Collins Fragmentation Function from LEP data?

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- Brief overview of factorization and transverse momentum
- Collins effect in electron-positron annihilation
- Effects of Sudakov factors
- Implications and conclusions

Factorization and transverse momentum

Leading twist (LT) factorization theorem:

$$\frac{d\sigma}{d\Omega dx_1 dx_2} = \sum_{a,b} \int_{x_1}^{1} dx \int_{x_2}^{1} d\bar{x}\, f_1^a(x)\, \frac{d\hat{\sigma}_{ab}}{d\Omega dx d\bar{x}}\, \overline{f}_1^b(\bar{x})$$

For $|\boldsymbol{q}_T| \equiv Q_T \ll Q$ (Collins, Soper & Sterman, NPB 250 (1985) 199)

$$\frac{d\sigma}{d\Omega dx_1 dx_2 d^2\boldsymbol{q}_T} = Y(x_1, x_2, Q, Q_T) +$$

$$\sum_{a,b} \int_{x_1}^{1} dx \int_{x_2}^{1} d\bar{x} \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, f_1^a(x, \boldsymbol{k}_T)\, \frac{d\hat{\sigma}_{ab}}{d\Omega dx d\bar{x} d^2\boldsymbol{q}_T}\, \overline{f}_1^b(\bar{x}, \boldsymbol{p}_T)$$

$Y(x_1, x_2, Q, Q_T)$ becomes important only when $Q_T \sim Q$

$$\frac{d\hat{\sigma}}{d\Omega dx d\bar{x} d^2\boldsymbol{q}_T} = H(x, \bar{x}, \boldsymbol{p}_T, \boldsymbol{k}_T, \boldsymbol{q}_T; Q) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{-i\boldsymbol{b}\cdot(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)}\, e^{-S(\boldsymbol{b})}$$

$e^{-S(\boldsymbol{b})}$ is a Sudakov form factor (exponentiation rather than cancellation of soft gluon contributions)

Factorization and transverse momentum

$$\frac{d\hat{\sigma}}{d\Omega dx d\bar{x} d^2\boldsymbol{q}_T} = H(x,\bar{x},\boldsymbol{p}_T,\boldsymbol{k}_T,\boldsymbol{q}_T;Q) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{-i\boldsymbol{b}\cdot(\boldsymbol{p}_T+\boldsymbol{k}_T-\boldsymbol{q}_T)}\, e^{-S(\boldsymbol{b})}$$

Transverse momentum dependence of the hard part will lead to $1/Q$ suppression (in contrast to intrinsic transverse momentum)
One can perform a collinear expansion of the hard part

$$H(x,\bar{x},\boldsymbol{p}_T,\boldsymbol{k}_T,\boldsymbol{q}_T;Q) \approx H(x,\bar{x};Q)$$

At tree level:

$$\int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{-i\boldsymbol{b}\cdot(\boldsymbol{p}_T+\boldsymbol{k}_T-\boldsymbol{q}_T)}\, e^{-S(\boldsymbol{b})} \rightarrow \delta^2(\boldsymbol{p}_T+\boldsymbol{k}_T-\boldsymbol{q}_T)$$

Need to consider: transverse momentum dependent functions
Ralston & Soper, NPB 152 (1979) 109

$$\Phi(x) \rightarrow \Phi(x,\boldsymbol{p}_T)$$
$$\Delta(z) \rightarrow \Delta(z,\boldsymbol{k}_T)$$

Leads to many azimuthal asymmetries, even unpolarized ones

e^+e^-: D.B., Jakob & Mulders, NPB 504 (97) 345; PLB 424 (98) 143

Fragmentation functions

Collins & Soper, NPB 194 (1982) 445

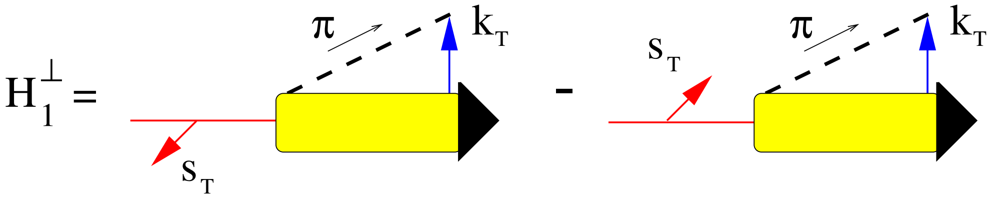
$$\Delta_{ij}(k) \quad = \quad \sum_X \int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z} \langle 0|\psi_i(x)|P,S;X\rangle\langle P,S;X|\overline{\psi}_j(0)|0\rangle$$

The unpolarized fragmentation correlation function $\Delta(z)$ is parameterized as

$$\Delta(z) = D_1(z)\,\frac{\gamma^+}{4}$$

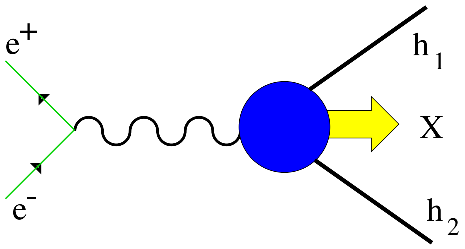
and $\Delta(z,\boldsymbol{k}_T)$ as

$$\Delta(z,\boldsymbol{k}_T) = D_1(z,\boldsymbol{k}_T)\,\frac{\gamma^+}{4} + i H_1^\perp(z,\boldsymbol{k}_T)\,\frac{\not{k}_T\gamma^+}{4}$$



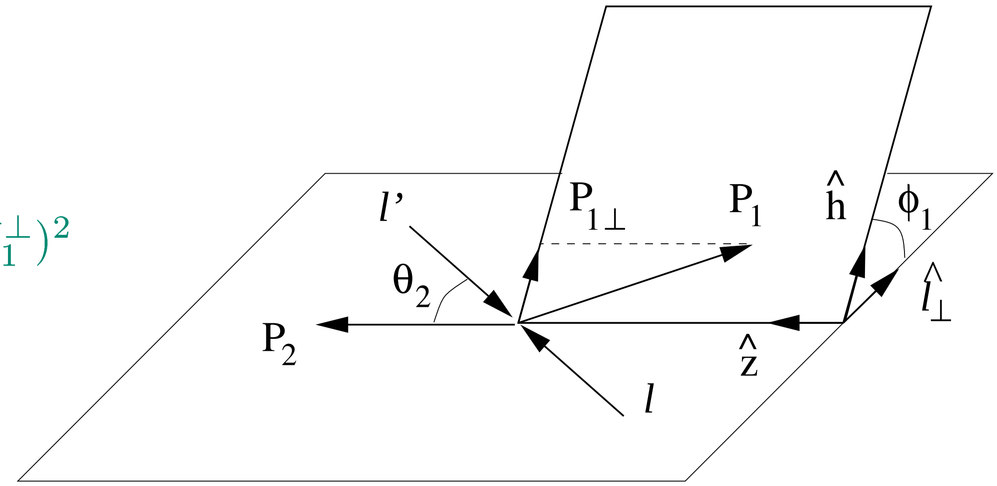
Apart from experimental indications that the Collins effect is nonzero at low energies (HERMES, SMC) and that it can account for the $p\,p^\uparrow \to \pi\,X$ SSA [see other talks at this workshop], there is also LEP data...

Collins effect in $e^+ e^- \to \pi^+ \pi^- X$



$$e^+ + e^- \to \pi^+ + \pi^- + X$$

$$\langle \cos(2\phi_1) \rangle \propto (H_1^\perp)^2$$



D.B., Jakob & Mulders, NPB 504 (97) 345; PLB 424 (98) 143

A confirmation of this asymmetry would confirm the Collins effect, without the need of polarization

A first indication of a nonzero correlation comes from a preliminary analysis of the 91-95 LEP1 data (DELPHI) by Efremov, Smirnova & Tkatchev, NPB (Proc. Suppl.) 79 (1999) 554

Collins effect in $e^+ e^- \to \pi^+ \pi^- X$

Problem is that the asymmetry expression used is the tree level result

$$\frac{d\sigma(e^+e^- \to \pi^+\pi^- X)}{d\Omega dz_1 dz_2 d^2\boldsymbol{q}_T} \propto \{1 + \cos(2\phi_1) A(\boldsymbol{q}_T)\}$$

with $(\boldsymbol{q}_T^2 \equiv Q_T^2 \ll Q^2)$

$$A(\boldsymbol{q}_T) \equiv \frac{B(y)\ \sum_a\ c_2^a\ \mathcal{F}\left[\left(2\,\boldsymbol{q}_T\!\cdot\!\boldsymbol{p}_T\ \boldsymbol{q}_T\!\cdot\!\boldsymbol{k}_T\ -\ \boldsymbol{q}_T^2\,\boldsymbol{p}_T\!\cdot\!\boldsymbol{k}_T\right) H_1^\perp \overline{H}_1^\perp\right]}{Q_T^2 M_1 M_2\ A(y)\ \sum_a\ c_1^a\ \mathcal{F}\left[D_1 \overline{D}_1\right]}$$

$$\begin{aligned} A(y) &= \left(\frac{1}{2} - y + y^2\right) \stackrel{cm}{=} \frac{1}{4}\left(1 + \cos^2\theta_2\right) \\ B(y) &= y\,(1-y) \stackrel{cm}{=} \frac{1}{4}\sin^2\theta_2 \end{aligned}$$

$$\begin{aligned} c_1^a &= \left({g_V^a}^2 + {g_A^a}^2\right) \\ c_2^a &= \left({g_V^a}^2 - {g_A^a}^2\right) \end{aligned}$$

Beyond tree level

At tree level the convolutions are defined as follows:

$$\mathcal{F}\left[D\overline{D}\right] \equiv \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\; \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) D^a(z_1, \boldsymbol{k}_T^2)\overline{D}^a(z_2, \boldsymbol{p}_T^2)$$

Beyond tree level and hence, beyond the range of intrinsic transverse momentum one has to take into account the effect of resummed perturbative QCD corrections

Resummation of soft gluons into Sudakov form factors results in a replacement (Collins, Soper & Sterman)

$$\delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) \to \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{-i\boldsymbol{b}\cdot(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)}\, e^{-S(b,Q)}$$

$$S(b,Q) = \int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[A(\alpha_s(\mu))\,\ln\frac{Q^2}{\mu^2} + B(\alpha_s(\mu))\right]$$

Beyond tree level

Tree level:

$$\mathcal{F}\left[D\overline{D}\right] \equiv \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) D^a(z_1, \boldsymbol{k}_T^2) \overline{D}^a(z_2, \boldsymbol{p}_T^2)$$

Inclusion of the Sudakov form factor leads to

$$\mathcal{F}\left[D\overline{D}\right] = \frac{1}{2\pi} \int_0^\infty db\, b\, J_0(bQ_T)\, e^{-S(b)}\, \tilde{D}(z_1, b)\, \tilde{\overline{D}}(z_2, b)$$

Collins (NPB 396 (1993) 161): "*The effect [of Sudakov form factors] is to broaden the transverse momentum distribution as Q increases, but in a spin-independent way: the broadening is due to recoil against the transverse momentum of soft gluon emission. This will have the effect of diluting the spin asymmetry [...].*

Sudakov form factors in polarized scattering: Weber (NPB 382 (1992) 63; NPB 403 (1993) 545)

Estimating the asymmetry

Assume Gaussian transverse momentum dependence:

$$D_1(z, z^2\boldsymbol{k}_T^2) = D_1(z)\; R^2 \exp(-R^2\boldsymbol{k}_T^2)/\pi z^2$$

we find

$$A(\boldsymbol{q}_T) \equiv \frac{B(y)\;\sum_a\;c_2^a\;H_1^{\perp a}(z_1)\;\overline{H}_1^{\perp a}(z_2)}{4M^4R^4\;A(y)\;\sum_a\;c_1^a\;D_1^a(z_1)\;\overline{D}_1^a(z_2)}\;\mathcal{A}(Q_T)$$

where

$$\mathcal{A}(Q_T) \equiv M^2\,\frac{\int_0^\infty db\, b^3\, J_2(bQ_T)\;\exp\left(-S(b_*) - S_{NP}(b)\right)}{\int_0^\infty db\, b\, J_0(bQ_T)\;\exp\left(-S(b_*) - S_{NP}(b)\right)}$$

Here we introduce the usual b regulator: $b_* = b/\sqrt{1+b^2/b_{\rm max}^2}$ and the nonperturbative Sudakov factor of Ladinsky-Yuan (PRD 50 (94) R4239), which uses $b_{\rm max} = 0.5\,\text{GeV}^{-1}$ and leads to

$$S_{NP}(b) = 2.05\,b^2 \qquad \text{at}\;\; Q = 90\,\text{GeV}$$

We restrict to S_{LL}

Gaussian form of $H_1^\perp(z, \boldsymbol{k}_T)$

One might be inclined to assume the maximally allowed function by saturating the bound satisfied by $H_1^\perp$

$$|\boldsymbol{k}_T|\ H_1^\perp(z, |\boldsymbol{k}_T|) \leq z\ M_h\ D_1(z, |\boldsymbol{k}_T|)$$

producing a $1/|\boldsymbol{k}_T|$ behavior of $H_1^\perp(z, \boldsymbol{k}_T)$. However, this is not consistent with the fact that the Collins effect should vanish in the limit $\boldsymbol{k}_T \to 0$.

Often used is an Ansatz based on a simple model by Collins

$$\frac{H_1^\perp(z, \boldsymbol{k}_T^2)}{D_1(z, \boldsymbol{k}_T^2)} = \eta\ \frac{M_C M_\pi}{\boldsymbol{k}_T^2 + M_C^2}$$

For the present purpose, the additional fall-off with $1/\boldsymbol{k}_T^2$ on top of the Gaussian fall-off is not needed.

We will restrict to a Gaussian fall-off and assume the simple form

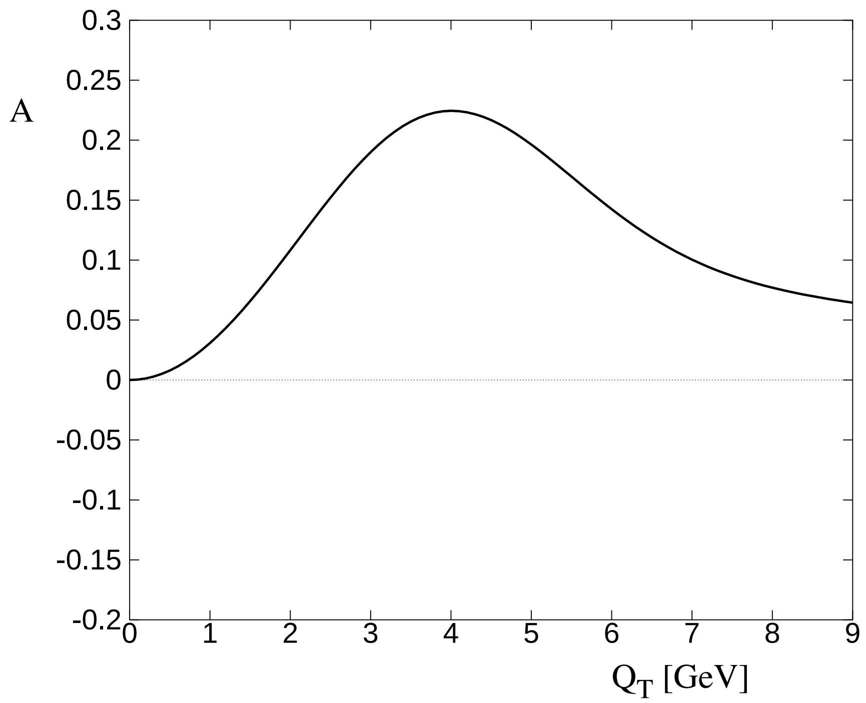
$$H_1^\perp(z, \boldsymbol{k}_T^2) = c(z)\, D_1(z, \boldsymbol{k}_T^2)$$

Implications

$$\mathcal{A}(Q_T) \equiv M^2 \, \frac{\int_0^\infty db\, b^3\, J_2(bQ_T)\, \exp\left(-S(b_*) - S_{NP}(b)\right)}{\int_0^\infty db\, b\, J_0(bQ_T)\, \exp\left(-S(b_*) - S_{NP}(b)\right)}$$

$\mathcal{A}(Q_T)$

$Q^2 = M_Z^2$



Features:

- Kinematic zero at $Q_T = 0$ as expected
- Maximum of 0.22 at $Q_T = 4\,\text{GeV}$

Comparison to tree level

Efremov *et al.* NPB (Proc. Suppl.) 74 (99) 49; 79 (99) 554

$$\frac{d\sigma}{d\cos\theta_2 d\phi_1} \propto \left(1 + \frac{6}{\pi} \left[\frac{H_1^{q\perp}}{D_1^q} \right]^2 \frac{v_q^2 - a_q^2}{v_q^2 + a_q^2} \frac{\sin^2\theta_2}{1+\cos^2\theta_2} \cos(2\phi_1) \right)$$

$$\frac{d\sigma}{d\cos\theta_2 d\phi_1} \propto \left(1 + A \left[\frac{H_1^{\perp a}(z_1)}{D_1^a(z_1)} \right]^2 \frac{{g_V^a}^2 - {g_A^a}^2}{{g_V^a}^2 + {g_A^a}^2} \frac{\sin^2\theta_2}{1+\cos^2\theta_2} \cos(2\phi_1) \right)$$

Note: numerator and denominator have separate $\sum_a$

$$\left. \begin{array}{lcl} A^{\text{efr}} & = & 6/\pi \\ A^{(0)} & = & 1/(2M^2R^2) \end{array} \right\} \Longrightarrow \left\{ \begin{array}{lcl} R^2 & \sim & 13\,\text{GeV}^{-2} \\ \langle \boldsymbol{p}_T^2 \rangle & \sim & (270\,\text{MeV})^2 \end{array} \right.$$

$$A^{\text{sud}} = 0.07$$

If $A^{\text{efr}} = 6/\pi$ yields $\overline{|H_1^\perp/D_1|} = 6\% \pm 2\%$,

then $A^{\text{sud}} = 0.07$ yields $\overline{|H_1^\perp/D_1|} = 30\% \pm 10\%$ [too large?]

Conclusions

- Leading twist factorization at $Q_T^2 \ll Q^2$ requires distribution and fragmentation functions as a function of transverse momentum
- Sudakov factors need to be included
- In the Collins effect $\cos(2\phi)$ asymmetry in $e^+ e^- \to \pi^+ \pi^- X$ a strong suppression due to Sudakov factors was demonstrated.
- This is relevant at $Q \sim M_Z$ and casts some doubt on the actual determination of the Collins fragmentation function from LEP data
- Tree level estimates tend to overestimate transverse momentum dependent azimuthal asymmetries