Set QCD: Quantum Chromodynamics

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General Category
Physics / Quantum Chromodynamics

Description
Lattice gauge theory is a discretization of quantum chromodynamics which is generally accepted to be the fundamental physical theory of strong interactions among the quarks as constituents of matter. The most time-consuming part of a numerical simulation in lattice gauge theory with Wilson fermions on the lattice is the computation of quark propagators within a chromodynamic background gauge field. These computations use up a major part of the world's high performance computing power.

Quark propagators are obtained by solving the inhomogeneous lattice Dirac equation

$$Ax = b$$

where $A = I - \kappa D$ with $0 \leq \kappa < \kappa_c$ is a large but sparse complex non-hermitian matrix representing a periodic nearest-neighbour coupling on a four-dimensional Euclidian space-time lattice, see Fig. 1.

Matrix Properties
From the physical theory it is clear that the matrix A should be positive real (all eigenvalues lie in the right half plane) for $0 \leq \kappa < \kappa_c$. Here, κ_c represents a critical parameter which depends on the given matrix D.

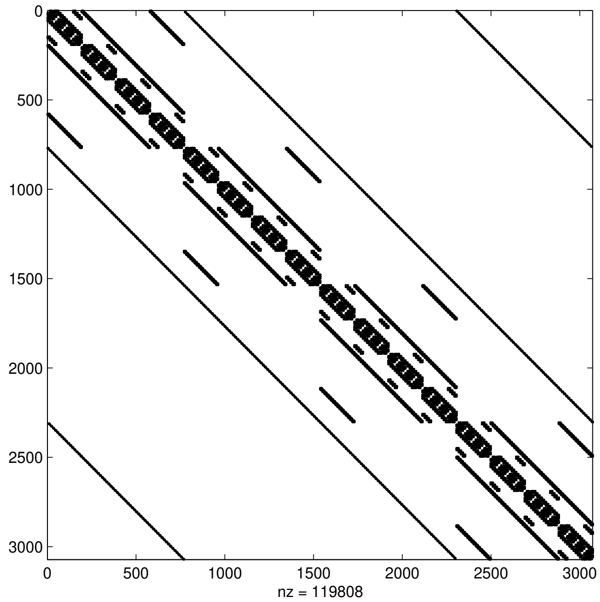

Figure 1: Sparsity pattern of Wilson fermion matrix $A = I - \kappa D$.

Denoting

$$\gamma_5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

the Wilson fermion matrix A is Γ_5 symmetric,

$$\Gamma_5 A = A^H \Gamma_5, \quad \Gamma_5 = I \otimes (\gamma_5 \otimes I_3).$$

Due to the nearest neighbour coupling, the matrix A has 'property A'. This means that with a red-black (or odd-even) ordering of the grid points the matrix becomes

$$A = I - \kappa D \text{ with } D = \begin{pmatrix} 0 & D_{oe} \\ D_{eo} & 0 \end{pmatrix}.$$

Set of QCD Matrices

The QCD matrices provided in the set QCD consist of realistic matrices D generated at different physical temperatures β, see Tab. 1.

matrix D	β	n	$nz(D)$	κ_c
conf5.0_00l4x4.1000.mtx	5.0	3072	119808	0.20611
conf5.0_00l4x4.1400.mtx	5.0	3072	119808	0.20328
conf5.0_00l4x4.1800.mtx	5.0	3072	119808	0.20265
conf5.0_00l4x4.2200.mtx	5.0	3072	119808	0.20235
conf5.0_00l4x4.2600.mtx	5.0	3072	119808	0.21070
conf6.0_00l4x4.2000.mtx	6.0	3072	119808	0.15968
conf6.0_00l4x4.3000.mtx	6.0	3072	119808	0.16453
conf5.4_00l8x8.0500.mtx	5.4	49152	1916928	0.17865
conf5.4_00l8x8.1000.mtx	5.4	49152	1916928	0.17843
conf5.4_00l8x8.1500.mtx	5.4	49152	1916928	0.17689
conf5.4_00l8x8.2000.mtx	5.4	49152	1916928	0.17835
conf6.0_00l8x8.2000.mtx	6.0	49152	1916928	0.15717
conf6.0_00l8x8.3000.mtx	6.0	49152	1916928	0.15649
conf6.0_00l8x8.8000.mtx	6.0	49152	1916928	0.15623

Table 1: Set of QCD matrices.

References

A survey of lattice gauge theory is given in

1. M. Creutz: Quarks, Gluons, and Lattices, Cambridge University Press, Cambridge (1986)

2. I. Montvay, G. Münster: Quantum Fields on the Lattice, Cambridge University Press (1994)

More background information can be found at various locations in the Web, search for High Energy Physics (HEP).

A list of HEP Web sites is available at CERN (European Laboratory for Particle Physics), see `http://www.cern.ch/Physics/HEP.html`.