mercantile Exchange).

only. The total sample size is 43,236 member/day observations for house accounts and 41,013 member/day observations for customer accounts.

We plot in Figure 1 the cumulative performance bond across all CMs averaged over each month. We see that both house account and customer account margins have experienced a positive trend until the third quarter of 2001. Another interesting feature of the data is the higher magnitude of the customer margins (i.e., cumulative B_C around 15 billion dollars per day) compared to house margins (i.e., cumulative B_H around 6 billion dollars per day). This difference between the magnitude of house and customer accounts is due in part to the gross margining regime under which customer accounts are administered. We display in Figure 2 the daily variation margins for house and customer accounts. We notice a clear change in the volatility of the variation margins between the first and second part of our sample period. Our preliminary analysis seems to indicate that the distribution of the performance bonds and variation margins are not stationary through time.

Since we are interested in potential default by a CM, we need to compare, on each day, the performance bond with the variation margin of each CM.[11] In particular, a situation where $B_H + V_H < 0$ is problematic since the CM may decide to default instead of paying the deficiency. In our analysis, we favor an alternative, however equivalent, expression that is the ratio of the variation margin and performance bond. In this case, default may occur if $V_H/B_H < -1$ or $V_C/B_C < -1$.[12] We plot in Figure 3 the time-series of the ratio of the variation margin and performance bond for each type of accounts. This variable indicates the fraction of the margin that is won or lost by a CM on a given day. Contrasting the V/B ratio for house and customer accounts turns to be very informative as this ratio is much more volatile for house accounts than it is for customer accounts. In our sample, there are 68 occurrences when the daily loss on the house account exceeds the posted margin whereas the corresponding number for customer accounts is four. The most severe event corresponds to a loss on proprietary trading that amounts to 173% of the posted margin. In dollar terms, the most severe daily loss (among these 68 occurrences) amounts around 44 million dollars. Interestingly, underwater accounts do not only affect a handful of CMs exhibiting extreme risk appetite as almost one third of our sample CMs have experienced

[11] More precisely, we contrast the performance bond set at the end of a given trading day, $B_{t^{end}}$, with the variation margin computed over the following trading day, V_{t+1}. In the following, we will drop the time index for simplicity. Note that V_{t+1} often differs from $B_{t+1^{end}} - B_{t^{end}}$ since both CM's positions and market volatility vary from one day to the next.

[12] A V/B ratio smaller than -1 is a necessary condition for defaulting but not a sufficient one. Reasons for not defaulting include the benefit of reduced clearing fee rate, reduced collateral cost (i.e., net margining system), membership value, present value of future rents, and reputation effect for parent companies.

at least one margin-exceeding loss.

We complement our visual inspection of the data with some descriptive statistics in Table 1. For each type of account (house vs. customer), we consider four variables of interest, namely the performance bond, the variation margin, as well as the level and the absolute value of the V/B ratio. We see that the average house performance bond is just shy of the \$100 million mark and that the average customer performance bond is almost three times larger. Comparing average V across types of accounts reveals that proprietary trading is on average more profitable than trading by customers, which is consistent with previous studies comparing the performance of various investor types (Grinblatt and Keloharju, 2000, and Frazzini and Lamont, 2007).[13] Although the average and median values of the V/B ratio are zero, the absolute value ratio is, on a typical day, around 15% for house accounts and 7% for customer accounts. In dollar terms, that amounts to 900 million dollars ($= 0.150 \times 6$ billion) for house accounts and 1.11 billion dollars ($= 0.074 \times 15$ billion) for customer accounts. On a technical note, we notice that the B and V variables are much more skewed (especially B_H and B_C) and exhibit fatter tails (especially V_H and V_C) than the V/B ratios.

In the three lower panels of Table 1, we split the three-year sample period into three equal-sized subperiods. Consistent with our analysis of Figures 1 and 2, we find that the average margin account has been growing through time and that the distributions of the variation margins are pathologically non-stationary. Differently, the first four moments of the V/B ratio are remarkably stable across subperiods. Of particular interest is the very low correlation between the V/B ratio and the level of the performance bond, i.e., 0.003 for house accounts and less than 0.001 for customer accounts. Furthermore, this correlation coefficient has never exceeded 1.3% or gone below -1.3% on a given sample year using either house or customer data. This modest in-sample correlation points toward linear independence between the V/B ratio and the margin account size, which is a very important property if one wants to aggregate data across CMs.

In Table 2, we present similar descriptive statistics by account size quantile. Specifically, the first (respectively fourth) quantile considers the 25% smallest (respectively largest) daily performance bonds. Consistent with this partition rule, the average performance

[13] We cannot infer the CM's overall profit/loss from house accounts since much of the trading activity is arbitraging between the futures and over-the-counter markets or cash markets (e.g., cash-futures arbitrage of the S&P 500 index, eurodollar-interest rate swap arbitrage). Thus a negative V might be matched by positive mark-to-market gains elsewhere.

bond grows exponentially from Panel A (smallest accounts) to Panel D (largest accounts). We note that the distribution of the V/B variable remains stable across size quantiles and that the correlation between V/B and B is always close to zero.

In the following sections, we are going to focus our analysis on the V/B ratio rather than on their sum, or on any of their component in isolation. We follow this strategy for the following reasons. First, the V/B ratio allows us to efficiently combine the information about variation margins and performance bonds (no loss of information) in a compact way. Second, the distribution of V/B is much more stable through time and across size quantiles than the distribution of B or V. Third, the very low correlation between V/B and B allows us to aggregate information across CMs without dramatically skewing the data because of account size differences.

The final step of our explanatory analysis of the data is constructing an histogram of the V/B variable. Consistent with our previous results, we note in Table 3 the higher variability of the V/B ratio for proprietary trading than for trading from customers. For instance, 76.2% (respectively 48.62%) of the time V_C/B_C (respectively V_H/B_H) remains within the]-0.1;0.1] range. Another key statistical difference between house and customer accounts is the thickness of the tails of the distribution of V/B. In our house accounts sample, 1 out of 185 observations lies in the extreme left tail (defined as relative losses exceeding 70% of the performance bond, i.e., $V/B < -0.7$) whereas the corresponding frequency is only 1 out of 1,250 observations for customer accounts. With 60 CMs, this implies one extreme (but not rare) event affecting a house account every three days ($\simeq$ 185/60). We also notice that, on average, observations in the tails of the distribution tend to be associated with smaller performance bond. However, as suggested by the maximum values, some sizable accounts also end up far in the tails of the distributions. We specifically analyze the extreme left tail of the distribution in Figure 4. We clearly see that V/B is much more negatively skewed for house accounts than it is for customer accounts. This result further supports the idea that the main source of default risk is proprietary trading and not trading by customers.

2.2 Tail Behavior

Modelling defaults in a derivatives exchange calls for a characterization of the far end of the left tail of the V/B distribution. To do so, we deviate from the standard approach in statistics that is to fit the entire distribution with a given distribution. The intuition is that data in the bulk or in the right tail of the distribution do not help much, and sometimes hurt, in characterizing the left tail of the distribution. Instead, we follow an extreme value approach in which only observations with losses exceeding a pre-specified threshold θ are used.[14]

The result we use is a theorem of Balkema and de Haan (1974) and Pickands (1975) that states that under moderately general conditions the distribution of the excess $z = X - \theta$ of a random variable X over a high threshold θ, conditional on the threshold being exceeded, converges to the generalized Pareto distribution G. This distribution depends on two parameters: a scale parameter σ and a tail shape parameter k, which both can be estimated by maximum likelihood (see Appendix for details). Its cumulative distribution function is:

$$G(z; \sigma, k) = \begin{cases} 1 - (1 - kz/\sigma)^{1/k} & k \neq 0 \\ 1 - e^{-z/\sigma} & k = 0 \end{cases} . \tag{1}$$

For $k > 0$, the above applies for $0 \leq z < \sigma/k$, with $G = 1$ for $z \geq \sigma/k$ and $G = 0$ for $z < 0$ (i.e., the range of z is bounded). The case $k = 0$ will occurs if sampling is from normal, lognormal and several other common distributions. The case $k < 0$ occurs if sampling from fatter tailed distributions. Differentiating G with respect to z gives the probability density function:

$$g(z; \sigma, k) = \frac{1}{\sigma}(1 - kz/\sigma)^{-1+1/k} \tag{2}$$

in the relevant domain. Note that in what follows, exceedance is interpreted as amount by which V/B falls below θ.

The choice of the threshold θ is a crucial step in characterizing the tail of the distribution. In general it should be set large enough that the data exceeding it is clearly in the relevant tail of the distribution, but small enough that the remaining sample is adequate for estimating the remaining parameters. In our test, we perform estimation for a range of θ's from -0.7 to -0.9. This corresponds to daily losses ranging from 70 to 90% of the posted margin. Furthermore, estimation of the tail distribution parameters is done separately for house and customer accounts. We report in Table 4 the maximum likelihood estimation results. For each threshold level, we display the number of observations in the tail n, the estimated parameters k and σ, along with their asymptotic standard errors. We see that out of the 43,236 member/day observations for house accounts, 233 observations lie in the

[14] Over the past decade, Extreme Value Theory has been increasingly used to model extreme events in finance, e.g. Longin (1996, 2000, 2005), Embrechts, Kluppelberg, and Mikosch (1997), Diebold et al. (1998), McNeil and Frey (2000), Neftci (2000), Longin and Solnik (2001), Poon, Rockinger, and Tawn (2004), and McNeil, Frey, and Embrechts (2005).

left tail of the distribution when the threshold is set to -0.7. Moreover, the number of exceedances mechanically decreases as the threshold becomes further negative, e.g. $n = 142$ when $\theta = -0.8$. Depending on the considered threshold, the tail shape parameter estimate ranges from 0.0107 to 0.2492 and the scale parameter ranges from 0.1321 to 0.3059. We also notice that, as expected, standard errors decrease with the sample size. When customer account data are used, the number of exceedances is much smaller than with house account data. In particular, when $\theta = -0.9$, the number of exceedances is as low as eight. This latest result confirms our analysis in Section 2.1 in which we document that extreme losses affect more frequently house accounts. The results from this section are going to play an important role in the valuation of the default insurance, which we next explore.

3 Default Insurance

3.1 Contract Features and Valuation Strategy

We now turn to the development of an insurance contract that protects the CCP against the default of one or several of its CMs. We assume that CM default occurs according to the following stochastic process. Default requires that (a) futures prices have changed sufficiently since the previous day that loss in margin account exceeds performance bond posted, and (b) member elects to default rather than pay the deficiency. Over the fixed term of T years, there is a policy deductible, or first loss amount born by the insured, D, and overall payout limit L. Premium is paid in advance for each payment interval, which have length t_p years, at an annual rate of p per dollar of policy limit, i.e., payment on each such date is $p \cdot t_p \cdot L$.[15] Another realistic feature of our contract is that accumulated default losses for purpose of the deductible is reset to zero at the end of each reset interval of length t_r.

In our model we make the following two assumptions. First, default by a CM can only be due to a deficiency in the house account. Second, account value change as a proportion of previous day performance bond is independent of the size of the performance bond. Both assumptions are consistent with the risk analysis presented in Section 2. Indeed, we have shown that deficiencies in house accounts are much more frequent than deficiencies in customer accounts. Furthermore, the independence assumption is motivated by the fact that empirical correlations between V/B and B are close to zero.

[15] It is conventional in the insurance industry to report the insurance premium in basis points per year on policy maximum payout.

The main building blocks of our model are as follows. Value changes of policy-relevant magnitude are in the negative tail of the distribution of proportional value changes of any given firm. As in Section 2, the tail is defined by proportional value loss exceeding a threshold θ. Value changes of policy-relevant magnitude are caused by the arrival of trigger events, which arrive as a Poisson process with constant arrival intensity λ. Upon arrival of a trigger event, proportional value loss for a given firm exceeds θ with a fixed probability π. In our model, exceedance of the threshold may be correlated across firms and a single exceedance correlation parameter ρ_θ characterizes the common pairwise correlation. Conditional upon a trigger event and exceedances occurring, the size of exceedance z for a given firm is a random draw from a generalized Pareto distribution with scale parameter σ and tail shape parameter k. We allow the size of exceedances to be correlated across firms and we use a single exceedance size correlation parameter ρ_z to characterize the common pairwise correlation. Firms with value loss exceeding performance bond, $z + \theta < -1$, are candidates for default. Default occurs independently across firms with probability π_1 for $-2 \leq z + \theta < -1$ and with probability π_2 for $z + \theta < -2$.

The nine parameters of the default process are thus $\lambda, \pi, \theta, k, \sigma, \rho_\theta, \rho_z, \pi_1, \pi_2$. Of these, θ and π are technical parameters and statistician imposed in the tail analysis. Given these, λ, k, σ are estimated by maximum likelihood methods from historical data. The remaining parameters $\rho_\theta, \rho_z, \pi_1, \pi_2$ are analyst imposed.[16] Note that since no defaults have occurred over our sample period, any empirical estimate of π_1, π_2 would of course be zero.

Fair actuarial pricing for the contract is the p satisfying:

$$p \cdot A(T, t_p, r) \cdot L = E(\text{NPV of Policy Payouts}) \tag{3}$$

where $A(T, t_p, r)$ is the present value of a \$1/year annuity for T years (readily calculated analytically), paid in advance at intervals t_p, in a constant interest rate r environment. L is the policy limit on which the premium is paid and p is the annual premium rate. The right hand side of the equation is estimated as the average net present value (NPV) of policy payouts over Monte Carlo simulations of the default process modelled above.[17]

For given λ, k, σ values, each path is simulated as follows. An exponentially distributed random time dt to next Poisson trigger event is drawn. This is done by drawing a uniform

[16] In Section 3.5, we investigate the effect on the insurance premium of changing the value of the key parameters.

[17] In Equation (3), future cash-flows are discounted using the risk-free rate since it is very difficult to make a case for having a positive or negative beta for this insurance contract.

$[0,1]$ random variable u, then setting $\Delta t = -\ln u/\lambda$. If time since start crosses a deductible reset date, cumulative loss is reset to zero. Standard normal random variables x_0, y_0 are drawn for the common factors influencing whether individual firms exceed the value loss thresholds and the amount by which they exceed. Looping through each of the M firms, an independent standard normal x_i is drawn and the correlated standard normal $x = \rho_\theta^{1/2} x_0 + (1-\rho_\theta)^{1/2} x_i$ formed. If $x < F^{-1}(\pi)$, where F^{-1} is the inverse normal cumulative distribution function, then firm i is determined to have exceeded loss threshold θ. This implies that exceedance occurs with probability π. If firm i exceeds the loss threshold, another independent standard normal y_i is drawn and the correlated standard normal $y = \rho_z^{1/2} y_0 + (1-\rho_z)^{1/2} y_i$ formed. An exceedance size is computed as $z = G^{-1}(F(y))$, where F is the cumulative normal distribution function and G^{-1} is the inverse of the generalized Pareto cumulative distribution function. Note that $F(y)$ will be a (correlated) uniform random variable, and thus z will be a correlated random variable following a generalized Pareto distribution. If the proportional loss exceeds the performance bond, $\theta + z < -1$, the firm is a candidate for default. Default or not is determined by a further independent uniform random draw, with default probability either π_1 or π_2 depending on the magnitude of $\theta + z$.

If default by firm i occurs, the default cost is set by drawing a performance bond level from the empirical distribution of B^H. The range of performance bond levels is partitioned into buckets from 0 to maximum observed level. The number of daily observations of firms in each bucket is counted over the year. This determines an empirical distribution function $F(S) = Pr\{s \leq S\}$ of firm sizes, assumed to be linear between bucket levels. The number of clearing members is assumed constant at $M = 60$, which is the average number of clearing members. Firm sizes for simulation are then the M sizes given by $s_i = Q^{-1}(i/M)$, where Q is the size distribution and $i = 1, \ldots, M$. In the event of a default by CM i, payment by the insurance company is set at the minimum of the default amount, the remaining aggregate policy limit, and the amount by which cumulative defaults to date exceed the deductible. Any payment is discounted back to the present at the risk-free rate and added to cumulative present value of payouts.

3.2 Valuation Results

We consider the following base case for the insurance policy: a three-year policy, with a \$500 million deductible, a \$500 million payout limit, a deductible reset interval of 0.25 years, and a premium payment interval of 0.25 years. The values of the other parameters are $\rho_\theta = \rho_z = 0.5$, $\pi_1 = 0.2$, $\pi_2 = 0.5$, and $r = 5\%$.[18] Assuming 250 trading days per year, annualized arrival rate of threshold exceedances per firm is estimated as $a = n/(250 \times 60 \times 3)$, where n is the number of exceedances of θ in the sample. Estimated arrival rate of trigger events is then $\lambda = a/\pi$ with $\pi = 0.25$. Furthermore, we take the size distribution as fixed at the average distribution over the most recent sample year (2001).

We report in Table 5 the estimated insurance premium for several threshold levels. Our main finding is that the premium is around two basis points per year on policy maximum payout, with standard errors approximately one-fifth of the point estimate. Given the $500 million payout limit assumed in our base case (see Panel A), a two basis point premium rate results in $100,000 premium payments per year. Since the term of the policy is three years, the total insurance premium is around $300,000. We also report the probability of any payout event over term of policy (p_{pay}), the probability of policy limit payout over term of policy (p_{max}), and the probability of at least one firm defaults during policy term (p_{def}). The marginal effects of having a policy payout limit and/or a deductible are investigated in Panels B-D of Table 5. As expected, these two features have dramatic effect on the insurance premium and on the probabilities of any payout or maximum payout. For instance, with a threshold equals to -0.8, waving the deductible increases the probability of any payout by a ratio of 19 and the insurance premium (or value) by a ratio of six. Furthermore, relaxing jointly the cap on the payout and setting the deductible to zero pushes the value of the insurance to around $2 million.

To further illustrate our simulation-based pricing strategy, we display in Figure 5 the value and the frequency of the simulated total default losses obtained from 40,000 simulated paths. In this experiment, we use our base case with a threshold of -0.8. There are 768 paths ($p_{def} = 1.92\%$) experiencing at least one default and, as a result, 39,232 paths (1 - $p_{def} = 98.08\%$) experiencing no default. Although some of the total losses are sizable – the maximum value is $2.08 billion – the majority of the losses remains below the $25 million mark.[19] Moreover, we find that 2% of the total default losses exceed $1 billion.

We investigate in Table 6 whether our pricing results are critically sensitive to the value of certain key parameters. In this comparative statics exercise, we recompute the value of the default premium, along with the associated probabilities (p_{pay}, p_{max}, and

[18] The relatively small values for π_1 and π_2 reflect the apparent strong economic incentive of CM for not defaulting.

[19] The discrepancy between DefMax in Panel A of Table 5 and the maximum value in Figure 5 is due to the fact that DefMax measures individual defaults whereas Figure 5 displays *total* or *cumulative* defaults over the entire path.

p_{def}), while changing the value of one parameter at the time. The value of the other parameters remains equal to their base-case value and the threshold we use is -0.8. We consider designated changes in parameters of plus or minus 50% of the base-case value (e.g. from $\rho_\theta = 0.5$ to $\rho_\theta = 0.75$ and 0.25). We also consider an extreme case in which we set all correlation parameters (ρ_θ and ρ_z) to zero. While the estimation results remain rather stable on the whole, several findings are worthwhile being particularly stressed. First, π_1 is highly influential, as one might expect. Second, changing the value of π_2 has no effect whatsoever on the results. The reason is that no V/B values smaller than -2 occurred in the simulations with this particular set of parameters. Indeed, as shown in Section 2.2, the most negative value for V/B is -1.9543 (i.e., the upper bound is 1.9543 = 0.2695 / 0.1379).

3.3 Value of the Federal Reserve Guarantee

The insurance contracts developed in this section allows us to put a dollar amount on the service provided by the central bank. Indeed, the Federal Reserve (or the U.S. Federal Treasury) may ultimately have to act as the *implicit* insurer of the clearing house in case of a major financial meltdown (Bernanke, 1990). The cost of this service is given by the fair value of a default insurance with no payout limit and a "deductible" equal to the total security deposit held by the CCP. In order to estimate the size of this deductible one needs to understand the default procedure in place at the Chicago Mercantile Exchange. In the event of a payment default that originates from a house account, the CCP takes the following actions (in this order): (1) transfer all customer positions to another CM; (2) take control or liquidate the positions in the defaulting house accounts; (3) apply the CM's security deposit and house performance bonds to the failed obligation – recall customer performance bonds may not be used by the CCP; (4) attach all other assets of the CM that are available to the CCP, such as membership and pledged shares; (5) invoke any applicable parent guarantee; (6) use lines of credit as a temporary liquidity facility; (7) apply the surplus fund and aggregate security deposit of the clearing membership; and (8) assess non-defaulting CMs for any unsatisfied obligations up to a pre-specified cap, expressed as a multiple of the aggregate security deposit.

As of December 31st, 2001 (i.e., end of our sample period), the available financial resources in the case of a default are $821 million (from (1) to (7)) with an additional $2,358 million in capital assessment powers (8).[20] Although the former amount is readily

[20] The $821 million figure is obtained by adding $3 million (market value of pledged shares and membership), $113 million (surplus funds), and $705 million (security deposits of clearing firms). Note also that

available to the CCP, the latter is more of a theoretical bound since there is no deposit yet. In particular, it is not completely clear which fraction could eventually be obtained in a timely manner. Furthermore, since the allocation would be based on each CM's share of the security deposit pool, the size of the assessment would be greatly reduced should the defaulting firms be the largest CMs. Finally, some CMs may decide to default on this extra capital assessment. For these reasons, we use in the following a "deductible" of $821 million.

We present the estimation results in Table 7. The value of the commercial equivalent of the Federal Reserve guarantee is between $200,000 and $380,000 for a three-year policy. The magnitude of the insurance premia is consistent with our results in Table 5 but are rather small, especially when compared to actual historical losses faced by the Federal Reserve. For instance, the ultimate cost of the U.S. Savings and Loan crisis of the 1980s and 1990s is estimated to have totaled around $160 billion, among which about $124 billion was directly paid by the U.S. government. However, the figures reported in Table 7 correspond to expected costs, not actual ones. Although the theoretical cost of this guarantee is modest, the value for the society is huge. Indeed, reducing the chance of a systemic collapse of the financial system is priceless. The various economics and financial disruptions triggered by a CCP failure would have a huge social cost.

A few additional remarks need to be done. First, our estimates are based on pre-merger data from 1999-2001. As a result, current figures would certainly need to be scaled up. Second, our sample does not include the most critical episode in the history of derivatives clearing, namely October 1987. We study the impact on our key results of including the Black Monday in Section 3.4. Third, our estimates do not account for defaults on customer accounts – an issue further discussed in Section 3.5.

3.4 Black Monday Effect

On October 19th, 1987, the Chicago Mercantile Exchange's S&P 500 futures contract dropped 80.75 points to 201.50. As mentioned by Bernanke (1990) and Bates and Crane (1999), rumors about possible clearinghouse failures spread out quickly on Black Monday which added to the sense of panic in the markets. In this section, we complement our original dataset with performance bonds and variation margins for all Chicago Mercantile

the CCP has a $500 million line of credit but this amount is not included by the CME in the definition of the total financial safeguard package (Source: Chicago Mercantile Exchange 10K form for fiscal year 2001).

Exchange CMs' house accounts on October 19th, 1987. This expanded sample allows us to quantify the impact of a major crisis on the fair value of a default insurance contract. From a regulator's point of view, information about an actual crisis situation can be of great interest.

The main concern to add one extreme day from 1987 to our sample is the fact that margins in the eighties were much smaller than in recent years. However, since the variable of interest is the V/B ratio, the size difference does not alter the analysis. Furthermore, as expected on a market crash, the number of underwater accounts is extremely high. We find that 15 CMs experienced a loss exceeding their posted margin on this particular day. However, as many of them concerned very small accounts, we eliminate from consideration member accounts with performance bonds less than $10 million. This conservative approach allows us to not unduly exaggerate the role of the crash in the analysis. We end up with four V/B ratios smaller than a -1 and five smaller than -0.7 (our larger threshold), namely -4.646, -4.482, -4.430, -4.052, -0.979. We note that these values are extremely high, without any comparison with the values observed in our original (no-crash) sample.

Armed with this expanded dataset, we re-conduct our tail analysis and present the results in the Panel A of Table 8. A direct effect of adding four outliers is the lower value of the tail shape parameter. The parameter estimate is also more stable across thresholds as it remains between -0.315 and -0.340. This is consistent with a much fatter left tail for the distribution of V/B. The estimated value of the scale parameter is significantly less affected by the inclusion of the market crash. Of particular interest is the impact of the extra data on the value of the insurance contract. We report the estimated insurance premium for our base case in Panel B of Table 8. Although the probability of having an extreme event remains virtually unchanged, the losses are much more severe. This is reflected in the probability of having payout event over policy term, the probability of policy limit payout over policy term, and in turn, by the insurance premium which is multiplied by more than three compared to Table 5 (Panel A). Note that the maximum default size appears to be a major contributor to the premium difference with the 1987 data. Indeed, the simulated default loss can be as high as $13.6 billion.

Finally, we recompute the value of the Federal Reserve guarantee in Panel C of Table 8. In line with the first two panels, our estimates differ drastically from our previous analysis. As for the Federal Reserve guarantee, the estimated fair value is multiplied on average by 10, with a value around $3 million. Thus, including information about actual financial crises turns out to be particularly important in assessing the fair cost of the protection provided by the Federal Reserve. This result is of primary importance since it is only in such extreme market conditions that the central bank eventually becomes insurer of last resort (Bernanke, 1990).

3.5 Default on a Customer Account

In this section, we discuss several potential extensions of the current analysis. So far, the only considered source of CM default is proprietary trading. However, it is plausible that the default originates from a customer position. As already mentioned, this event is far less likely to occur for two reasons. First, customer accounts are administered under gross margining and second, the CM's house performance bonds are pledged to cover any deficiency on the customer account. Technically speaking, the second reason implies that one would have to switch from V_C/B_C to $V_C/(B_C + B_H)$ in case of payment default on a customer account. This prevents us from using our model with customer account data.

Although CCPs segregate customer performance bonds from house performance bonds, customer performance bonds for each CM are pooled together. If a default occurred in the CM's customer account, the CCP applies toward the default all customer performance bond deposits and positions in the defaulting CM's customer origin account. Accordingly, positions and performance bonds deposited by customers not causing the default are potentially at risk, i.e., they can be liquidated (Jordan and Morgan, 1990). A potential adverse effect of this is that investors may be reluctant to channel their trades through a CM that has large, risk-seeker customers. This could be particulary acute in absence of proprietary trading since, in that case, no house performance bond is pledged. In order to reduce the perverse effect of information asymmetry, such a CM could purchase a default insurance like the one developed in this paper to protect its non-defaulting customers, and in turn, increase its market share.

4 Conclusion

A clearing house failure is a major systemic risk event. In this paper, we take an inside look at a central counterparty clearing house, with a special emphasis on its exposure to default risk. Using actual daily data on margin requirements and variation margins for all the clearing members of the Chicago Mercantile Exchange's clearing house, we shed some light on the potential sources of default risk within the clearing house. In particular, we identify many occurrences when a member's daily loss exceeds his posted margin. We document that underwater accounts are much more frequent for house accounts than it is for customer accounts, which suggests that the major source of default risk is proprietary trading. Furthermore, we quantify the default risk by characterizing the left tail of the distribution of the ratio between variation margin and performance bond.

In this paper, we also develop, and price, a realistic insurance contract covering the loss to the clearing house from default by one or several clearing members. This financial innovation allows a substitution of the implicit insurer of the clearing house, i.e., the central bank, by an explicit insurer, i.e., one or several insurance companies. The estimate of the insurance premium can also be interpreted as the fair cost of the service provided by the central bank as an implicit insurer of the clearing house.

The insurance policy designed in this paper could be employed in different contexts. Indeed, it could be used to compute the fair premium of an insurance policy covering the loss to a brokerage firm from default by one or several customers, especially if the firm is also a CM. A first beneficial effect of such a contract would be to protect the performance bond of the firm. A second benefit would be to protect the performance bond of non-defaulting customers in the case of a default by another customer of this CM. Finally, although our insurance contract has been developed for derivatives exchanges, it could be calibrated in a similar way to OTC de-facto clearing institutions, such as prime brokers.

Appendix: Estimation of Tail Distribution Parameters

Suppose we have a large set of N independent observations of a random variable X. Choose a tail estimation threshold θ large enough so that the observations exceeding θ are credibly in the upper tail, but not so large that the number of observations is insufficient to confidently identify k and σ. Let n be the number of observations exceeding θ. There is a tradeoff here between bias, from not being sufficiently far in the tail for the asymptotic distribution result to hold, and variance, from having too few observations to estimate k and σ.

Let X_i, $i = 1, ..., n$, denote the observations above the θ selected, and $z_i = X_i - \theta$ the corresponding exceedances. Using the i.i.d. assumption, the log of the likelihood function for this data is the sum of the log density functions from Equation (2):

$$\mathcal{L} = -n \ln \sigma - (1 - \frac{1}{k}) \sum_{i=1}^{n} \ln(1 - k z_i / \sigma). \tag{A1}$$

Maximum likelihood estimates are the values of k and σ that maximize this expression. First order conditions for a maximum of $\mathcal{L}$ with respect to k and σ are:

$$\mathcal{L}_\sigma = 0 = -n/\sigma - (1 - \frac{1}{k}) \sum_i \frac{k z_i / \sigma^2}{1 - k z_i / \sigma} \tag{A2}$$

$$\mathcal{L}_k = 0 = -\frac{1}{k^2} \sum_i \ln(1 - k z_i / v) + (1 - \frac{1}{k}) \sum_i \frac{z_i / \sigma}{1 - k z_i / \sigma}. \tag{A3}$$

We define a new variable $c \equiv k/\sigma$ and substitute into Equation (A2). The pair of equations can be partially solved to:

$$k = -\frac{1}{n} \sum_i \ln(1 - c z_i) \tag{A4}$$

$$0 = n + \left(1 + \frac{n}{\sum_i \ln(1 - c z_i)}\right) \sum_i \frac{c z_i}{1 - c z_i}. \tag{A5}$$

Note that the last equation contains only the single unknown c. Although not analytically solvable, it is readily numerically solvable for $\hat{c}$ (e.g., by Newton's method or the secant method). Then k can be computed from Equation (A4), and $\hat{\sigma} = \hat{k}/\hat{c}$.

References

[1] Bailey, W. and E. Ng (1991), Default Premiums in Commodity Markets: Theory and Evidence, *Journal of Finance*, 46, 1071-1093.

[2] Bank for International Settlements (2007), New Developments in Clearing and Settlement Arrangements for OTC Derivatives, Committee on Payments and Settlements Systems, Bank for International Settlements.

[3] Bates, D. and R. Craine (1999), Valuing the Futures Market Clearinghouse's Default Exposure during the 1987 Crash, *Journal of Money, Credit, and Banking*, 31, 348-372.

[4] Bernanke, B. (1990), Clearing and Settlement during the Crash, *Review of Financial Studies*, 3, 133-151.

[5] Booth, G. G., J. P. Broussard, T. Martikainen, and V. Puttonen (1997), Prudent Margin Levels in the Finnish Stock Index Futures Market, *Management Science*, 43, 1177-1188.

[6] Brennan, M. J. (1986), A Theory of Price Limits in Futures Markets, *Journal of Financial Economics*, 16, 213-233.

[7] Commodity Futures Trading Commission (1998), Review of the Financial and Sales Practice Compliance Program of the Chicago Mercantile Exchange, Report of the Division of Trading and Markets.

[8] Commodity Futures Trading Commission (2001), Review of Standard Portfolio Analysis of Risk ("SPAN") Margin System, Report of the Division of Trading and Markets.

[9] Cotter, J. (2001), Margin Exceedences for European Stock Index Futures using Extreme Value Theory, *Journal of Banking and Finance*, 25, 475-502.

[10] Cotter, J. and K. Dowd (2006), Extreme Spectral Risk Measures: An Application to Futures Clearinghouse Margin Requirements, *Journal of Banking and Finance*, 30, 3469-3485.

[11] Day, T. E. and C. M. Lewis (2004), Margin Adequacy and Standards: An Analysis of the Crude Oil Futures Market, *Journal of Business*, 77, 101-135.

[12] Embrechts, P., C. Kluppelberg, and T. Mikosch (1997), *Modelling Extremal Events for Insurance and Finance*, Springer-Verlag, Berlin.

[13] Fenn, G. W. and P. Kupiec (1993), Prudential Margin Policy in a Futures-Style Settlement System, *Journal of Futures Markets*, 13, 389-408.

[14] Figlewski, S. (1984), Margins and Market Integrity: Margin Setting for Stock Index Futures and Options, *Journal of Futures Markets*, 4, 385-414.

[15] Frazzini, A. and O. Lamont (2007), Dumb Money: Mutual Fund Flows and the Cross-Section of Stock Returns, *Journal of Financial Economics*, forthcoming.

[16] Gay, G. D., W. C. Hunter, and R. W. Kolb (1986), A Comparative Analysis of Futures Contract Margins, *Journal of Futures Markets*, 6, 307-324.

[17] Gemmill, G. (1994), Margins and the Safety of Clearing Houses, *Journal of Banking and Finance*, 18, 979-996.

[18] Grinblatt, M. and M. Keloharju (2000), The Investment Behavior and Performance of Various Investor-Types: A Study of Finland's Unique Data Set, *Journal of Financial Economics*, 55, 43-67.

[19] Jordan, J. V. and G. E. Morgan (1990), Default Risk in Futures Markets: The Customer-Broker Relationship, *Journal of Finance*, 45, 909-933.

[20] Knott, R. and M. Polenghi (2006), Assessing Central Counterparty Margin Coverage on Futures Contracts using GARCH Models, Working Paper, Bank of England.

[21] Kroszner, R. S. (2006), Central Counterparty Clearing: History, Innovation, and Regulation, Speak at the European Central Bank and Federal Reserve Bank of Chicago Joint Conference on Central Counterparty Clearing, Frankfurt, Germany.

[22] Kupiec, P. (1994), The Performance of S&P 500 Futures Product Margins under the SPAN Margining System, *Journal of Futures Markets*, 14, 789-811.

[23] Kupiec, P. and P. White (1996), Regulatory Competition and the Efficiency of Alternative Derivative Product Margining Systems, *Journal of Futures Markets*, 16, 943-968.

[24] Longin, F. (1996), The Asymptotic Distribution of Extreme Stock Market Returns, *Journal of Business*, 63, 383-408.

[25] Longin, F. (2000), From Value at Risk to Stress Testing: The Extreme Value Approach, *Journal of Banking and Finance*, 24, 1097-1130.

[26] Longin, F. (2005), The Choice of the Distribution of Asset Returns: How Extreme Value Theory Can Help?, *Journal of Banking and Finance*, 29, 1017-1035.

[27] Longin, F. and B. Solnik (2001), Extreme Correlation of International Equity Markets, *Journal of Finance*, 56, 649-676.

[28] McNeil, A. J. and R. Frey (2000), Estimation of Tail Related Risk Measures for Heteroscedastistic Financial Time Series: An Extreme Value Approach, *Journal of Empirical Finance*, 7, 271-300.

[29] McNeil, A. J., R. Frey, and P. Embrechts (2005), *Quantitative Risk Management: Concepts, Techniques and Tools*, Princeton University Press, Princeton, NJ.

[30] Poon, S. H., M. Rockinger, and J. Tawn (2004), Extreme Values Dependency in International Stock Markets: Diagnostics, Models, and Financial Implications, *Review of Financial Studies*, 17, 581-610.

Table 1: Descriptive Statistics

	B^H	V^H	$\frac{V^H}{B^H}$	$\frac{\lvert V^H \rvert}{B^H}$	B^C	V^C	$\frac{V^C}{B^C}$	$\frac{\lvert V^C \rvert}{B^C}$
Panel A: 1999-2001								
Mean	98.78	0.47	0.003	0.150	267.78	-0.53	-0.002	0.074
Median	13.96	0.00	0.000	0.105	40.48	0.00	0.000	0.045
Std-Dev	226.79	43.92	0.217	0.156	546.68	71.93	0.117	0.090
Skewness	4.84	0.74	0.137	2.662	3.41	-0.89	0.205	3.494
Kurtosis	35.54	116.71	8.774	15.949	17.16	138.70	16.133	28.966
Corr(B^i,•)	1.000	0.042	0.003	-0.040	1.000	-0.021	0.000	-0.010
Panel B: Year 1999								
Mean	68.22	-0.24	0.001	0.131	212.95	0.37	-0.001	0.068
Median	14.48	0.00	0.000	0.092	38.84	0.00	0.000	0.042
Std-Dev	129.94	23.69	0.190	0.137	444.41	44.96	0.106	0.082
Skewness	3.20	-0.38	0.033	2.587	3.36	0.80	-0.122	3.153
Kurtosis	15.38	55.80	8.338	14.366	16.22	71.78	12.281	20.194
Corr(B^i,•)	1.000	-0.029	-0.013	-0.027	1.000	0.040	0.013	-0.045
Panel C: Year 2000								
Mean	83.37	0.96	0.006	0.165	252.22	-1.19	-0.003	0.081
Median	14.09	0.00	0.003	0.118	40.50	0.00	-0.001	0.048
Std-Dev	176.04	42.02	0.233	0.165	505.05	74.77	0.131	0.102
Skewness	3.74	-0.04	0.257	2.500	3.39	-0.80	0.472	3.788
Kurtosis	20.94	77.54	7.830	14.315	16.51	120.15	19.163	34.778
Corr(B^i,•)	1.000	0.060	0.012	-0.013	1.000	-0.048	-0.008	-0.006
Panel D: Year 2001								
Mean	148.00	0.72	0.003	0.156	346.59	-0.82	-0.002	0.072
Median	13.36	0.00	0.000	0.109	41.81	0.00	0.000	0.045
Std-Dev	325.83	59.86	0.226	0.163	671.42	91.07	0.112	0.086
Skewness	3.88	0.94	0.046	2.792	3.06	-1.03	0.071	3.031
Kurtosis	21.32	85.06	9.460	17.586	13.95	109.57	11.710	19.294
Corr(B^i,•)	1.000	0.047	0.004	-0.079	1.000	-0.025	0.000	0.009

Notes: This table presents descriptive statistics for house (H) and customer (C) margin accounts from 71 clearing members from the Chicago Mercantile Exchange's clearing house. There are 60 clearing members with both house and customer accounts, nine with an house account only, and two with a customer account only. For house margin accounts, we focus on the end-of-the-day performance bond (B^H), end-of-the-day variation margin (V^H), their ratio (V^H/B^H), and the ratio of the absolute variation margin and performance bond ($|V^H|/B^H$). We consider similar variables for customer margin accounts (B^C, V^C, V^C/B^C, and $|V^C|/B^C$). For each variable, the sample size is 43,236 member/day observations for house accounts and 41,013 member/day observations for customer accounts. We compute the mean, median, standard-deviation (all in million of dollars for the B and V variables), skewness, kurtosis, and correlation between the performance bond and the other variables. The descriptive statistics are computed for the entire sample period in Panel A (1999-2001) and then for each year separately in Panels B-D (1999, 2000, and 2001).

Table 2: Descriptive Statistics by Size Quartiles

	B^H	V^H	$\frac{V^H}{B^H}$	$\frac{\lvert V^H \rvert}{B^H}$	B^C	V^C	$\frac{V^C}{B^C}$	$\frac{\lvert V^C \rvert}{B^C}$
Panel A: First Quartile (Smallest 25% Performance Bonds)								
Mean	0.35	0.00	0.007	0.172	2.08	0.00	-0.001	0.080
Median	0.19	0.00	0.000	0.108	1.82	0.00	0.000	0.047
Std-Dev	0.38	0.12	0.261	0.197	1.55	0.28	0.128	0.100
Skewness	1.16	0.95	0.126	2.677	0.58	0.12	0.090	3.179
Kurtosis	3.27	43.92	9.354	14.528	2.38	24.19	12.784	19.970
Corr(B^i,●)	1.000	0.03	-0.001	-0.071	1.000	-0.01	0.000	-0.123
Panel B: Second Quartile								
Mean	6.89	0.01	0.003	0.148	18.88	-0.07	-0.004	0.082
Median	6.67	0.01	0.001	0.105	15.68	0.00	0.000	0.052
Std-Dev	3.52	1.69	0.211	0.150	10.33	2.80	0.125	0.095
Skewness	0.22	-0.18	0.113	2.546	0.57	-0.13	-0.119	2.827
Kurtosis	1.99	13.50	8.199	15.067	2.04	21.45	10.091	16.002
Corr(B^i,●)	1.000	-0.01	-0.023	0.058	1.000	-0.01	0.002	0.044
Panel C: Third Quartile								
Mean	32.86	-0.03	0.000	0.149	107.54	-0.26	-0.002	0.071
Median	27.19	-0.01	0.000	0.108	81.43	-0.01	0.000	0.042
Std-Dev	16.78	7.53	0.209	0.147	65.36	14.47	0.116	0.092
Skewness	0.92	-0.02	0.113	2.217	0.90	-0.80	0.913	4.820
Kurtosis	2.73	10.08	6.595	10.909	2.41	31.67	30.117	57.434
Corr(B^i,●)	1.000	-0.01	-0.006	-0.014	1.000	-0.03	-0.013	0.004
Panel D: Fourth Quartile (Largest 25% Performance Bonds)								
Mean	355.04	1.90	0.003	0.134	942.58	-1.76	-0.002	0.063
Median	256.89	-0.14	-0.001	0.102	741.92	-0.01	0.000	0.039
Std-Dev	340.29	87.49	0.181	0.121	759.90	143.09	0.095	0.072
Skewness	3.03	0.32	0.174	2.006	1.81	-0.43	-0.074	2.428
Kurtosis	14.78	29.63	5.914	11.713	6.59	35.41	8.095	11.827
Corr(B^i,●)	1.000	0.04	0.014	0.006	1.000	-0.02	0.000	0.152

Notes: This table presents size-based descriptive statistics for house and customer margin accounts. For house margin accounts, we focus on the end-of-the-day performance bond (B^H), end-of-the-day variation margin (V^H), their ratio (V^H/B^H), and the ratio of the absolute variation margin and performance bond ($|V^H|/B^H$). We consider similar variables for customer margin accounts (B^C, V^C, V^C/B^C, and $|V^C|/B^C$). For each variable, the sample size is 43,236 member/day observations for house accounts and 41,013 member/day observations for customer accounts. We compute the mean, median, standard-deviation (all in million of dollars for the B and V variables), skewness, kurtosis, and correlation between the performance bond and the other variables. The descriptive statistics are computed for four quartiles, from the smallest 25% performance bonds in Panel A to the largest 25% performance bonds in Panel D.

Table 3: Performance Bonds and V/B Ratios

		House Accounts				Customer Accounts			
from	to	$E(B \mid \bullet)$	$Max(B \mid \bullet)$	Obs	Freq	$E(B \mid \bullet)$	$Max(B \mid \bullet)$	Obs	Freq
-∞	-0.7	33.5	1,604	234	0.54%	87.5	842	32	0.08%
-0.7	-0.6	37.6	620	164	0.38%	144.2	2,851	27	0.07%
-0.6	-0.5	69.5	1,989	338	0.78%	146.9	3,851	59	0.14%
-0.5	-0.4	74.6	1,930	620	1.43%	144.7	3,625	151	0.37%
-0.4	-0.3	75.9	2,233	1,302	3.01%	225.1	3,639	357	0.87%
-0.3	-0.2	101.9	2,183	2,690	6.22%	292.5	4,140	1,080	2.63%
-0.2	-0.1	107.6	2,280	5,663	13.10%	278.8	4,421	3,455	8.42%
-0.1	0	98.7	2,440	10,938	25.30%	264.4	4,508	15,656	38.17%
0	0.1	97.3	2,608	10,082	23.32%	269.0	4,815	15,598	38.03%
0.1	0.2	110.7	2,636	5,555	12.85%	289.4	4,173	3,127	7.62%
0.2	0.3	109.3	2,291	2,819	6.52%	248.7	4,086	914	2.23%
0.3	0.4	102.1	2,134	1,424	3.29%	256.1	3,719	312	0.76%
0.4	0.5	65.2	1,979	627	1.45%	195.3	3,671	131	0.32%
0.5	0.6	56.7	1,916	340	0.79%	112.6	1,416	66	0.16%
0.6	0.7	53.2	2,152	160	0.37%	234.3	3,302	17	0.04%
0.7	+∞	24.0	845	280	0.65%	47.4	170	31	0.08%

Notes: This table presents summary statistics for the performance bonds and the histograms of the ratio of the variation margin (V) and performance bond (B). $E(B \mid \bullet)$ denotes the expected value of the performance bond B conditional on B being within a given range (e.g., between $-\infty$ and -0.7 in the first row). $Max(B \mid \bullet)$ denotes the maximum value of the performance bond B conditional on B being within a given range (e.g., between -0.7 and -0.6 in the second row). Both $E(B \mid \bullet)$ and $Max(B \mid \bullet)$ are measured in million of dollars. Obs is the number of observations in each range. Freq is the frequency in percent in each range.

Table 4: EVT-based Tail Analysis

θ	n	k	s.e.(k)	σ	s.e.(σ)
Panel A: House Accounts					
-0.7	233	0.0107	0.0648	0.2243	0.0207
-0.8	142	0.1379	0.0723	0.2695	0.0297
-0.9	90	0.2492	0.0791	0.3059	0.0395
Panel B: Customer Accounts					
-0.7	32	0.1321	0.1534	0.1672	0.0389
-0.8	15	0.2658	0.1896	0.1946	0.0609
-0.9	8	0.1850	0.2882	0.1562	0.0705

Notes: This table presents the maximum-likelihood coefficient estimates of a generalized Pareto distribution $G(V/B)$, namely a tail shape parameter k and a scale parameter σ. Estimation is done alternatively for house accounts in Panel A and customer accounts in Panel B. The exceedances are defined using three different thresholds, $\theta =$ -0.7, -0.8, -0.9. n is the number of exceedances used in the estimation, i.e., number of V/B ratios smaller than θ. s.e.(k) and s.e.(σ) denote asymptotic standard errors of the tail shape and scale parameters respectively.

Table 5: Insurance Price Estimates

θ	λ	p_{pay}	p_{max}	p_{def}	bp/yr	s.e.(%)	Value	DefMax
Panel A: Base Case								
−0.7	0.0052	0.0011	0.0004	0.0218	2.18	0.18	305.9	3,122
−0.8	0.0032	0.0010	0.0003	0.0192	1.87	0.19	262.2	1,859
−0.9	0.0020	0.0011	0.0003	0.0151	2.27	0.17	318.2	1,527
Panel B: No Policy Payout Limit								
−0.7	0.0052	0.0011	0.0000	0.0218	-	0.24	612.9	3,122
−0.8	0.0032	0.0010	0.0000	0.0192	-	0.22	403.7	1,859
−0.9	0.0020	0.0011	0.0000	0.0151	-	0.22	508.1	1,527
Panel C: No Deductible								
−0.7	0.0052	0.0218	0.0011	0.0218	11.04	0.07	1,547.2	3,122
−0.8	0.0032	0.0192	0.0010	0.0192	11.77	0.07	1,649.9	1,859
−0.9	0.0020	0.0151	0.0011	0.0151	10.32	0.07	1,446.5	1,527
Panel D: No Policy Payout Limit and No Deductible								
−0.7	0.0052	0.0218	0.0000	0.0218	-	0.10	2,160.0	3,122
−0.8	0.0032	0.0192	0.0000	0.0192	-	0.08	2,023.6	1,859
−0.9	0.0020	0.0151	0.0000	0.0151	-	0.09	1,968.9	1,527

Notes: This table presents the estimated insurance premium for several threshold levels θ. The λ parameter is the annualized Poisson arrival rate of trigger events that cause account loss in the tail range, p_{pay} is the probability of any payout event over term of policy, p_{max} is the probability of policy limit payout over term of policy, p_{def} is the probability of at least one firm defaults during policy term, bp/yr denotes the Monte Carlo estimate of actuarially fair price expressed in basis points per year on policy maximum payout, paid in advance at premium payment interval, and s.e.(%) is the Monte Carlo sampling error of price expressed as proportion of estimated fair price. Value gives the present value of the insurance liability (and premiums) in thousands of dollars, i.e., policy limit $\times$ bp/yr $\times$ discount factor for three years. DefMax is the largest default in million of dollars occurring in the 40,000 simulations. In Panel A, we consider the following base case: a three-year policy, with a \$500 million deductible, a \$500 million payout limit, a deductible reset interval of 0.25 years, a premium payment interval of 0.25 years, $\rho_\theta = \rho_z = 0.5$, $\pi_1 = 0.2$, $\pi_2 = 0.5$, r = 5%, and M = 60. In Panels B-D, we consider several variations of the base case.

Table 6: Comparative Statics Analysis

	λ	p_{pay}	p_{max}	p_{def}	bp/yr	s.e.(%)	Value	DefMax
Base Case	0.0032	0.0010	0.0003	0.0192	1.87	0.19	262.2	1,859
$\rho_\theta = 0.75$	0.0032	0.0013	0.0005	0.0153	2.64	0.16	369.4	1,608
$\rho_\theta = 0.25$	0.0032	0.0009	0.0003	0.0216	1.72	0.20	241.3	2,487
$\rho_z = 0.75$	0.0032	0.0010	0.0004	0.0173	1.99	0.19	278.9	2,555
$\rho_z = 0.25$	0.0032	0.0009	0.0002	0.0211	1.53	0.20	214.2	1,652
$\rho_\theta = \rho_z = 0$	0.0032	0.0008	0.0003	0.0286	1.61	0.21	225.8	2,406
$\pi_1 = 0.30$	0.0032	0.0017	0.0006	0.0218	3.32	0.15	466.0	1,859
$\pi_1 = 0.10$	0.0032	0.0005	0.0002	0.0143	1.07	0.25	150.1	1,859
$\pi_2 = 0.75$	0.0032	0.0010	0.0003	0.0192	1.87	0.19	262.2	1,859
$\pi_2 = 0.25$	0.0032	0.0010	0.0003	0.0192	1.87	0.19	262.2	1,859
$M = 80$	0.0032	0.0013	0.0004	0.0207	2.58	0.16	361.5	1,947
$M = 40$	0.0032	0.0006	0.0003	0.0155	1.52	0.23	212.8	2,176

Notes: This table presents the results of a comparative static analysis of the estimated insurance premium. The λ parameter is the annualized Poisson arrival rate of trigger events that cause account loss in the tail range, p_{pay} is the probability of any payout event over term of policy, p_{max} is the probability of policy limit payout over term of policy, p_{def} is the probability of at least one firm defaults during policy term, bp/yr denotes the Monte Carlo estimate of actuarially fair price expressed in basis points per year on policy maximum payout, paid in advance at premium payment interval, and s.e.(%) is the Monte Carlo sampling error of price expressed as proportion of estimated fair price. Value gives the present value of the insurance liability (and premiums) in thousands of dollars, i.e., policy limit $\times$ bp/yr $\times$ discount factor for three years. DefMax is the largest default in million of dollars occurring in the 40,000 simulations. The first row displays the base case results with a threshold $\theta = -0.8$, a three-year policy, a \$500 million deductible, a \$500 million payout limit, a deductible reset interval of 0.25 years, a premium payment interval of 0.25 years, $\rho_\theta = \rho_z = 0.5$, $\pi_1 = 0.2$, $\pi_2 = 0.5$, r = 5%, and M = 60. In subsequent rows, we consider several variations of the base case where the successive shocks are displayed in the first column.

Table 7: Cost of the Federal Reserve Guarantee

θ	λ	p_{pay}	p_{def}	s.e.(%)	Value	DefMax
−0.7	0.0052	0.0005	0.0218	0.31	382.5	3,122
−0.8	0.0032	0.0004	0.0192	0.30	202.9	1,859
−0.9	0.0020	0.0005	0.0151	0.30	265.2	1,527

Notes: This table presents the estimated insurance premium for several threshold levels θ. The λ parameter is the annualized Poisson arrival rate of trigger events that cause account loss in the tail range, p_{pay} is the probability of any payout event over term of policy, p_{def} is the probability of at least one firm defaults during policy term, and s.e.(%) is the Monte Carlo sampling error of price expressed as proportion of estimated fair price. Value gives the present value of the insurance liability (and premiums) in thousands of dollars, i.e., policy limit × bp/yr × discount factor for three years. DefMax is the largest default in million of dollars occurring in the 40,000 simulations. The features of the contract and the value of the key parameteres are as follows: a three-year policy, a $821 million deductible, no payout limit, a deductible reset interval of 0.25 years, a premium payment interval of one year, $\rho_\theta = \rho_z = 0.5$, $\pi_1 = 0.2$, $\pi_2 = 0.5$, r = 5%, and M = 60.

Table 8: Estimation Results When the Black Monday is Included

Panel A: EVT-based Tail Analysis

θ	n	k	s.e.(k)	σ	s.e.(σ)
-0.7	238	-0.3153	0.0853	0.1873	0.0197
-0.8	147	-0.3201	0.1089	0.2147	0.0288
-0.9	95	-0.3403	0.1375	0.2395	0.0402

Panel B: Insurance Price Estimates

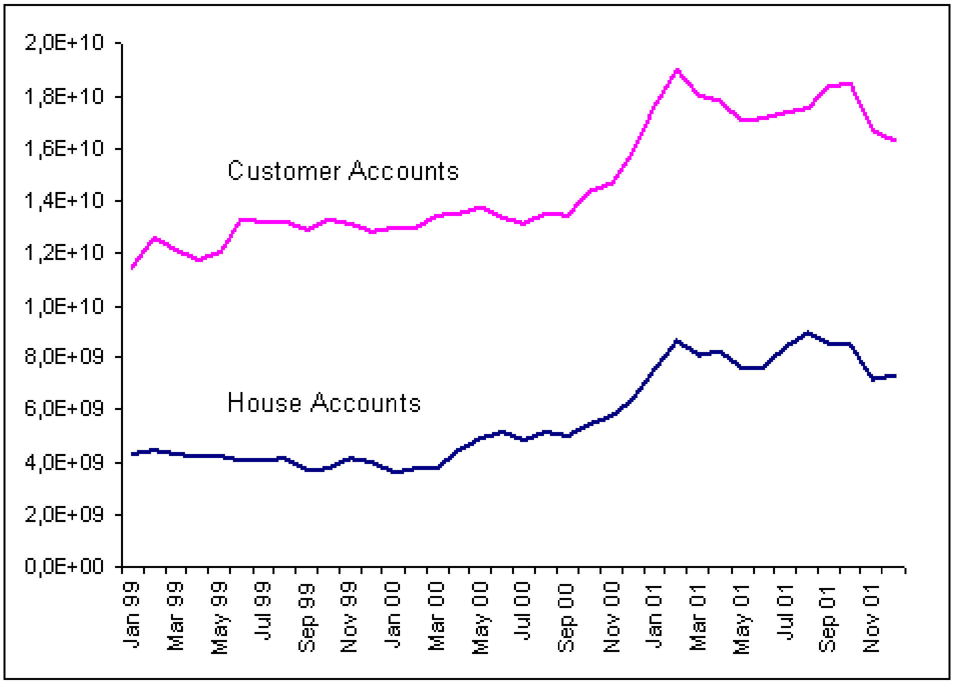
θ	λ	p_{pay}	p_{max}	p_{def}	bp/yr	s.e.(%)	Value	DefMax
-0.7	0.0053	0.0030	0.0016	0.0239	7.37	0.10	1,033.2	10,962
-0.8	0.0033	0.0029	0.0015	0.0186	7.25	0.10	1,015.8	13,623
-0.9	0.0021	0.0030	0.0018	0.0163	7.39	0.10	1,036.4	6,547

Panel C: Value of the Federal Reserve Guarantee

θ	λ	p_{pay}	p_{def}	s.e.(%)	Value	DefMax
-0.7	0.0053	0.0021	0.0239	0.22	2,657.3	10,962
-0.8	0.0033	0.0020	0.0185	0.21	2,703.6	13,623
-0.9	0.0021	0.0019	0.0163	0.16	3,055.1	6,547

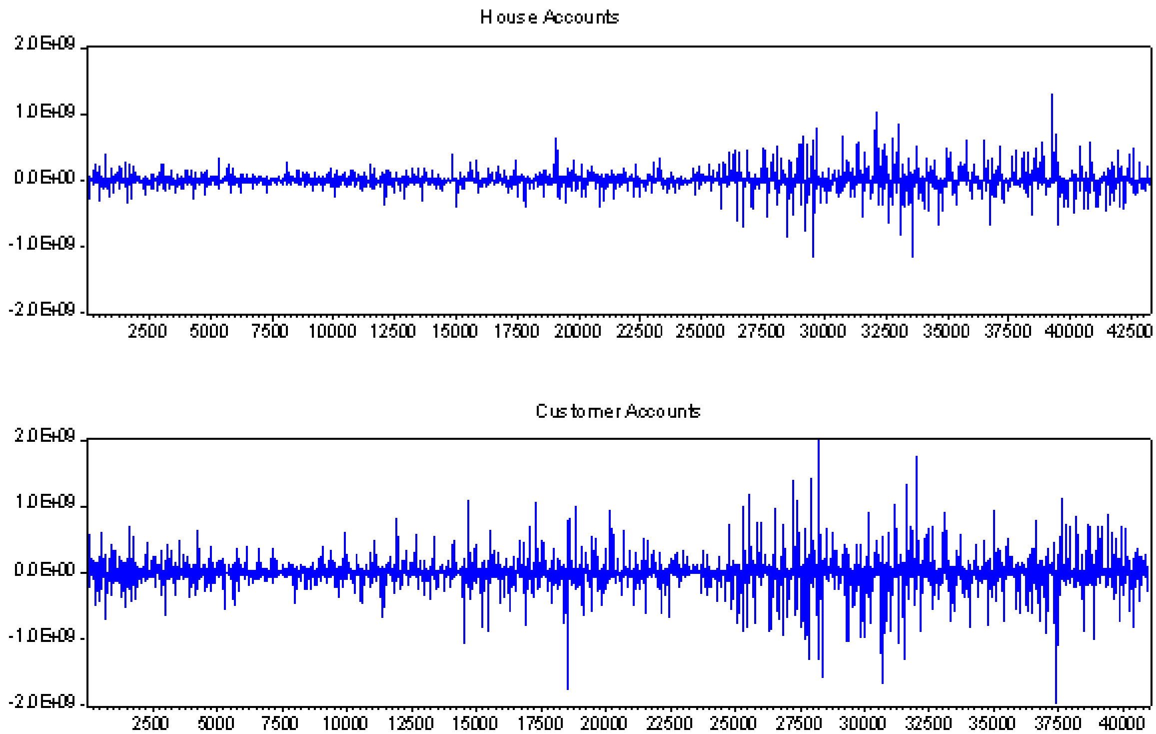
Notes: This table presents the estimation results for the tail analysis (Panel A), the insurance price (Panel B), and the value of the Federal Reserve guarantee (Panel C) when the sample includes the data from the Black Monday of October 19th, 1987. Panel A replicates Panel A in Table 4. The analysis is conducted for house accounts only. The panel presents the maximum-likelihood coefficient estimates of a generalized Pareto distribution $G(V/B)$, namely a tail shape parameter k and a scale parameter σ. The exceedances are defined using three different thresholds, $\theta =$ -0.7, -0.8, -0.9. n is the number of exceedances used in the estimation, i.e., number of V/B ratios that are smaller than θ. s.e.(k) and s.e.(σ) denote asymptotic standard errors of the tail shape and scale parameters respectively. Panel B replicates Panel A in Table 5. The panel presents the estimated insurance premium for several threshold levels. The λ parameter is the annualized Poisson arrival rate of trigger events that cause account loss in the tail range, p_{pay} is the probability of any payout event over term of policy, p_{max} is the probability of policy limit payout over term of policy, p_{def} is the probability of at least one firm defaults during policy term, bp/yr denotes the Monte Carlo estimate of actuarially fair price expressed in basis points per year on policy maximum payout, paid in advance at premium payment interval, and s.e.(%) is the Monte Carlo sampling error of price expressed as proportion of estimated fair price. Value gives the present value of the insurance liability (and premiums) in thousands of dollars, i.e., policy limit $\times$ bp/yr $\times$ discount factor for three years. DefMax is the largest default in million of dollars occurring in the 40,000 simulations. We consider the following base case: a three-year policy, with a \$500 million deductible, a \$500 million payout limit, a deductible reset interval of 0.25 years, a premium payment interval of 0.25 years, $\rho_\theta = \rho_z = 0.5$, $\pi_1 = 0.2$, $\pi_2 = 0.5$, r = 5%, and M = 60. Panel C replicates Table 7. The features of the contract and the value of the key parameters are as in Panel B except that there are a \$821 million deductible and no payout limit.

Figure 1: **Daily Total Performance Bond**



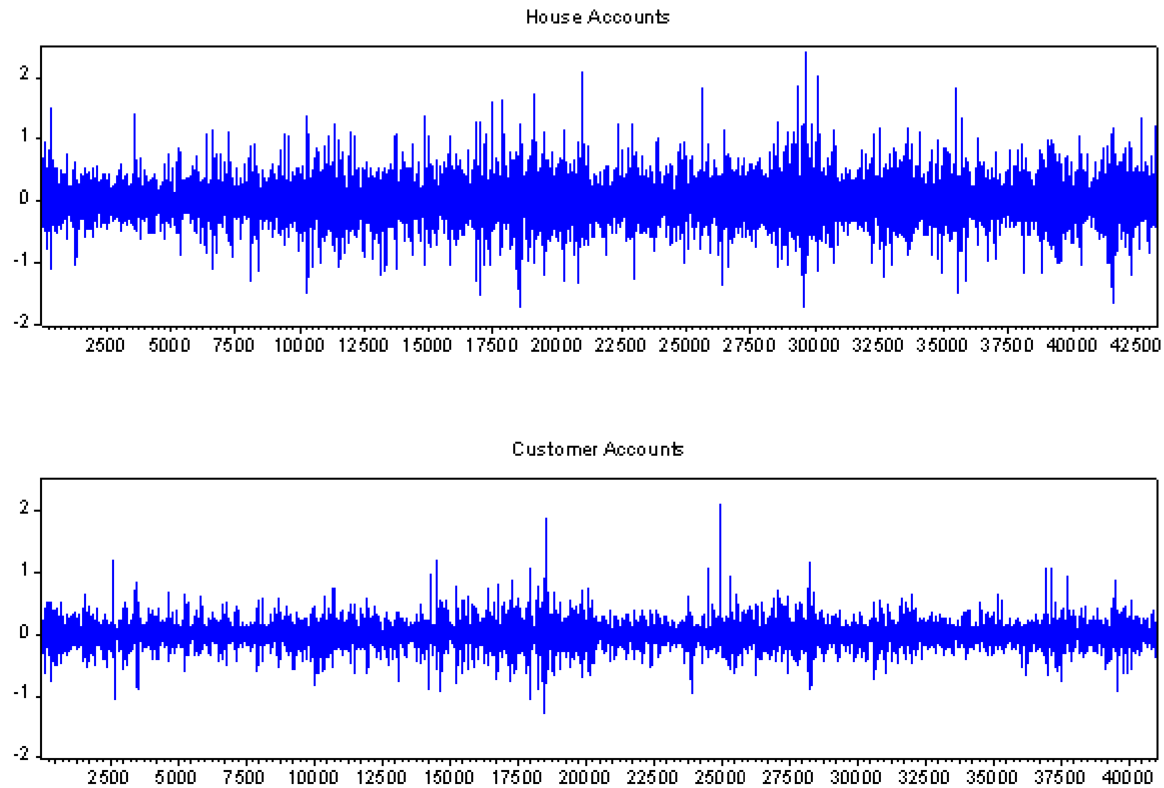
Notes: This figure displays the cumulative performance bond (in dollars) across all clearing members ($\Sigma_i B_{it}$) averaged over each month. The lower line is for house accounts and the upper line is for customer accounts.

Figure 2: **Daily Variation Margins**



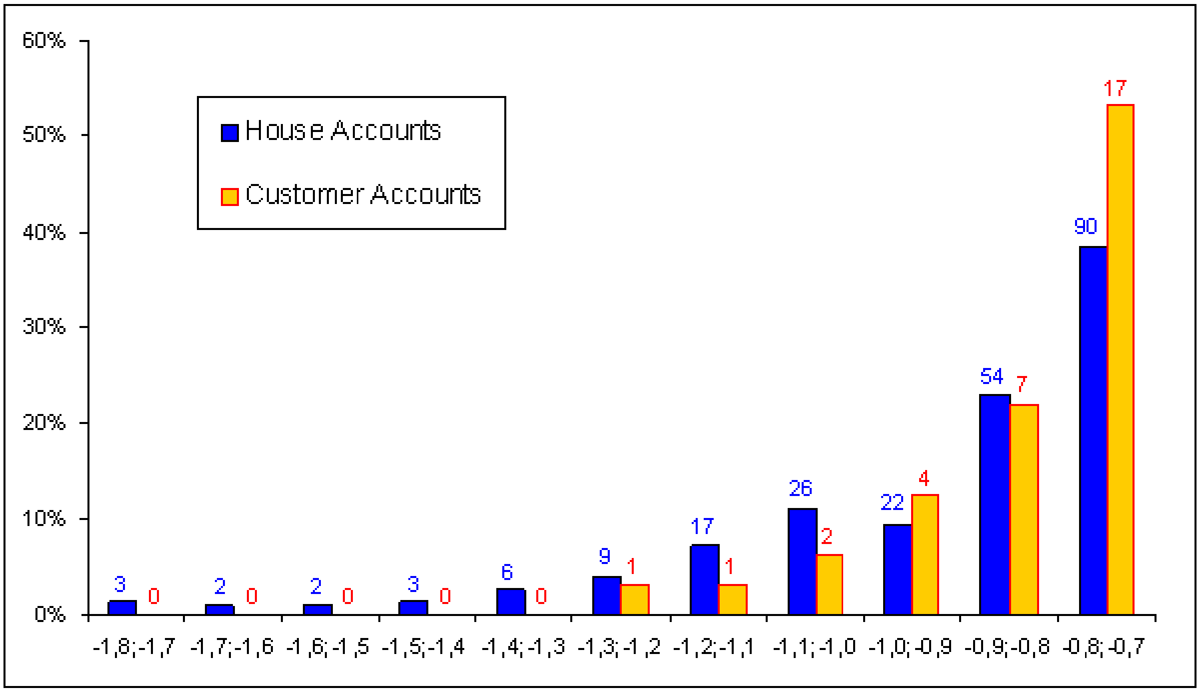
Notes: This figure displays all daily variation margins (in dollars) between January 4th, 1999 and December 31st, 2001. The top panel is for house accounts and the lower panel is for customer accounts. On each sample day, we stack all clearing members' variation margins. For instance, the first 63 observations in Panel A are the daily variation margins of the 63 clearing members posting margins on January 4th, 1999. The following 63 observations are the daily variation margins of the 63 clearing members posting margins on January 5th, 1999, and so on. The total number of observations is 43,236 observations in Panel A and 41,013 observations in Panel B.

Figure 3: **Ratio of the Variation Margins and Performance Bonds**



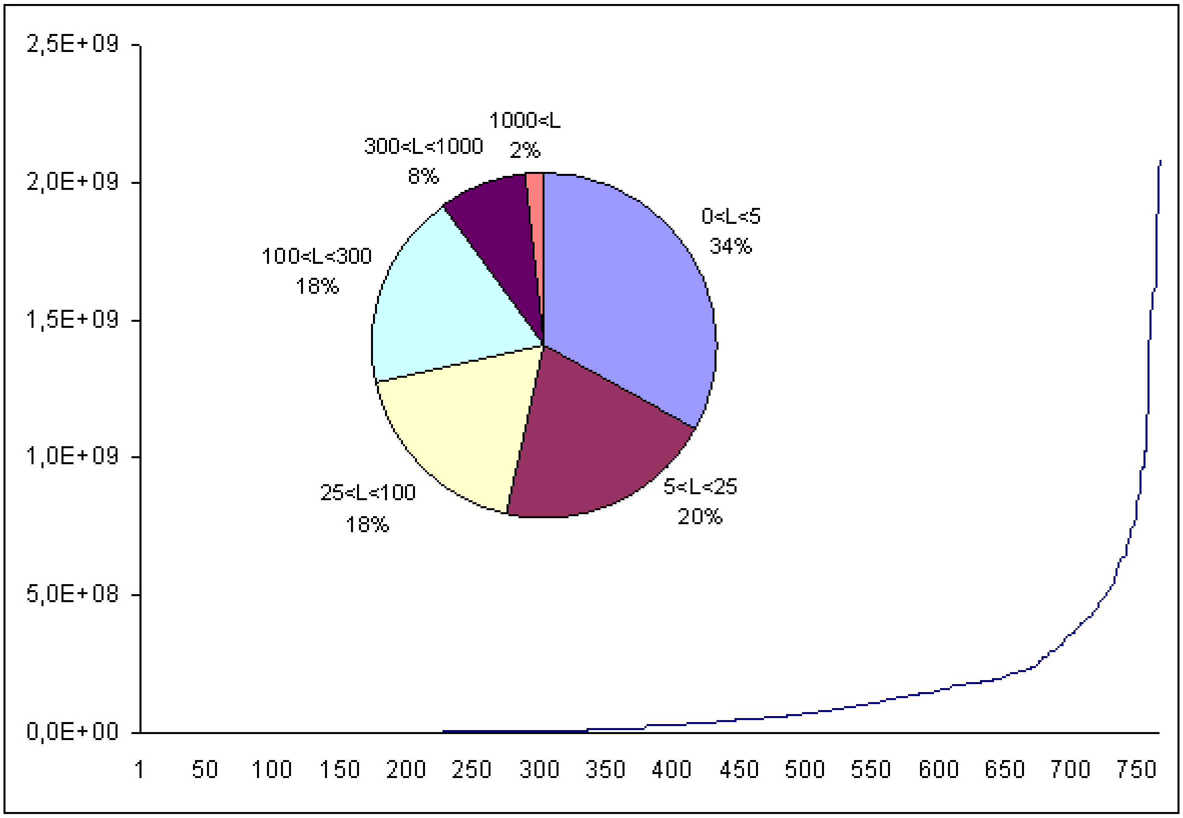
Notes: This figure displays the ratio of the daily variation margins (V) over the daily performance bond (B) for all clearing members between January 4th, 1999 and December 31st, 2001. The top panel is for house accounts and the lower panel is for customer accounts. On each sample day, we stack all clearing members' V/B ratios. For instance, the first 63 observations in Panel A are the daily variation margins of the 63 clearing members posting margins on January 4th, 1999. The following 63 observations are the daily variation margins of the 63 clearing members posting margins on January 5th, 1999, and so on. The total number of observations is 43,236 observations in Panel A and 41,013 observations in Panel B.

Figure 4: **Left-Tail Histogram of V/B for House and Customer Accounts**



Notes: The bars represent the frequency of observations in a particular bucket, i.e., number of observations in this bucket (displayed above each bar) divided by the total number of observations in the tail. The dark shaded bars are for house accounts and the light shaded bars are for customer accounts.

Figure 5: **Simulated Default Losses**



Notes: This figure displays the value and the frequency of the simulated total default losses obtained from 40,000 simulated paths. In this experiment, we use our base case with θ = -0.8 (refer to caption of Table 5 for details). The number of paths experiencing no defaults is 39,232. The exponentially increasing line shows the 768 non-zero total default losses (in dollars) sorted from smallest to largest. The superimposed pie chart displays the size distribution of the non-zero total default losses, labelled L and measured in million of dollars.