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Homework #9 Solutions

$B^o - \bar{B}^o$Mixing

Question #1

Variables:

- Mixing parameter

$$r = \frac{\#\ of\ events\ with\ wrong\ sign\ (mixing)}{\#\ of\ events\ with\ right\ sign\ (no\ mixing)} = \frac{n_w}{n_r}$$

  That is, the wrong sign leptons and the right sign leptons are produced with $r\ :\ 1$ ratio for a given neutral B.

- total number of decays $N = n_r + n_w$
- Probability of right sign decay $P_r = \frac{n_r}{N} = \frac{1}{\frac{N}{n_r}} = \frac{1}{1+r}$
- Probability of wrong sign decay $P_w = \frac{n_w}{N} = 1 - P_r = \frac{r}{1+r}$

1. $B^o - \bar{B}^o$produced by by e^+e^- annihilation far above the $\Upsilon(4S)$. Their oscillations are treated independently.

$$\frac{N(\ell^+\ell^+) + N(\ell^-\ell^-)}{N(\ell^+\ell^-)} = \frac{P_r P_w + P_w P_r}{P_r P_r + P_w P_w} = \frac{2 \cdot \frac{1}{r+1} \cdot \frac{r}{r+1}}{\frac{r^2+1}{(r+1)^2}} = \frac{2r}{r^2+1}$$

   - The nominator represents one mixing out of the two, that is,
     - $B^o \to \bar{B}^o,\ \bar{B}^o$, thus the rate down by r
     - $B^o,\ \bar{B}^o \to B^o$, thus the rate down by r
   - The denominator represents "no mixing" + "both mixing" ($B^o \to \bar{B}^o$, $\bar{B}^o \to B^o$, thus the rate down by r^2)

2. $B^o - \bar{B}^o$produced by $\Upsilon(4S)$: when it decays into $B^o - \bar{B}^o$, there are no other particles produced, and they are completely corelated. That is, when one of them oscillates, the other does, too such that always $B^o - \bar{B}^o$is observed at a given time until one of them decays.

   First semileptonic decay fixes the identity of the second B at the time of decay. The second B oscillates then until its decay.

   - N^+ = # of events with ℓ^+ as first lepton
     At this time, B that decayed was B^o, thus the second B was $\bar{B}^o$.
   - N^- = # of events with ℓ^- as first lepton
     At this time, B that decayed was $\bar{B}^o$, thus the second B was B^o.

$$\frac{N(\ell^+\ell^+) + N(\ell^-\ell^-)}{N(\ell^+\ell^-)} = \frac{N^+ \cdot n_w + N^- \cdot n_w}{N^+ \cdot n_r + N^- \cdot n_r} = \frac{(N^+ + N^-)n_w}{(N^+ + N^-)n_r} = \frac{n_w}{n_r} = r$$

Question #2

- $x_d = \frac{\Delta m_{B_d^o}}{\Gamma_{B_d^o}} = \frac{\Delta m_d}{\Gamma_d}$
- ARGUS :
  - $r_d = \frac{x_d^2}{2+x_d^2} = 0.21 \pm 0.08$
    $\Rightarrow r_d \cdot (2 + x_d^2) = x_d^2 \Rightarrow 2r_d = x_d^2(1 - r_d) \Rightarrow x_d^2 = \frac{2r_d}{1-r_d} = 0.53 \pm 0.25$
- PDG :
  -

$$\begin{aligned}
\Delta m_d &= (0.479 \pm 0.012) \cdot 10^{12} \text{ [h} - \text{bar/s]} && (0.1) \\
\tau_{B^o} &= (1.540 \pm 0.024) \cdot 10^{-12} \text{ [s]} = \text{h} - \text{bar}/\Gamma_{B^o} \\
x_d &= \Delta m_d / \Gamma_{B^o} \\
&= \Delta m_d \cdot \tau_{B^o} / [\text{h} - \text{bar}] \\
&= (0.479 \pm 0.012) \cdot 10^{12} \text{ [h} - \text{bar/s]} \cdot (1.540 \pm 0.024) \cdot 10^{-12} \text{ [s/h} - \text{bar]} \\
&= (0.479 \pm 0.012) \times (1.540 \pm 0.024) \\
&= 0.738 \pm 0.022 \\
\text{where } \delta x_d &= 0.738 \cdot \sqrt{(\frac{0.012}{0.479})^2 + (\frac{0.024}{1.540})^2} = 0.022
\end{aligned}$$

$$
\begin{aligned}
r_d &= \frac{x_d^2}{2 + x_d^2} \\
\delta r &= \frac{2x_d \delta x_d}{2 + x_d^2} - \frac{x_d^2 \cdot 2x_d \delta x_d}{(2 + x_d^2)^2} \\
&= 2x_d \delta x_d \cdot \frac{2 + x_d^2 - x_d^2}{(2 + x_d^2)^2} = \frac{4x_d \delta x_d}{(2 + x_d^2)^2} \\
&= 0.010 \\
r_d &= 0.214 \pm 0.010
\end{aligned}
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