Appendix B. WEIGHT FUNCTIONS

1. Neutron Weight Functions in GROUPR

Most of the weight functions available in GROUPR are shown in Figs. 1 and 2. As discussed in the GROUPR chapter of this manual, the accuracy of a set of multigroup constants is determined by the group structure and the weight function. If only a few groups are used, it is necessary to have a weight function that is similar to the flux in the kinds of nuclear systems to be calculated with that particular group structure. If the flux is not well enough known, or if it varies strongly between different regions of the device, it is necessary to use more groups. The weight functions provided in the code are representative of a few typical nuclear systems, as described below. For systems with unusual spectra, or for very high accuracy calculations, the user should consider entering custom weight functions.

IWT=2 The weight function is constant (not shown). This option is usually chosen for very fine group structures such as the 620-group or 640-group dosimetry structures.

IWT=3 The weight function is proportional to $1/E$ (not shown). The slowing-down of neutrons in water gives a $1/E$ flux from about 1 eV up to 100 keV, or so. This weight function is traditionally used for calculating resonance integrals, but it is not useful at the lower and higher energies needed for a full set of transport constants. The graph of this function is a flat line on a flux-per-unit-lethargy plot, such as the one in Fig. 1.

IWT=4 This weight function combines a thermal Maxwellian at low energies, a $1/E$ function at intermediate energies, and a fission spectrum at high energies to obtain a function appropriate for several different applications. The temperatures of the Maxwellian and fission parts and the energies where the components join can be chosen by the user. Therefore, option 4 can be used to produce typical thermal reactor weight functions like those shown in Figs. 1 and 2, a pure fission spectrum for calculating some kinds of dosimetry cross sections, or a pure thermal spectrum for getting effective thermal average cross sections. The function for IWT=4 shown in the figure was produced using a thermal temperature of 0.025 eV joined to $1/E$ at 0.1 eV, and a fission temperature of 1.40 MeV joined to $1/E$ at 820.3 keV.

IWT=5 This is a mid-life PWR (pressurized water reactor) flux spectrum with a fusion peak added (see below for a discussion of the fusion peak used). Note the peaks and dips resulting from oxygen resonances and windows at high

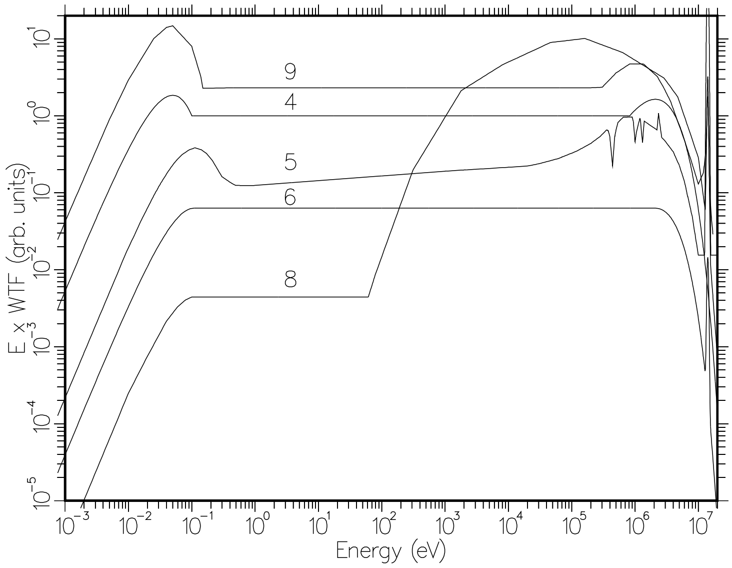

Figure 1: Built-in neutron weight functions of GROUPR on a logarithmic flux-per-unit-lethargy plot that emphasizes the low energy range.

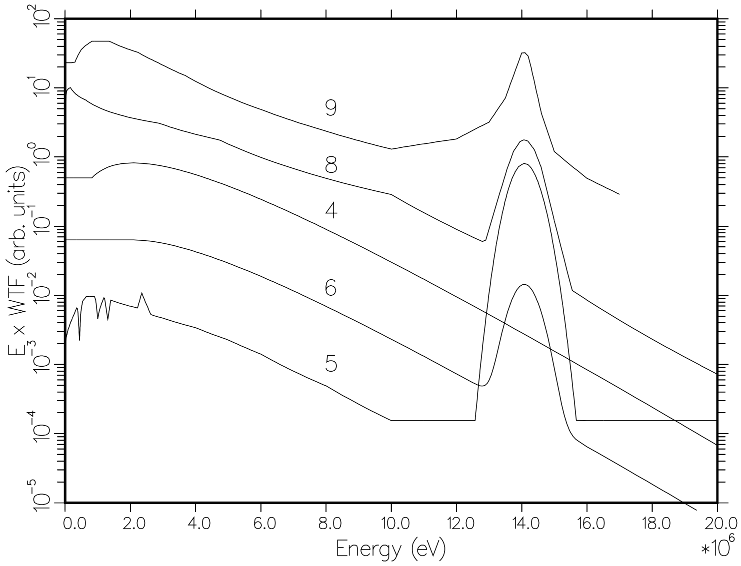

Figure 2: Built-in neutron weight functions on a linear plot that emphasizes the high-energy range.

energies, and the hardening apparent in the epithermal region. The thermal part of the function is also hardened with respect to a simple Maxwellian shape. The dips that should show up in the eV range due to resonances and U-238 have been removed to allow the self-shielding method to work without risk of counting the shielding effects twice. This weight function has been used for several libraries[1] prepared for the Electric Power Research Institute (EPRI), and for the MATXS7 library used with the TRANSX code[2].

IWT=6 This function is similar to option 4, but the breakpoints were chosen to keep the curves continuous. The thermal Maxwellian is calculated for 300 K. In this case, the fusion peak (see below) was added to the high-energy tail of the fission spectrum for smoothness.

IWT=7 This option is reserved for future use.

IWT=8 This function is intended for cross sections libraries to be used for fast reactor analysis (typically, fast breeder designs), but is also useful for fusion-blanket problems. It has a fusion peak at high energies, followed by a fission spectrum, a slowing-down spectrum typical of a fast reactor, and a thermal tail. The tail is provided to help give reasonable results in shields far from the core; its characteristic temperature is 300 K. Note the sharp drop in the flux as energy decreases from 19 keV. This region is important for U-238 absorption, and this drop-off helps to give good group constants for fast reactors. Of course, it would be entirely wrong for a thermal reactor.

IWT=9 This is another typical thermal+$1/E$+fission+fusion function that has been used for many libraries at the Los Alamos National Laboratory in the 30-group structure. The CLAW-3 and CLAW-4 libraries are available from the Radiation Shielding Information Center at the Oak Ridge National Laboratory.

IWT=10 This is the same as the CLAW weight function (IWT=9), but the shape of the thermal part is automatically recalculated to follow a Maxwellian law for temperature T.

IWT=11 This is the weight function used in the VITAMIN-E library. It has the following segments: a .0253-eV thermal Maxwellian below 0.414 eV, a $1/E$ law from .414 eV to 2.12 MeV, a 1.415-MeV fission spectrum from 2.12 to 10 MeV, another $1/E$ section from 10 to 12.52 MeV, a fusion peak (25 keV width) between 12.52 and 15.68 MeV, and a final $1/E$ section for all higher energies. The shape of the fusion peak is almost identical to IWT=5 (see Fig. B-2). The low-energy part of this weight function is not shown.

2. Shape of the Fusion Peak

Because of the finite width of the distribution of ion energies in a D-T fusion plasma, the emitted 14-MeV neutrons clearly will not have a delta-function energy spectrum. In fact, owing to the presence of a cross-product term in the kinematic relations, the typical ion-energy spread of the few tens of kilovolts is magnified into a neutron-energy spread of around 1 MeV.

For an assumed isotropic Maxwellian plasma, the neutron peak shape (for example, see the review article by Lehner[3]) is given by

$$S(E) = C \int_0^\infty \exp\{-b[v^2 + v_0(g)^2] - cg^2\} \sinh(2bvv_0)\, \frac{g^3\sigma(g)}{v_0(g)}\, dg \ . \tag{1}$$

Here $S(E)$ is the number of neutrons emitted with laboratory energy between E and $E + dE$, C is a normalization factor, v is the laboratory velocity corresponding to energy E, and v_0 is the velocity of the neutron in the CM system. Both v_0 and the fusion cross section σ are determined by the relative velocity g of the reacting ions, the integration variable in Eq. (1). The coefficient b is equal to $M/2kT$, where M is the total mass of the reacting ions and kT is the plasma temperature. Similarly, c is $\mu/2kT$, where μ is the reduced mass of the ion system.

The only approximation involved in Lehner's derivation of Eq. (1) is that all particles may be treated nonrelativistically. At 14 MeV, the relativistic factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \ , \tag{2}$$

is very close to unity (1.015), and it varies negligibly over the range of interest (say, 13 to 17 MeV). It is sufficient then to invoke relativistic mechanics in defining the location of the 14-MeV peak but not in discussing the shape. This effect moves the peak toward lower neutron energies, but only by about 20 keV.

Although the expression for $S(E)$ in Eq. (1) is accurate, it has the disadvantage of requiring a numerical integration at each point E in the energy spectrum. For this reason, we consider what simplifications can be made without serious loss of accuracy. In the energy range around the 14-MeV peak, the product $2bvv_0$ in Eq. (1) has a numerical value of about 6000. Thus, the hyperbolic sine can obviously be replaced by just the positive exponential term. If we make this change in Eq. (1), we can write the following, still nearly exact, expression for the neutron spectrum:

$$S_1(E) = C_1 \int_0^\infty \exp[-b(v - v_0)^2]\, P(v_0)\, dv_0 \ . \tag{3}$$

Here we also have inverted the function $v_0(g)$ and changed the integration variable. The spectrum then is a linear superposition of velocity exponentials with slightly different peak locations. For normal plasma temperatures, the velocity distribution $P(v_0)$ is very narrow, since the expression for v_0 is dominated by the nuclear Q-value (17.586 MeV) rather than the contribution from the ion kinetic energy (typically around 50 keV). Thus, it seems reasonable to approximate $P(v_0)$ as a delta function

$$P(v_0) \approx \delta(v_0 - v_p) \ . \tag{4}$$

This gives a second approximate form,

$$S_2(E) = C_2 \exp[-b(v - v_p)^2] \ , \tag{5}$$

where v_p has the obvious meaning of the laboratory neutron velocity at the center of the peak. We shall refer to this as the velocity exponential form of the neutron energy spectrum. An expression essentially identical to Eq. (5) was given in an early paper by Nagle and coworkers[4].

In order to examine the accuracy of the velocity exponential form, we have calculated $S(E)$ from Eq. (1) and $S_2(E)$ from Eq. (5) at 20 keV, a typical plasma temperature in current fusion-reactor concepts. In performing the numerical integration over g in Eq. (1), we used numerical values for the D-T fusion cross section taken from the compilation by Jarmie and Seagrave[5]. In evaluating $S_2(E)$ using Eq. (5), a value of v_p was chosen so as to force agreement between $S_2(E)$ and $S(E)$ at 17 MeV. As discussed by Muir[6], the overall agreement is remarkably good, the maximum error over the range from 13.5 to 17 MeV being about 2%. The value of v_p thus derived corresponds to a peak-center energy E_p of 14.07 MeV. This value includes the small ($\sim$ 20 keV) relativistic correction mentioned above.

If we approximate the mass of the D+T system as 5 times the neutron mass, then we obtain the recommended peak shape

$$S_{\mathrm{rec}}(E) = \exp[-\frac{5}{kT}(\sqrt{E} - \sqrt{E_p})^2] \ , \tag{6}$$

where E_p=14.07 MeV. The functional form in Eq. (6) was used to calculate the fusion peak shapes appearing in two of the data statements in `GENWTF` (namely, those utilized for IWT=5 and 8) and is also used explicitly in `GETWTF` to calculate analytically the weight function for IWT=6. In all three cases, kT=25 keV is used as an average or typical fusion-reactor plasma temperature. See Fig. 2 for a graphical display of the resulting weight functions in the 14-MeV region.

3. Photon Weight Functions in GAMINR

The main features of a typical photon spectrum are a $1/E$ region at intermediate energies, a drop-off at low energies caused by photoelectric absorption, and a shoulder at high energies corresponding to the maximum Q-value for capture. The relative size and importance of these features depends on the composition of the system. With light materials like water and concrete, the $1/E$ portion is very obvious and extends to low energies. As heavy materials are added, the low-energy range is suppressed. The highest-energy photons in a system are normally primary photons from capture, with maximum energies of about $Q + (A/A{+}1)E$, where E is the neutron energy and A is the atomic weight ratio to the neutron for the absorber. Thus, the shape of the high-energy part of the photon flux may be different for fusion and fission systems, with energies above 10 MeV being important for fusion.

IWT=2 The weight function is constant (not shown). This option is most useful for fine-group libraries, or for benchmark comparisons of different codes where simplicity is useful.

IWT=3 The weight function is basically $1/E$, however low- and high-energy roll-offs are included to keep the first and last group from being unrealistically biased by unimportant regions of the cross-section energy range. See Fig. 3.

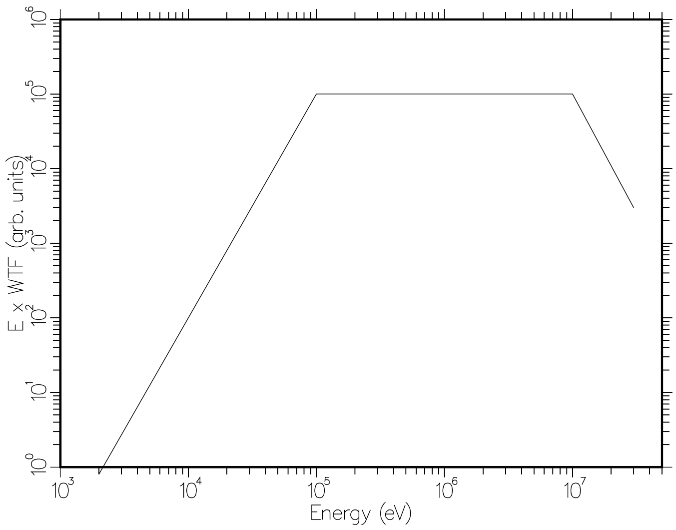

Figure 3: Built-in photon weight function of GAMINR showing the high- and low-energy roll-offs.

4. References

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