1 Final Limits and Conclusion

Based on the observed zero event and with an expected background of 0.485 of an event, the following limits can be set at 90% C.L. using Feldman-Cousins (FC) statistics.:

- From the 1997 PNN2 analysis only:

$$90\% \text{ C.L. on } Br(K^+ \to \pi^+ \nu\bar{\nu}) < 1.94 \times 1.667 \times 10^{-9} = 3.23 \times 10^{-9}$$

- Comibing both 1996 and 1997 PNN2 analysis, we can set a 90% C.L. limit on the branching ratio as:

$$\begin{gathered} \text{Number of events observed} = 1 \\ \text{Total Background} = 1.22 \pm 0.24 \\ SES_{96} = (1.169 \pm 0.029) \times 10^{-9} \\ SES_{97} = (1.667 \pm 0.043) \times 10^{-9} \end{gathered}$$

  Therefore, the following 90% C.L. upper limit can be set using FC statistics. A simple way of combining the two analysis:

$$Br(K^+ \to \pi^+ \nu\bar{\nu}) < 3.14 \times \left(\frac{1}{1.169 \times 10^{-9}} + \frac{1}{1.667 \times 10^{-9}}\right)^{-1} = 3.14 \times 6.87 \times 10^{-10} = 2.16 \times 10^{-9}$$

  The previous published limit from the 1996 PNN2 analysis alone set an upper limit on this branching ratio at 4.2×10^{-9}. Therefore, combining these two analysis give an upper limit which is roughly a factor of two better than that was achieved before.

- Assuming a scalar form factor for $K^+ \to \pi^+ + nothing$ and using FC statistics the following limits can be set:
  1) Using only the PNN1 signal region: The single event sensitivity for PNN1 95-98 analysis is 8.325×10^{-11}. We use the UMC acceptance of 0.0335 for PNN1 from 95-97 analysis. For a scalar form factor, the UMC acceptance was calculated to be 0.0020 in the PNN1 region. Therefore, the single event sensitivity from PNN1 analysis for a scalar spectrum is $\frac{0.0335}{0.0020} \times 8.325 \times 10^{-11} = 1.394 \times 10^{-9}$. Using the observation of 2 events with expected background of 0.15, we obtain the 90% C.L. on scalar spectrum of $K^+ \to \pi^+ + nothing$ as:

$$90\% \text{ C.L. on } Br(K^+ \to \pi^+ + nothing)_{scal} < 5.76 \times 1.394 \times 10^{-9} = 8.03 \times 10^{-9}$$

However, this method is perhaps not correct, because here we assumed that the two signal events (from vector interaction) in PNN1 region against a background of 0.15 are background events for scalar type interaction. However, if we have enough confidence that the two events that we observed in PNN1 region are signal events and coming from vector interaction, then we can assume that for scalar type interaction there was 0 background events against an expected background of 0.15. Therefore, we can put a limit on scalar spectrum of $K^+ \to \pi^+ + nothing$ as:

$$90\% \text{ C.L. on } Br(K^+ \to \pi^+ + nothing)_{scal} < 2.29 \times 1.394 \times 10^{-9} = 3.19 \times 10^{-9}$$

2) Using only the PNN2 signal region from 1996 analysis: The single event sensitivity for PNN2 1996 analysis is 1.17×10^{-9}. We use the UMC acceptance of 0.0808 for the PNN2 region. For a scalar form factor, the UMC acceptance was calculated to be 0.0635 in the PNN2 region. Therefore, the single event sensitivity from 1996 PNN2 analysis for a scalar spectrum is $\frac{0.0808}{0.0635} \times 1.17 \times 10^{-9} = 1.489 \times 10^{-9}$. Using the observation of 1 events with expected background of 0.73, we obtain the 90% C.L. on scalar spectrum of $K^+ \to \pi^+ + nothing$ as:

$$90\% \text{ C.L. on } Br(K^+ \to \pi^+ + nothing)_{scal} < 3.6 \times 1.489 \times 10^{-9} = 5.36 \times 10^{-9}$$

3) Using only the PNN2 signal region from 1997 analysis: The single event sensitivity for PNN2 1997 analysis is 1.667×10^{-9}. We use the UMC acceptance of 0.0808 for the PNN2 region. For a scalar form factor, the UMC acceptance was calculated to be 0.0635 in the PNN2 region. Therefore, the single event sensitivity from 1997 PNN2 analysis for a scalar spectrum is $\frac{0.0808}{0.0635} \times 1.667 \times 10^{-9} = 2.12 \times 10^{-9}$. Using the observation of 0 event with expected background of 0.485, we obtain the 90% C.L. on scalar spectrum of $K^+ \to \pi^+ + nothing$ as:

$$90\% \text{ C.L. on } Br(K^+ \to \pi^+ + nothing)_{scal} < 1.94 \times 2.12 \times 10^{-9} = 4.12 \times 10^{-9}$$

4) Using only the PNN2 signal region from 1996-97 analysis: The 1996 and 1997 PNN2 analysis can be combined to put limit on the scalar spectrum of $K^+ \to \pi^+ + nothing$. Using the observation of 1 event with expected background of 1.22, we obtain the 90% C.L. on scalar spectrum of $K^+ \to \pi^+ + nothing$ as:

$$Br(K^+ \to \pi^+ + nothing)_{scal} < 3.14 \times (\frac{1}{1.489\times 10^{-9}} + \frac{1}{2.12\times 10^{-9}})^{-1} = 3.14 \times 8.747 \times 10^{-10}$$

$$\text{or, } Br(K^+ \to \pi^+ + nothing)_{scal} < 2.75 \times 10^{-9}$$

5) combining 1995-98 PNN1 results and the PNN2 results from 1996-97 analysis: The PNN1 results from the 1995-98 analysis and the results from the 1996-97 PNN2 analysis can be combined to put limit on the scalar spectrum of $K^+ \to \pi^+ + nothing$. Using the observation of 3 event with expected background of 1.37, we obtain the 90% C.L. on scalar spectrum of $K^+ \to \pi^+ + nothing$ as:

$$Br(K^+ \to \pi^+ + nothing)_{scal} < 6.05 \times (\frac{1}{1.394\times 10^{-9}} + \frac{1}{8.747\times 10^{-10}})^{-1} = 6.05 \times 5.374 \times 10^{-10}$$

$$\text{or, } Br(K^+ \to \pi^+ + nothing)_{scal} < 3.25 \times 10^{-9}$$

But again assuming only one observed background event, against an expected background of 1.37, we obtain the following limit on the scalar spectrum of $K^+ \to \pi^+ + nothing$ as:

$$Br(K^+ \to \pi^+ + nothing)_{scal} < 2.99 \times (\frac{1}{1.394\times 10^{-9}} + \frac{1}{8.747\times 10^{-10}})^{-1} = 2.99 \times 5.374 \times 10^{-10}$$

$$\text{or, } Br(K^+ \to \pi^+ + nothing)_{scal} < 1.61 \times 10^{-9}$$

The previous published 90% C.L. upper limit (1996 PNN2 paper) on $Br(K^+ \to \pi^+ + nothing)$ was 4.7×10^{-9} assuming a scalar form factor. This result was also obtained by combining the results from the PNN1 and PNN2 regions. However, in this result it was assumed that there are 3 background events (two from PNN1 and one from 1996 PNN2). Assuming only one background event (from PNN2 region), the limit would have been 2.51×10^{-9}.

- Assuming a tensor form factor for $K^+ \to \pi^+ + nothing$ and using FC statistics the following limits can be set:
  1) Using only the PNN1 signal region: The single event sensitivity for PNN1 95-98 analysis is 8.325×10^{-11}. We use the UMC acceptance 0.0335 for PNN1 from 1995-97 analysis. For a tensor form factor, the UMC acceptance was calculated to be 0.0046 in the PNN1 region. Therefore, the single event sensitivity from PNN1 analysis for a tensor spectrum is $\frac{0.0335}{0.0046} \times 8.325 \times 10^{-11} = 6.06 \times 10^{-10}$. Using the observation of 2 events with expected background of 0.15, we obtain the 90% C.L. on tensor spectrum of $K^+ \to \pi^+ + nothing$ as:

$$90\% \text{ C.L. on } Br(K^+ \to \pi^+ + nothing)_{tens} < 5.76 \times 6.06 \times 10^{-10} = 3.49 \times 10^{-9}$$

Again, assuming 0 background events against an expected background of 0.15, we obtain the 90% C.L. on tensor spectrum of $K^+ \to \pi^+ + nothing$ as:

$$90\% \text{ C.L. on } Br(K^+ \to \pi^+ + nothing)_{tens} < 2.29 \times 6.06 \times 10^{-10} = 1.39 \times 10^{-9}$$

2) Using only the PNN2 signal region from 1996 analysis: The single event sensitivity for PNN2 1996 analysis is 1.17×10^{-9}. We use the UMC acceptance of 0.0808 for the PNN2 region. For a tensor form factor, the UMC acceptance was calculated to be 0.0954 in the PNN2 region. Therefore, the single event sensitivity from 1996 PNN2 analysis for a tensor spectrum is $\frac{0.0808}{0.0954} \times 1.17 \times 10^{-9} = 9.91 \times 10^{-10}$. Using the observation of 1 events with expected background of 0.73, we obtain the 90% C.L. on tensor spectrum of $K^+ \to \pi^+ + nothing$ as:

$$90\% \text{ C.L. on } Br(K^+ \to \pi^+ + nothing)_{tens} < 3.6 \times 9.91 \times 10^{-10} = 3.6 \times 10^{-9}$$

3) Using only the PNN2 signal region from 1997 analysis: The single event sensitivity for PNN2 1997 analysis is 1.667×10^{-9}. We use the UMC acceptance of 0.0808 for the PNN2 region. For a tensor form factor, the UMC acceptance was calculated to be 0.0954 in the PNN2 region. Therefore, the single event sensitivity from 1997 PNN2 analysis for a tensor spectrum is $\frac{0.0808}{0.0954} \times 1.667 \times 10^{-9} = 1.41 \times 10^{-9}$. Using the observation of 0 event with expected background of 0.485, we obtain the 90% C.L. on scalar spectrum of $K^+ \to \pi^+ + nothing$ as:

$$90\% \text{ C.L. on } Br(K^+ \to \pi^+ + nothing)_{tens} < 1.94 \times 1.41 \times 10^{-9} = 2.74 \times 10^{-9}$$

4) Using only the PNN2 signal region from 1996-97 analysis: The 1996 and 1997 PNN2 analysis can be combined to put limit on the tensor spectrum of $K^+ \to \pi^+ + nothing$. Using the observation of 1 event with expected background of 1.22, we obtain the 90% C.L. on tensor spectrum of $K^+ \to \pi^+ + nothing$ as:

$$Br(K^+ \to \pi^+ + nothing)_{tens} < 3.14 \times \left(\frac{1}{9.91 \times 10^{-10}} + \frac{1}{1.41 \times 10^{-9}}\right)^{-1} = 3.14 \times 5.82 \times 10^{-10}$$

$$\text{or, } Br(K^+ \to \pi^+ + nothing)_{tens} < 1.83 \times 10^{-9}$$

5) combining 1995-98 PNN1 results and the PNN2 results from 1996-97 analysis: The PNN1 results from the 1995-98 analysis and the results from the 1996-97 PNN2 analysis can be combined to put limit on the tensor spectrum of $K^+ \to \pi^+ + nothing$. Using the observation of 3 event with expected background of 1.37, we obtain the 90% C.L. on tensor spectrum of $K^+ \to \pi^+ + nothing$ as:

$$Br(K^+ \to \pi^+ + nothing)_{tens} < 6.05 \times (\tfrac{1}{6.06\times10^{-10}} + \tfrac{1}{5.82\times10^{-10}})^{-1} = 6.05 \times 2.97 \times 10^{-10}$$

$$\text{or, } Br(K^+ \to \pi^+ + nothing)_{tens} < 1.80 \times 10^{-9}$$

Again, assuming only one observed background event, against an expected background of 1.37, we obtain the 90% C.L. on tensor spectrum of $K^+ \to \pi^+ + nothing$ as:

$$Br(K^+ \to \pi^+ + nothing)_{tens} < 2.99 \times (\tfrac{1}{6.06\times10^{-10}} + \tfrac{1}{5.82\times10^{-10}})^{-1} = 2.99 \times 2.97 \times 10^{-10}$$

$$\text{or, } Br(K^+ \to \pi^+ + nothing)_{tens} < 8.88 \times 10^{-10}$$

The previous published 90% C.L. upper limit (1996 PNN2 paper) on $Br(K^+ \to \pi^+ + nothing)$ was 2.5×10^{-9} assuming a tensor form factor. This result was also obtained by combining the results from the PNN1 and PNN2 regions. However, this result again assumed 3 background events against an expected background of 1.37. Assuming only one observed background event (from 1996 PNN2), the limit would have been 1.31×10^{-9}.

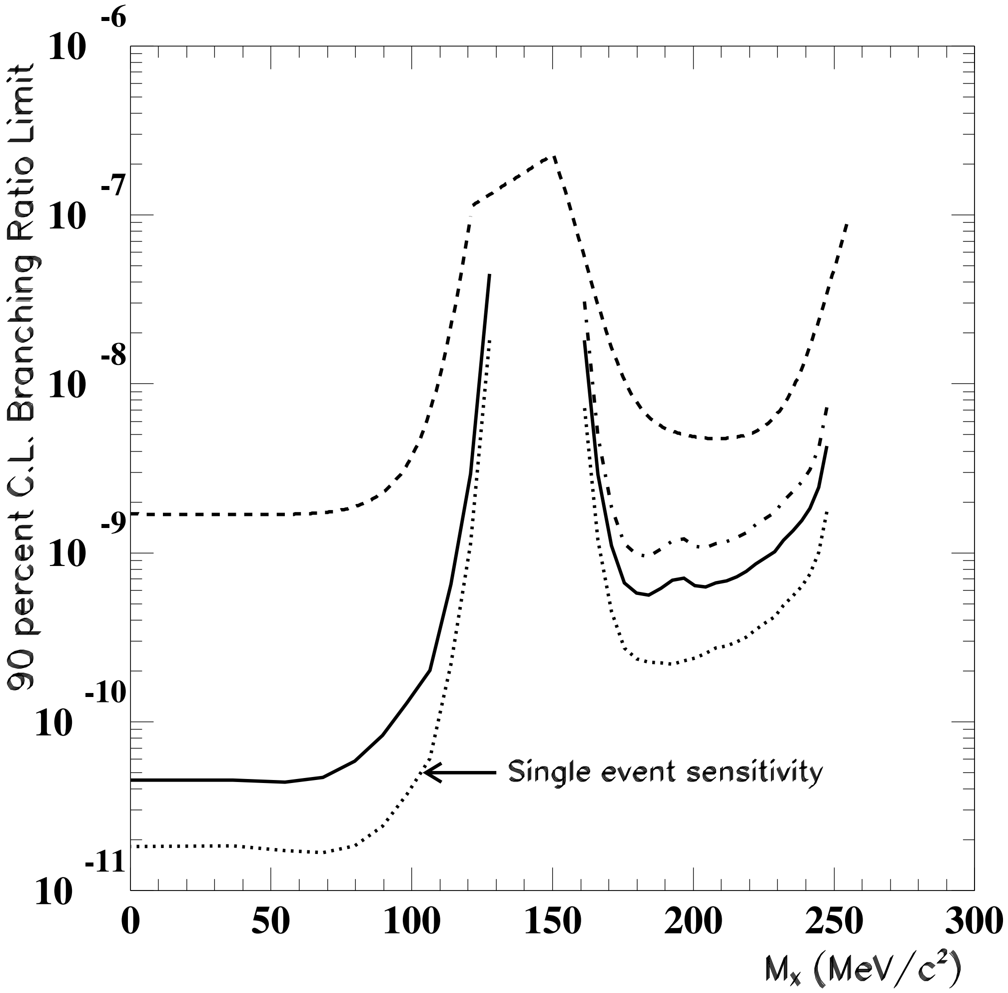

Figure 1: Limit on the branching ratio of $K^+ \to \pi^+ X$ as a function of mass of X. The 1997 PNN2 analysis only scales down the limit for $M_X > 150(MeV/c^2)$.