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by

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Translated from the Russian

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NUMERICAL SOLUTION OF HYPERBOLIC EQUATIONS AND SYSTEMS BY A METHOD OF THE RUNGE-KUTTA TYPE. II

by

Nguyen Kong Tuy

In the first part of the article the two-iteration algorithms of the Runge-Kutta type were applied to the solution of the Cauchy problem for hyperbolic equations and systems with two independent variables, where the initial data are given along the line segment $x + y = \text{const}$.

In the second part of this article an analogous problem is discussed for one equation with Cauchy data along the curve segment.

Below are used designations of the article [1] and the numbering of its paragraphs, formulas and figures is continued.

6. Statement of the Problem

Let there be an equation

$$u_{xy} = f(x, y, u, p, q) \tag{41}$$

$\left(u \text{ is an unknown function of } x, y;\ p = u_x,\ q = u_y\right)$ with initial conditions given along the segment AB of a certain curve Γ_0 in the form of:

$$u^0 = u^0(x), \qquad p^0 = p^0(x), \qquad q^0 = q^0(x) \quad ; \tag{42}$$

u^0, p^0, q^0, f are assumed to be continuous and differentiable a sufficient number of times. The term Γ_0 is assumed to be a sufficiently smooth line given in the form

$$y = y^0(x) \quad \text{or} \quad x = x^0(y) \quad , \tag{43}$$

while $x^0(y)$, $y^0(x)$ are decreasing functions.

Let us construct in the region of ABC of the definition of the solution a grid in the following (Figure 3). Let us subdivide AC into sufficiently small intervals of length h; from the points of division we will plot straight lines parallel to the axis Oy, and from their intersections with Γ_0, those parallel to the axis Ox, and we will distribute the nodes of the grid according to layers. We will connect the nodes of the n^{th} layer by the curve Γ_n. The curve Γ_n can be considered a segment of the initial curve Γ_0 displaced by nh in the direction Ox so that we have the equation Γ_n in the form

$$y^n(x) = y^0(x - nh) = y^0(z) \quad , \qquad z = x - nh \ .$$

In making calculations of the n^{th} layer we will use, for simplicity, the equation of the curve Γ_{n-1} in the form (43), and we will denote the earlier found values u, p, q in the nodes of the $(n - 1)^{st}$ layer, as well as the initial data, by u^0, p^0, q^0.

For an elementary curvilinear triangle MNP of the grid with vertices $M(x_1, y_1)$, $N(x_2 = x_1 + h,\ y_2 = y^0(x_1 + h))$ on the $(n - 1)^{st}$ layer and the vertex $P(x_2, y_1)$ on the n^{th} layer (Figure 4), the following relationships are known:

$$u(P) = u^0(x_1) + \int_{x_1}^{x_1 + h} p^0(x)\,dx + \iint F(x, y)\,dxdy \ , \qquad (44.a)$$

$$p(P) = p^0(x_2) + \int_{y^0(x_1 + h)}^{y^0(x_1)} F(x_1 + h, y)\,dy \ , \qquad (44.b)$$

$$q(P) = q^0(x_1) + \int_{x_1}^{x_1 + h} F(x, y_1)\,dx \ , \qquad (44.c)$$

where the multiple integral in (44.a) is taken over the region MNP. As in [1], for the increments $\Delta^* u = (P) - u(M)$, $\Delta^* p = p(P) - p(M)$, $\Delta^* q = q(P) - q(M)$, by using (44) we can find expansions in powers of h at the point M:

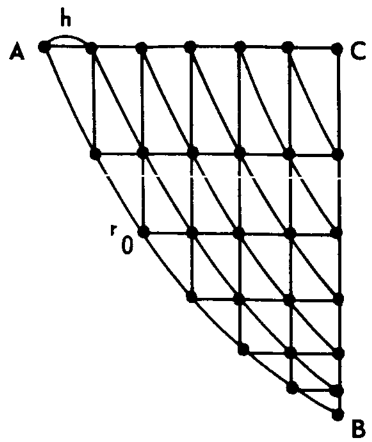

FIGURE 3

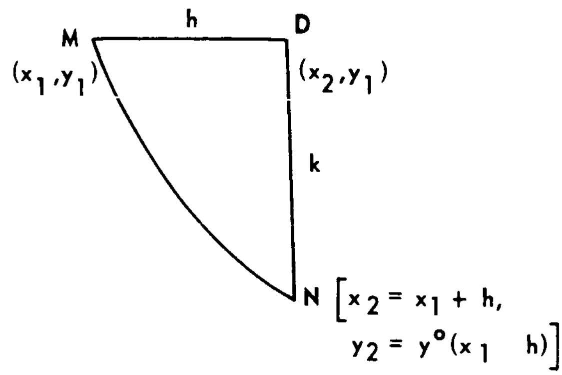

FIGURE 4

$$\Delta^* u = hp^0(x_1) + \frac{h^2}{2!}\frac{dp^0(x_1)}{dx} + \frac{h^3}{3!}\frac{d^2p^0(x_1)}{dx^2}$$

$$- \frac{h^2}{2!}\frac{dy^0(x_1)}{dx} F(M) - \frac{h^3}{3!}\frac{d^2y^0(x_1)}{dx^2} F(M)$$

$$- \frac{h^3}{3!}\frac{dy^0(x_1)}{dx} \quad 2\frac{\partial F(M)}{\partial x} + \frac{dy^0(x_1)}{dx}\frac{\partial F(M)}{\partial y} \quad + O(h^4) \ , \tag{45.a}$$

$$\Delta^* p = h\frac{dp^0(x_1)}{dx} + \frac{h^2}{2!}\frac{d^2p^0(x_1)}{dx^2} - h\frac{dy^0(x_1)}{dx} F(M)$$

$$- \frac{h^2}{2!}\frac{d^2y^0(x_1)}{dx^2} F(M) - \frac{h^2}{2!}\frac{dy^0(x_1)}{dx}$$

$$\times \left(2\frac{\partial F(M)}{\partial x} + \frac{dy^0(x_1)}{dx}\frac{\partial F(M)}{\partial y}\right) + O(h^3) \ , \tag{45.b}$$

$$\Delta^* q = hF(M) + \frac{h^2}{2!}\frac{\partial F(M)}{\partial x} + O(h^3) \ . \tag{45.c}$$

We set forth the problem of the construction of algorithms of the Runge-Kutta type which give approximate values of the transformations Δu, Δp, Δq.

7. Formal Algorithms of the Runge-Kutta Type

We will denote $\int_{x_1}^{x_1+h} p^0(x)\,dx = I(h)$. (For calculations of $I(h)$ see Paragraph 4). We introduce values of k:

$$k = k(h) = y^0(x_1) - y^0(x_1 + h)\ (h \geq 0)\ .$$

Upon the assumption that $y^0(x)$ is a decreasing function, the values of k will be positive. In Figure 4, $k(h)$ is equal to the length of NP. (Construction of the grid in Paragraph 6 and introduction of values of $k(h)$ is borrowed from [2]. From the determination of $k(h)$ we have:

$$k(h) = O(h)\ , \qquad k'(0) = \left.\frac{dk(h)}{dh}\right|_{h=0} = -\frac{dy^0(x_1)}{dx}\ ,$$

$$k''\ (0) = \frac{d^2y^0(x_1)}{dx^2}\ . \tag{46}$$

Assume

$$S = \frac{hk}{2} + \frac{h^3}{12}\frac{d^2y^0(x_1)}{dx^2}\ .$$

Let us note that S is equal to the area of the curvilinear triangle MNP, correct to the order $0(h^4)$.

At the first iteration we assume that:

$$u^1 = u^0(x_1) + I(h) + SF_1\ ,$$

$$F_1 = \omega_1 F(M) + \tau_1 F(N)\ , \qquad \omega_1, \tau_1 \geq 0\ , \qquad \omega_1 + \tau_1 = 1\ , \tag{47.a}$$

$$p^1 = p^0(x_2) + kF_1^{\wedge}\ , \qquad F_1^{\wedge} = F(N)\ , \tag{47.b}$$

$$q^1 = q^0(x_1) + hF_1^{-}\ , \qquad F_1^{-} = F(M)\ . \tag{47.c}$$

We form the elementary increments:

$$\left.\begin{aligned} \Delta^1 u &= u^1 - u^0(x_1) \\ \Delta^1 p &= p^1 - p^0(x_1) \\ \Delta^1 q &= q^1 - q^0(x_1) \end{aligned}\right\}\ , \tag{47.d}$$

$$\left.\begin{aligned} \delta^1 u &= u^1 - u^0(x_2) \\ \delta^1 p &= p^1 - p^0(x_2) \\ \delta^1 q &= q^1 - q^0(x_2) \end{aligned}\right\} . \tag{47.e}$$

Furthermore, we introduce the intermediate points $M_2^j(j = u, q)$, $N_2^j(j = u, p)$:

$$M_2^j = \Big(x_1 + p^j h \ ; \quad y_1 + \sigma^j k \ ; \quad u^0(x_1) + \alpha^j \Delta^1 u \ ;$$

$$p^0(x_1) + \beta^j \Delta^1 p \ ; \quad q^0(x_1) + \gamma^j \Delta^1 q\Big) , \tag{48.a}$$

$$N_2^j = \Big(x_2 + p^{-j} h \ ; \quad y_2 + \sigma^{-j} k \ ; \quad u^0(x_2) + \alpha^{-j} \delta^1 u \ ;$$

$$p^0(x_2) + \beta^{-j} \delta^1 p \ ; \quad q^0(x_2) + \gamma^{-j} \delta^1 q\Big) , \tag{48.b}$$

where p^j, σ^j, α^j, ..., γ^{-j} are numerical parameters.

At the second iteration we assume that:

$$u^2 = u^0(x_1) + I(h) + SF_2 \ ,$$

$$F_2 = \omega_2 F\left(M_2^u\right) + \tau_2 F\left(N_2^u\right) \ ; \quad \omega_2, \tau_2 \geq 0 \ , \quad \omega_2 + \tau_2 = 1 \ , \tag{49.a}$$

$$p^2 = p^0(x_2) + kF_2^{\hat{}} \ , \quad F_2^{\hat{}} = F\left(N_2^p\right) , \tag{49.b}$$

$$q^2 = q^0(x_1) + hF_2^{-} \ , \quad F_2 = F\left(M_2^q\right) , \tag{49.c}$$

$$\left.\begin{aligned} \Delta^2 u &= u^2 - u^0(x_1) \ , \\ \Delta^2 p &= p^2 - p^0(x_1) \ , \\ \Delta^2 q &= q^2 - q^0(x_1) \end{aligned}\right\} . \tag{49.d}$$

Finally, for Δu, Δp, Δq we take the following linear combinations:

$$\left.\begin{aligned} \Delta u &= \lambda_1 \Delta^1 u + \lambda_2 \Delta^2 u \ , \\ \Delta p &= \mu_1 \Delta^1 p + \mu_2 \Delta^2 p \ , \\ \Delta \rho &= \nu_1 \Delta^1 q + \nu_2 \Delta^2 q \end{aligned}\right\} \tag{50}$$

$\left(\lambda_i, \mu_i, \nu_i \text{ are numerical parameters}\right).$

8. Numerical Determination of Parameters

For $\overset{i}{\Delta}u$, $\overset{i}{\Delta}p$, $\overset{i}{\Delta}q$ are valid the following expansions in powers of h at the point M:

$$\overset{i}{\Delta}u = hp^0(x_1) + \frac{h^2}{2!}\frac{dp^0(x_1)}{dx} + \frac{h^3}{3!}\frac{d^2p^0(x_1)}{dx^2} + \frac{h^2}{2!} k'(0)F(M)$$
$$+ \frac{h^3}{3!}\left[\frac{3}{2}k''(0) + \frac{1}{2}\frac{d^2y^0(x_1)}{dx^2}\right. F(M)$$
$$+ \frac{h^3}{3!} 3k'(0)\left(\frac{dF_i}{dh}\right)_M + O(h^4) \quad , \tag{51.a}$$

$$\overset{i}{\Delta}p = h\frac{dp^0(x_1)}{dx} + \frac{h^2}{2!}\frac{d^2p^0(x_1)}{dx^2} + hk'(0)F(M)$$
$$+ \frac{h^2}{2!}k''(0)F(M) + \frac{h^2}{2!}2k'(0)\left(\frac{d\hat{F}_i}{dh}\right)_M + O(h^3) \quad , \tag{51.b}$$

$$\overset{i}{\Delta}q = hF(M) + \frac{h^2}{2!}2\left(\frac{d\bar{F}_i}{dh}\right)_M + O(h^3) \quad . \tag{51.c}$$

By comparing (50) and (51) and substituting $k'(0)$, $k''(0)$ in (46) we obtain expansions in powers of h at the point M for Δu, Δp, Δq:

$$\Delta u = (\lambda_1 + \lambda_2)\left[hp^0(x_1) + \frac{h^2}{2!}\frac{dp^0(x_1)}{dx} + \frac{h^3}{3!}\frac{d^2p^0(x_1)}{dx^2}\right]$$
$$- (\lambda_1 + \lambda_2)\frac{h^2}{2!}\frac{dy^0(x_1)}{dx}F(M) - (\lambda_1 + \lambda_2)\frac{h^3}{3!}\frac{d^2y^0(x_1)}{dx^2}F(M)$$
$$- \frac{h^3}{3!}\frac{dy^0(x_1)}{dx} \times 3\left(\lambda_1\frac{dF_1}{dh} + \lambda_2\frac{dF_2}{dh}\right)_M + O(H^4) \quad , \tag{52.a}$$

$$\Delta p = (\mu_1 + \mu_2)\left[h\,\frac{dp^0(x_1)}{dx} + \frac{h^2}{2!}\,\frac{d^2p^0(x_1)}{dx^2}\right]$$

$$- (\mu_1 + \mu_2)\left[h\,\frac{dy^0(x_1)}{dx}\,F(M) - \frac{h^2}{2!}\,\frac{d^2y^0(x_1)}{dx^2}\,F(M)\right]$$

$$- \frac{h^2}{2!}\,\frac{dy^0(x_1)}{dx}\,2\left(\mu_1\,\frac{dF_{\hat{1}}}{dh} + \mu_2\,\frac{dF_{\hat{2}}}{dh}\right)_M + O(h^3)\ , \qquad (52.b)$$

$$\Delta q = (\nu_1 + \nu_2)hF(M) + \frac{h^2}{2!}\,2\left(\nu_1\,\frac{dF_{\bar{1}}}{dh} + \nu_2\,\frac{dF_{\bar{2}}}{dh}\right)_M + O(h^3)\ . \quad (52.c)$$

The selection of parameters is effected so that the corresponding expansions of (52) and (45) coincide for an arbitrary function $F(x,y)$ and arbitrary step h. As a result of comparison of (52) and (45) we obtain the following conditions:

$$\lambda_1 + \lambda_2 = \mu_1 + \mu_2 = \nu_1 + \nu_2 = 1\ , \qquad (53)$$

$$3\left(\lambda_1\,\frac{dF_1}{dh} + \lambda_2\,\frac{dF_2}{dh}\right)_M = 2\,\frac{\partial F(M)}{\partial x} + \frac{dy^0(x_1)}{dx}\,\frac{\partial F(M)}{\partial y}\ , \qquad (54)$$

$$2\left(\mu_1\,\frac{dF_{\hat{1}}}{dh} + \mu_2\,\frac{dF_{\hat{2}}}{dh}\right)_M = 2\,\frac{\partial F(M)}{\partial x} + \frac{dy^0(x_1)}{dx}\,\frac{\partial F(M)}{\partial y}\ , \qquad (55)$$

$$2\left(\nu_1\,\frac{dF_{\bar{1}}}{dh} + \nu_2\,\frac{dF_{\bar{2}}}{dh}\right)_M = \frac{\partial F(M)}{\partial x}\ . \qquad (56)$$

For the right sides of (54) through (56) we have:

$$\frac{\partial F}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial u}\,p + \frac{\partial f}{\partial p}\,\frac{\partial p}{\partial x} + \frac{\partial f}{\partial q}\,f\ ,$$

$$\frac{\partial F}{\partial y} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial u}\,q + \frac{\partial f}{\partial p}\,f + \frac{\partial f}{\partial q}\,\frac{\partial q}{\partial y}\ ,$$

where $$\frac{\partial p}{\partial x} = \lim_{h \to 0}\,\frac{\Delta^* p}{h} = \frac{dp^0(x)}{dx} - \frac{dy^0(x)}{dx}\,f\ ; \qquad (57)$$

$$\frac{\partial q}{\partial y} = \lim_{h \to 0} \frac{q^0(x_1) + \Delta^* q - q^0(x_1 + h)}{k}$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{q^0(x_1) + \Delta^* q - q^0(x_1 + h)}{k/h}$$

$$= \left[\frac{dq^0(x)}{dx} - f\right] \Bigg/ \left(\frac{dy^0(x)}{dx}\right) . \tag{57}$$

From (57) it follows that:

$$2 \frac{\partial F(M)}{\partial x} + \frac{dy^0(x_1)}{dx} \frac{\partial F(M)}{\partial y}$$

$$= \Bigg[2 \frac{\partial f}{\partial x} + \frac{dy^0}{dx} \frac{\partial f}{\partial y} + \frac{\partial f}{\partial u} \left(2p^0 + \frac{dy^0}{dx} q^0\right) + \frac{\partial f}{\partial p}\left(2 \frac{dp^0}{dx} - \frac{dy^0}{dx} f\right) + \frac{\partial f}{\partial q}\left(\frac{dq^0}{dx} + f\right)\Bigg]_M . \tag{58}$$

For the calculation of the left sides of (54) through (56) it is necessary to find $(d/dh)_M$ for $F(N)$, $F\left(M_2^j\right.$ $\left.,\right) F\left(N_2^j\right)$:

$$\frac{dF(N)}{dh} = \frac{dF(x_1 + h,\ y_1 - k(h))}{dh} = \frac{\partial F(N)}{dx} - \frac{dk(h)}{dh} \frac{\partial F(N)}{\partial y} ,$$

so that on the strength of (46) and (57)

$$\left[\frac{dF(N)}{dh}\right]_M = \Bigg[\frac{\partial f}{\partial x} + \frac{dy^0}{dx} \frac{\partial f}{\partial y} + \frac{\partial f}{\partial u}\left(p^0 + \frac{dy^0}{dx} q^0\right) + \frac{\partial f}{\partial p} \frac{dp^0}{dx} + \frac{\partial f}{\partial q} \frac{dq^0}{dx}\Bigg]_M . \tag{59}$$

Furthermore, from (51) it follows that:

$$\left(\frac{d\Delta^1 u}{dh}\right)_M = p^0(x_1) \quad , \qquad \left(\frac{d\Delta^1 p}{dh}\right)$$

$$\left(\frac{d\Delta^1 q}{dh}\right)_M = F(M) \quad ,$$

and from comparisons of $\delta^1 u$, $\delta^1 p$, $\delta^1 q$ wit

$$\left(\frac{d\delta^1 u}{dh}\right)_M = -q^0($$

$$\left(\frac{d\delta^1 p}{dh}\right)_M = -\frac{dy}{}$$

$$\left(\frac{d\delta^1 q}{dh}\right)_M = -\frac{dq^0(x_1)}{dx} + F(M) \quad . \tag{61}$$

From (48.a), using (60), we have

$$\left[\frac{dF\left(M_2^u\right)}{dh}\right]_M = \left[\rho^j \frac{\partial f}{\partial x} - \sigma^j \frac{dy^0}{dx}\frac{\partial f}{\partial y} + \alpha^j p^0 \frac{\partial f}{\partial u} + \beta^j\left(\frac{dp^0}{dx} - \frac{dy^0}{dx} f\right)\frac{\partial f}{\partial p} + \gamma^j \frac{\partial f}{\partial q}\right]_M \quad . \tag{62}$$

From (48.b), where $x_2 = x_1 + h$, $y_2 = y_1 - k(h)$, using (61), we have

$$\begin{aligned}\left[\frac{dF\left(N_2^u\right)}{dh}\right]_M = \Bigg[&\left(1 + \rho^{-j}\right)\frac{\partial f}{\partial x} + \left(1 - \sigma^j\right)\frac{dy^0}{dx}\frac{\partial f}{\partial y} \\ &+ \left(p^0 + q^0 \frac{dy^0}{dx} - \alpha^{-j} q^0 \frac{dy^0}{dx}\right)\frac{\partial f}{\partial u} \\ &+ \left(\frac{dp^0}{dx} - \beta^{-j}\frac{dy^0}{dx} f\right)\frac{\partial f}{\partial p} \\ &+ \left(\frac{dq^0}{dx} - \gamma^{-j}\frac{dq^0}{dx} + \gamma^{-j} f\right)\frac{\partial f}{\partial q}\Bigg]_M \quad .\end{aligned} \tag{63}$$

Let us consider the condition (54). Its right side is given in (58) and the left side can be expressed in the following form, using for this (47.a), (49.a), (59), (62), and (63):

$$3\,\frac{\partial f}{\partial x}\,(\lambda_1\tau_1 + \lambda_2\tau_2 + \lambda_2\omega_2\rho + \lambda_2\tau_2\bar{\rho})$$

$$+\ 3\,\frac{\partial f}{\partial y}\,(\lambda_1\tau_1 + \lambda_2\tau_2 - \lambda_2\omega_2\sigma - \lambda_2\tau_2\bar{\sigma})\,\frac{dy^0}{dx}$$

$$+\ 3\,\frac{\partial f}{\partial u}\Big[(\lambda_1\tau_1 + \lambda_2\tau_2 + \lambda_2\omega_2\alpha)\,p^0$$

$$+\ (\lambda_1\tau_1 + \lambda_2\tau_2 - \lambda_2\tau_2\bar{\alpha})\,q^0\,\frac{dy^0}{dx}\Big]$$

$$+\ 3\,\frac{\partial f}{\partial p}\Big[(\lambda_1\tau_1 + \lambda_2\tau_2 + \lambda_2\omega_2\beta)\,\frac{dp^0}{dx}$$

$$-\ (\lambda_2\omega_2\beta + \lambda_2\tau_2\bar{\beta})\,\frac{dy^0}{dx}\,f\Big]$$

$$+\ 3\,\frac{\partial f}{\partial q}\Big[(\lambda_1\tau_1 + \lambda_2\tau_2 - \lambda_2\tau_2\gamma)\,\frac{dq^0}{dx}$$

$$+\ (\lambda_2\omega_2\gamma + \lambda_2\tau_2\bar{\gamma})\,f\Big]\,. \qquad (64)$$

Here, all values are given at the point M and, for simplicity, all indices $j = u$ are omitted.

As in [1] we assume:

$$\bar{\alpha} = \alpha\ , \qquad \bar{\beta} = \beta\ , \qquad \bar{\gamma} = \gamma\ , \qquad \bar{\rho} = \rho - 1\ , \qquad \bar{\sigma} = \sigma + 1\ . \quad (65)$$

Taking into account (53), (65) and equating (64) to the right part of (58), we will obtain for the parameters at Δu a system of equations (24) from [1]. Thus, the results of Table 1 of the article [1] remain valid for the parameters at Δu.

By considering analogously the conditions (55), (56) we will obtain for the parameters at Δp, Δq the systems of equations (25), (26) from [1], so that for the parameters at Δp, Δq the results of Table 2 of the article [1] remain valid.

The scheme of calculations according to the proposed algorithms will have a form analogous to (27), (28) from [1].

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13. ABSTRACT

In the first part of this article the two-iteration algorithms of the Runge-Kutta type were applied to the solution of the Cauchy problem for hyperbolic equations and systems with two independent variables, where the initial data are given along the line segment $x + y = \text{const.}$

In the second part of this article an analogous problem is discussed for one equation with Cauchy data along the curve segment.

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