**HiRho CCD INTERNAL NOTE**

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**Effect of antireflective quartz coating on a silicon photodiode**

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## 1. Introduction

To understand better the quantum efficiency of LBL-produced photodiodes (process prototypes for high-resistivity silicon CCD's), Steve Holland coated the back surface of several diodes with $SiO_2$. The thickness $d$ was such that $nd = \lambda_0/4$, with $\lambda_0$ was chosen to be 480 nm. The quantum efficiencies of coated and uncoated diodes were measured by Bill Moses. In this note we compare a calculation of the expected effect with Moses's results. The calculation is limited by our assumption that silicon has a real index of refraction which can be calculated from the published reflectivity of pure silicon. The results are surprisingly good, and tend to corroborate the suspicion that the loss in quantum efficiency blueward of about 600 nm is not a surface effect.

## 2. Expectation

The multiple reflections in the coating are shown in Fig. 1 [1]. We consider only normal reflection, but show an angle of incidence so that the rays can be labeled. We explicitly use the fact that the reflected amplitude fraction ($r$) for a ray incident from either side of an interface is independent of direction, except that there is a sign change in going from the rarer to denser medium. The transmitted fraction is taken as $t$ in going from rarer to denser, and as $t'$ in going from denser to rarer.

Each successive reflection is retarded by $\phi = 4\pi nd/\lambda$ from the previous one. If $nd = \lambda_0/4$, then successive reflections are retarded by $\lambda_0/2$, and there is maximal destructive interference. In general, we take $\phi = \pi\lambda_0/\lambda$, and multiply successive reflections by $e^{i\phi}$. The total reflected amplitude is then

$$\begin{aligned} R &= -ar - atr't'e^{i\phi}\left(1 - rr'e^{i\phi} + r^2r'^2e^{2i\phi} - r^3r'^3e^{3i\phi} + \ldots\right) \\ &= -ar - \frac{atr't'e^{i\phi}}{1 + rr'e^{i\phi}} \ . \end{aligned}$$

Stokes used a time-reversal argument to show that $tt' = (1 - r^2)$ [2]. We can use this to finish the reduction:

$$\begin{aligned} R &= -a\left[r + r'\frac{(1 - r^2)\,e^{i\phi}}{1 + rr'e^{i\phi}}\right] \\ &= -a\left[\frac{r + r'e^{i\phi}}{1 + rr'e^{i\phi}}\right] \end{aligned} \tag{1}$$

$$\frac{|R|^2}{|a|^2} = \frac{r^2 + 2rr'\cos\phi + r'^2}{1 + 2rr'\cos\phi + r^2r'^2}$$

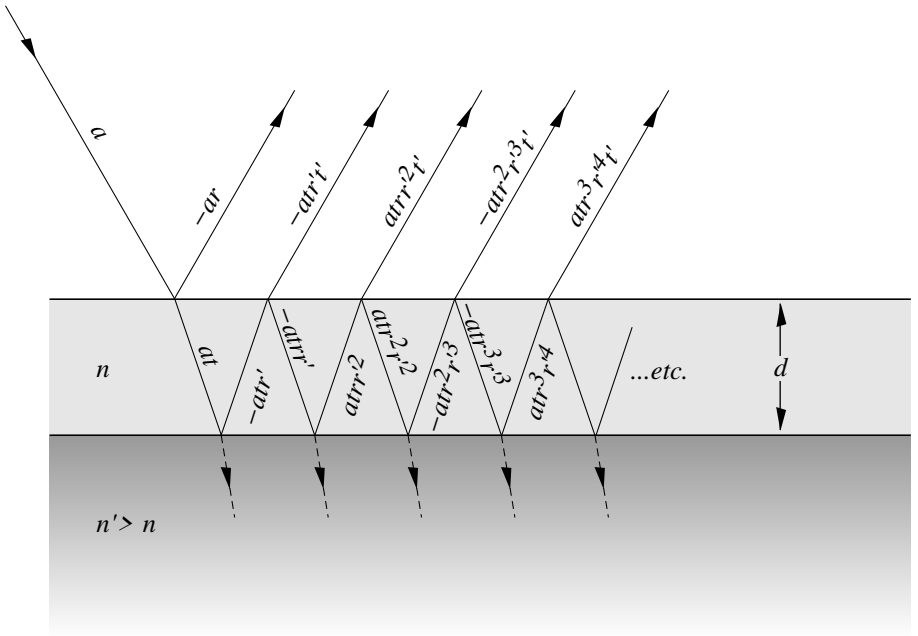


**Figure 1:** Amplitudes of successive rays in multiple reflections from a thin coating with index $n$ and thickness $d$. Successive reflections are retarded by a phase $\phi$, where $\phi = 4\pi nd/\lambda$.

In passing, we note that if $r = r'$ and $\phi = (2m+1)\pi$, then $R = 0$, as expected. Similarly, if $r = 0$ (coating made of of air), the $R = r'$, the result with no coating.

Our starting point for numerical work is a published curve for the reflectivity of pure silicon [3]. The usual relationship for the fraction of the intensity (or power) fraction reflected at normal incidence from an interface in which the light goes from a medium with index $n$ to a medium with index $n'$,

$$\text{Reflection coefficient for power} = \left(\frac{n'-n}{n'+n}\right)^2 ,$$

can be solved for $n'$ in the case $n = 1$. For the quartz overcoating, $n = 1.46$. Then $r$ and $r'$ are given by

$$r = \frac{n-1}{n+1}$$
$$r' = \frac{n'-n}{n'+n} .$$

(Note that in this case $n > 1$ and $n' > n$.) Eq. (1) can then be solved for fraction of the intensity reflected. The published reflectivity of uncoated silicon and our calculated reflectivity for quartz-coated silicon are shown by the solid lines in Fig. 2. On the same plot are shown Moses' measured quantum efficiencies (QE) for back side illuminated (BSI) diodes, with and without the antireflection (AR) coating.

What if the change in reflectivity is calculated correctly, but for some reason 100% of the photons which are not reflected are *not* converted to electron-hole pairs which are not subsequently collected? Following a suggestion by Saul Perlmutter, we multiply the calculated ratio of reflectivities with and without the coating by the measured quantum efficiency in the absence of the AR coating, to obtain the dotted curve labeled "No AR, scaled to AR."

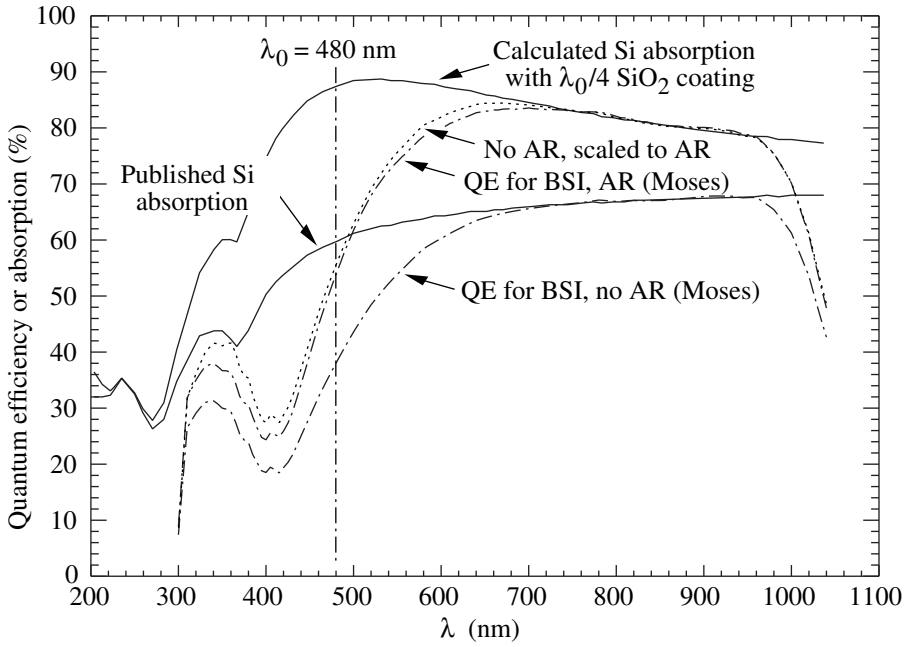


**Figure 2:** Experimental and theoretical effect of a $\lambda_0/4$ surface coating with $\lambda_0 = 480$ nm. Moses' experimental results are for Holland's high-resistivity diodes, backside illuminated, with and without antireflective coatings.

## 3. Discussion

The agreement between approximately 740 and 950 nm is rather remarkable, and may be fortuitous. The dropoff above 950 nm occurs because the photons no longer have enough enough energy to create electron-hole pairs, and the dropoff below 400 nm is the result of the silicon becoming absorptive. In the region between about 500 nm and 740 nm, the difference between the measured QE and the fraction of the photons penetrating the silicon is not understood, and is the subject of considerable concern. Several remedies are being explored. On the basis of this analysis, we predict that one of them, polishing the back surface, will have little effect.

Given the agreement in the 740–950 nm region, why does the scaled dotted curve not agree better with the measured QE in the AR case? The discrepancy is too small for this to be a serious matter, but we note that the measurements were made on different chips. There is no reason for them to be identical at this level.

In Fig. 3 we show the relative bandpasses of some of the standard broadband photometry filters used in astronomy. (The letters stand for blue, visible, red, and infrared.) It is evident that B and V are badly matched to our QE response, and in fact B is badly matched to even an ideal silicon photodiode response. The LBL images used for the deep supernova search are obtained with an R filter, and in its acceptance region the present photodiode has relatively high quantum efficiency. The results of folding the predicted and measured responses with the R response are shown in Fig. 4.

Can one do better with a different film thickness? An example is shown in Fig. 5, where the calculation was made for a coating thickness of (600 nm)/4. The effect of film thickness is

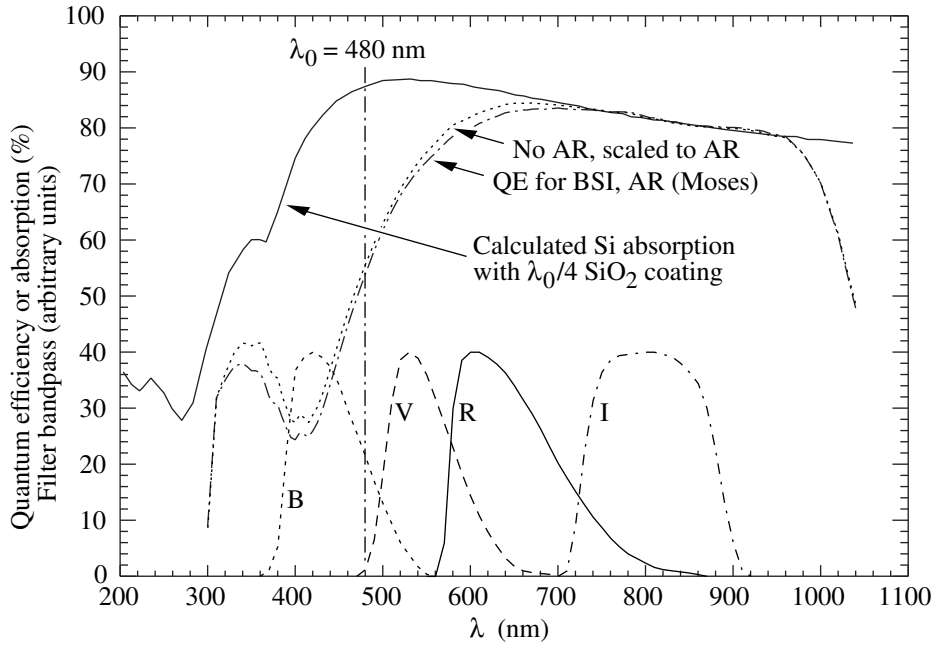


**Figure 3:** Nearly the same as Fig. 2, but showing the bandpasses of the standard wideband filters.

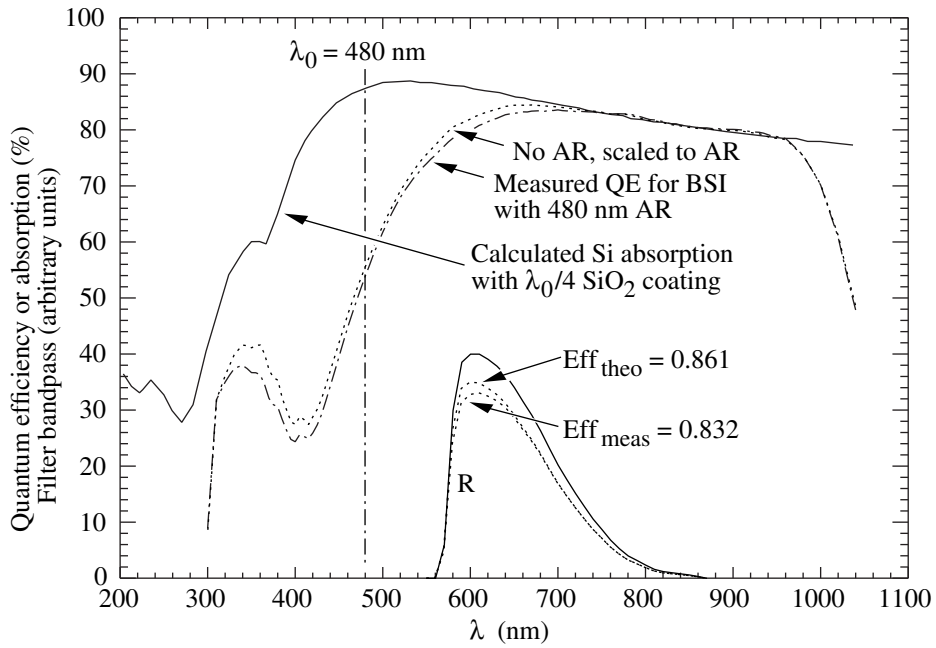


**Figure 4:** R-filter response convoluted with theoretical and experimental quantum efficiencies for a diode coated with $\lambda_0/4$ of $SiO_2$ at $\lambda_0 = 480$ nm.

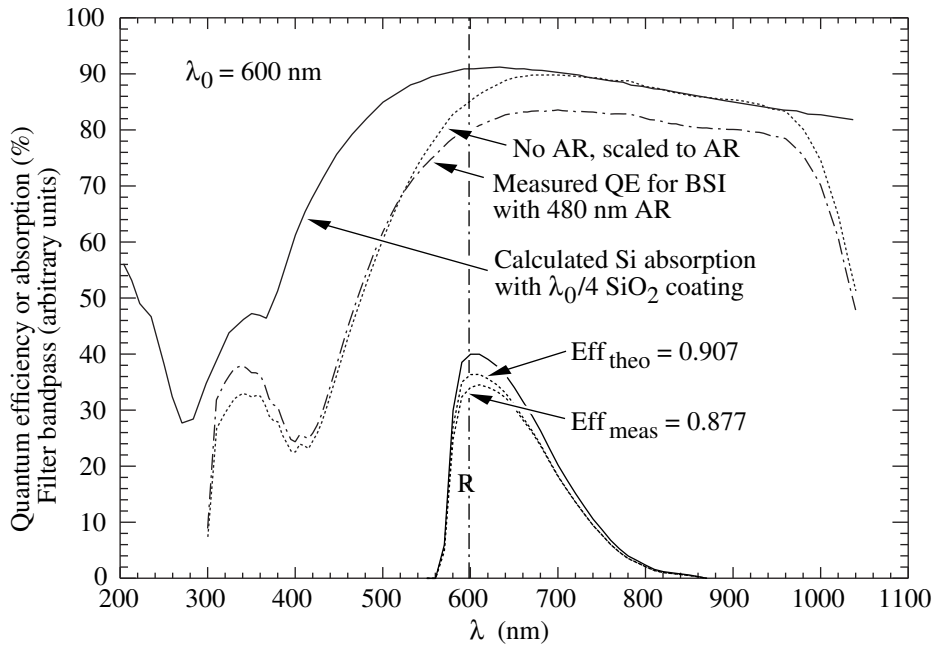


**Figure 5:** R filter response convoluted with theoretical and experimental quantum efficiencies for a diode coated with $\lambda_0/4$ of $SiO_2$ at $\lambda_0 = 600$ nm.

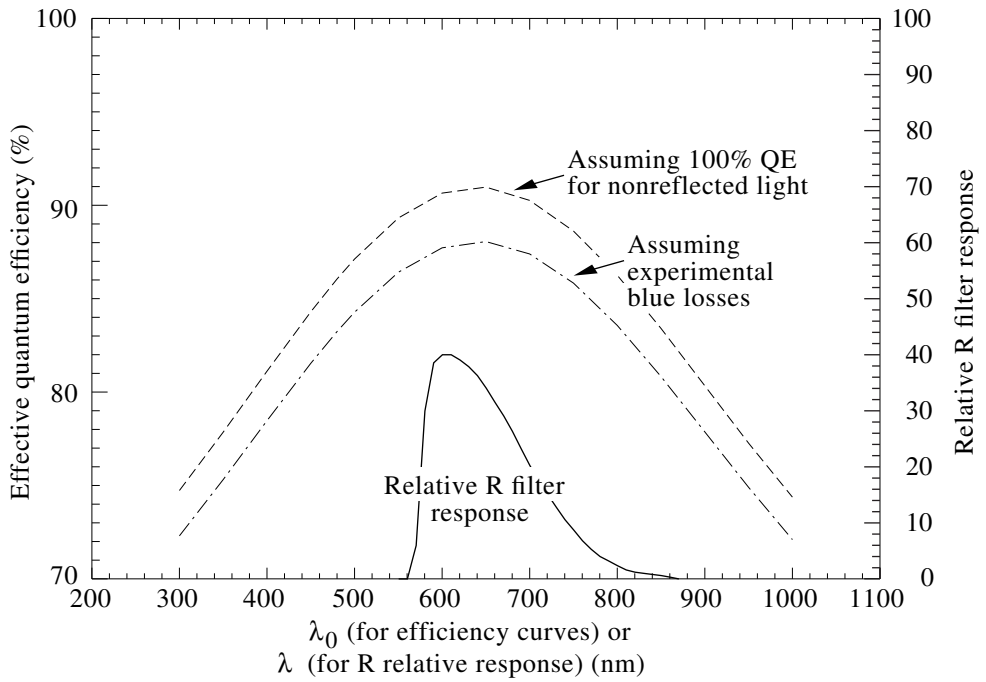


**Figure 6:** Experimental and theoretical quantum efficiencies for a diode coated with $\lambda_0/4$ of $SiO_2$ as a function of $\lambda_0$.

explored further in Fig. 6, where, as before, results are shown for our theoretical response and for the measured response. Changing the coating thickness over a fairly wide range has little effect.

We are more concerned over the combination (a) of the red filter response covering a region where the QE varies by a fair amount and (b) the fact that filters bought at different times from different manufacturers vary among themselves. This is particularly insidious for the deep supernova search program, since the photometry is done at different telescopes, with different filters and CCD's. Point (a) means that our actual R-band acceptance is different from that used in obtaining calibration standards.

## 4. Better coatings

Once the $SiO_2$ coating has been applied, the silicon is protected and other coatings ($Si_3N_4$, $CaF_2$, *etc.*) can be applied to tailor the transmission as desired. This is standard practice and can be done, but reinvention in this mature field is not something we should spend time on. Our goal was to understand the behavior of a single quarter-wave coating in order to separate optical effects from effects having to do with the conversion of photons to electron-hole pairs which are subsequently collected.

## 5. Conclusions

In a broad spectral region, the agreement between measured quantum efficiency and expected absorption argues for 100% conversion of photons to detected electron-hole pairs once the photons cross the interface. To sufficient accuracy, this includes the R photometry band, and the present performance is good enough to do useful science. On this basis, we should proceed with the CCD development. But the device would be even better if the response at shorter wavelengths could be improved, and it is worrisome not to understand the problem.

The "blue problem" is emphasized by the comparison of Moses' QE measurements for coated and uncoated diodes with the QE of a calibrated Hamamatsu S-2506 *pin* photodiode, as shown in Fig. 7. The S-2506 is probably AR coated but has an epoxy window, so its response is between our two cases in the far red, where we think things are understood. Its response does not show the dropoff to a minimum at 400 nm, and it is definitely a useful device in the V and even B bands.

If we understand the standard diode as naked silicon, plus an antireflective coating with little wavelength dependence, plus the epoxy window, then it might be sensible to scale our "no AR" results that they agree with the standard photodiode quantum efficiency in the far red. This is done in Fig. 7 by scaling our "no AR" results by 1.14. Our "blue problem" area is shaded.

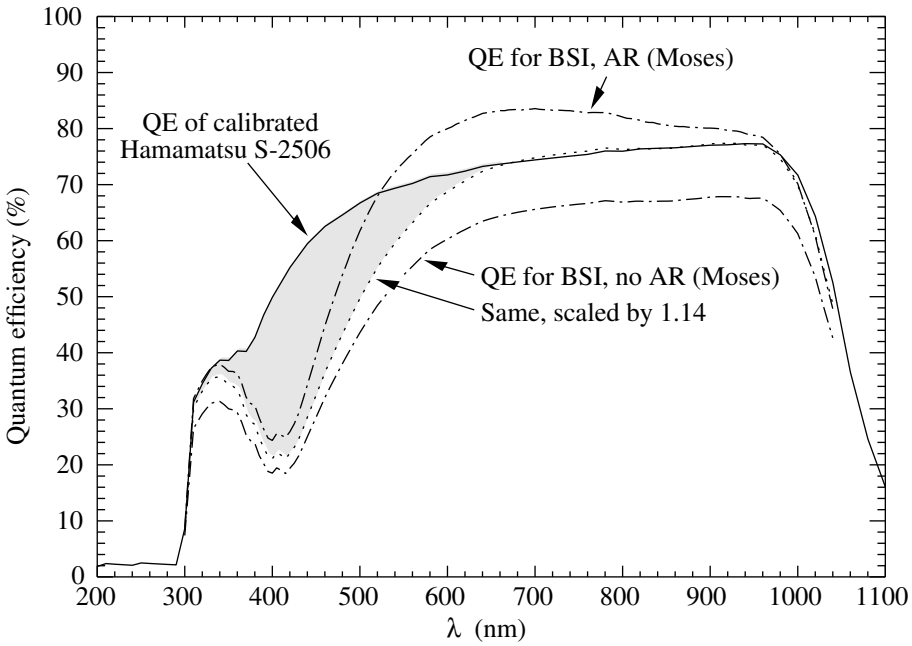


**Figure 7:** Measured quantum efficiencies of back side illuminated photodiodes, with and without antireflection coatings ($\lambda_0/4$ at $\lambda_0 = 480$ nm). For comparison purposes, the "no AR" case is also shown scaled by 1.14. The "blue problem" is indicated by the shaded region, where the blue response is suppressed.

## References

1. F. A. Jenkins and H. E. White, *Fundamentals of Optics*, McGraw-Hill, New York, 1950.

2. G. Stokes, *Mathematical and Physical Papers*, Vol. 2, pp. 89*ff*, especially p. 91.

3. From a photocopy from somewhere or another, obtained from Steve Holland.