Coin Tossing
Physics 24 – Freshman Seminar
Spring 2000

In our experiment in class each student "flipped" a coin ten times. The results are given in the Table.

number of heads	0	1	2	3	4	5	6	7	8	9	10	total
number of occurences	0	0	0	1	4	3	6	1	1	0	0	16
contribution to $\sum m_i$	0	0	0	3	16	15	36	7	8	0	0	85
contribution to $\sum m_i^2$	0	0	0	9	64	75	216	49	64	0	0	477

Table 1: Our coin-tossing results. The mean was $\mu = 85/16 = 5.3125$. The square of the standard deviation was $\sigma^2 = (477 - 16 \times 5.3125^2)/16 = 25.438/16 = 1.59$.

The standard deviation is a measure of how far away measurements are from the mean, on average. This can be calculate in either of two ways:

$$\begin{aligned} \sigma^2 &= \sum_i (m_i - \mu)^2 \\ \sigma^2 &= \sum_i (m_i^2 - \mu^2) \end{aligned} \tag{1}$$

Here m_i represents the number of heads that the ith student got. It is not hard to show that if the odds of getting heads is p and the odds of getting tails is $q = 1 - p$, when we flip a coin N times we expect to find $\mu = Np$ heads on average. For our case $N = 10$ and $p = 0.5$ so we expected five heads, on average. We found 5.3125. Pretty close.

The second most important thing about a distribution after its mean (average) is how far the answers range away from that mean. This is what σ gives us. The prediction of simple probability theory is

$$\sigma = \sqrt{Npq} \tag{2}$$

For our case, $N = 10$, $p = q = 0.5$, we anticipate $\sigma = 1.581$ This is close to what we found. Here the spread is pretty large. For example it is 31% of the mean. If we flip the coin 100 times we expect $\mu = 50$, but $\sigma = \sqrt{100 \times 0.5 \times 0.5} = 5$. Now σ is only 10% of the mean.

It is typical that that fractional uncertainty in a measurement decreases as $\sqrt{N}$ if there are N measurements.