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THE $t\bar{t}$ THRESHOLD AT A MUON COLLIDER*

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ABSTRACT

The beam energy spread is a major issue in future attempts to study the $t\bar{t}$ threshold at e^+e^- colliders. Muon colliders are expected to naturally have narrow band beams making them an ideal place to study the excitation curve. We present the parameter determinations that are possible from measuring the total cross section near threshold at a $\mu^+\mu^-$ collider.

1. Introduction

The production of top quarks near threshold has been studied vigorously in the last few years[1−15]. This process provides a test of perturbative QCD, as well as an accurate measurement of the top quark mass m_t and the strong gauge coupling constant α_s. Since theoretical problems are for the most part under control, smaller effects such as the top-Higgs Yukawa coupling and the top quark width might be probed. These previous studies have concentrated on the capabilities of e^+e^- machines.

Recently attention has been devoted to the possibility of a muon collider[16−21]. The impact on top quark threshold studies arises in two ways: (1) there is decreased initial state radiation (ISR) at a muon collider compared to an electron collider, and (2) the intrinsic beam spread and beamstrahlung of a muon collider is expected to be much smaller than at e^+e^- machines. These effects are attributed to the fact that the muon is much heavier than the electron, and result in a threshold curve which is more steeply rising. This offers the possibility of an improved measurement of the relevant parameters affecting the shape of the cross section. By the time a muon collider would actually be constructed, the top mass will be measured to a few GeV and a relatively inexpensive special-purpose ring could be constructed for optimizing the luminosity at $\sqrt{s} = 2m_t$. The utility of a narrow band beam has also been demonstrated for s-channel Higgs studies[19−21].

2. The Top Quark Threshold Region

Fadin and Khoze demonstrated that the top quark threshold cross section is calculable since the large top quark mass puts one in the perturbative regime of

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QCD, and the large top quark width effectively screens nonperturbative effects in the final state. There are two (equivalent) ways to obtain the total cross section near threshold: solving for a three-point Green's function[4] in either coordinate or momentum space. Here we solve the Schrödinger's equation

$$\left[-\frac{\Delta}{m_t} + V(r) - \left(E + i\frac{\Gamma_\Theta}{2}\right)\right] G(\mathbf{x}; E) \quad = \quad \delta^3(\mathbf{x}) \; , \tag{1}$$

where Γ_Θ is the (running) toponium width, and $E = \sqrt{s} - 2m_t$. The potential $V(r)$ is given for small r by two-loop perturbative QCD and for large r by a fit to quarkonia spectra. The total cross section is then proportional to $\mathrm{Im}\, G(\mathbf{x} = 0; E)$. In addition to the usual Yukawa term $V_H(r)$ in the potential at a muon collider, there is an additional s-channel Higgs contribution to the cross section since the muon has a larger Yukawa coupling than does the electron. This contribution is much smaller than the usual photon and Z exchanges considered here.

3. Initial State Radiation

The larger mass of muon compared to the electron implies that the initial state radiation in the process $\mu^+\mu^- \to t\bar{t}$ is reduced compared to $e^+e^- \to t\bar{t}$. Figure 1 shows the effects of including initial state radiation (but not beam effects) for a top quark mass of 180 GeV. There is a reduction of the cross section as well as a smearing out of the small resonance peak.

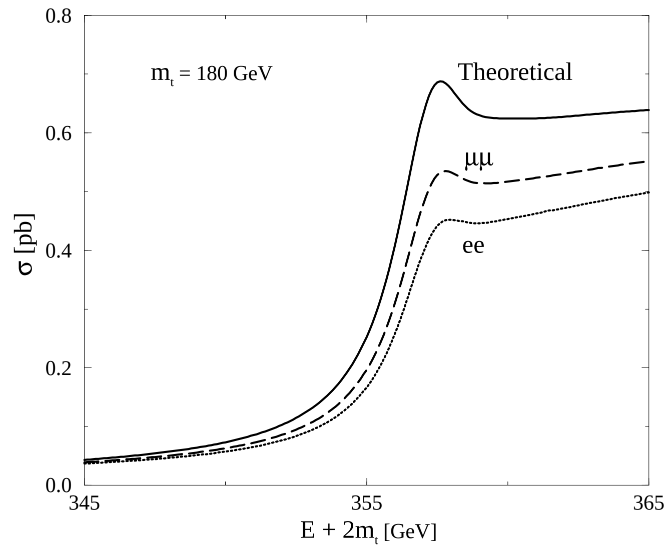

Fig. 1. Initial state radiation reduces the total cross sections and smears the threshold region. The effect is smaller for muons since they are heavier than electrons. The strong coupling is taken to be $\alpha_s(M_Z) = 0.12$.

4. Beam Effects

The beam energy spread is the major problem with precision measurements of the top threshold region at an e^+e^- collider. Ref. [5] demonstrated the effects of beam smearing for some proposed machine designs, and argued that a narrow beam was essential for studying the top quark threshold region. A muon collider should provide a naturally very narrow beam with rms deviation $R \lesssim 0.1\%$. The energy spectrum of each muon beam is expected to be roughly Gaussian in shape, but for $t\bar{t}$ studies the exact shape is not important if $R \lesssim 0.1\%$. We take $R = 0.1\%$ here, but the results are not changed significantly if the resolution is improved. We simulate the beam spread of the e^+e^- machine by taking $R = 1\%$; significant detailed studies have taken more realistic parameters for the electron collider[5,11], and have included beamstrahlung effects. The corresponding rms deviation σ in $\sqrt{s}$ is given by

$$\sigma = (0.25\ \text{GeV}) \left(\frac{R}{0.1\%}\right) \left(\frac{\sqrt{s}}{360\ \text{GeV}}\right) , \tag{2}$$

which for a muon collider (with $R \lesssim 0.1\%$) is of the same order as the measurement one hopes to make in the top mass. For large values of R ($\gtrsim 0.5\%$), the shape of the cross section in the threshold region depends on the beam profile; we assume here that the e^+e^- collider has a Gaussian profile for simplicity. A high resolution determination of the collider energy profile would be desirable to deconvolute the smearing of the threshold curve. For a muon collider on the other hand, no such measurement of the beam profile would be necessary. Figure 2 shows the effects of including beam smearing of the threshold curve.

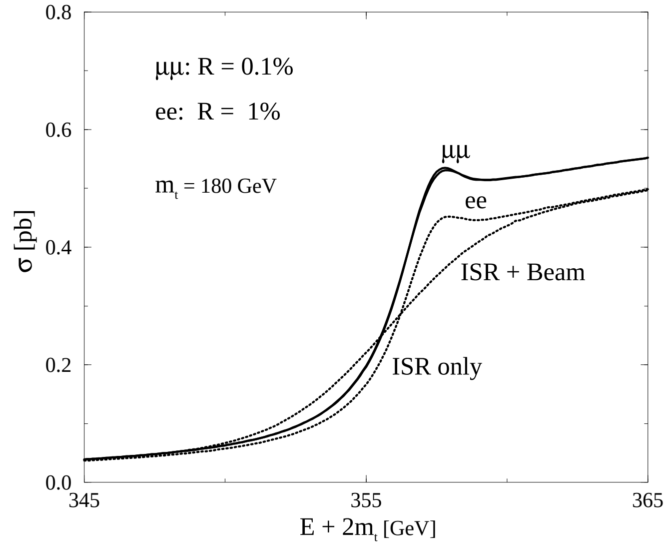

Fig. 2. The threshold curves are shown for $\mu^+\mu^-$ and e^+e^- machines including ISR and with and without beam smearing. Beam smearing has only a small effect at a muon collider, whereas at an electron collider the threshold region is significantly smeared. The strong coupling is taken to be $\alpha_s(M_Z) = 0.12$.

5. Parameter Determination

The $t\bar{t}$ threshold shape is a sensitive function of the strong coupling constant and top quark mass as shown in Figs. 3 and 4 respectively.

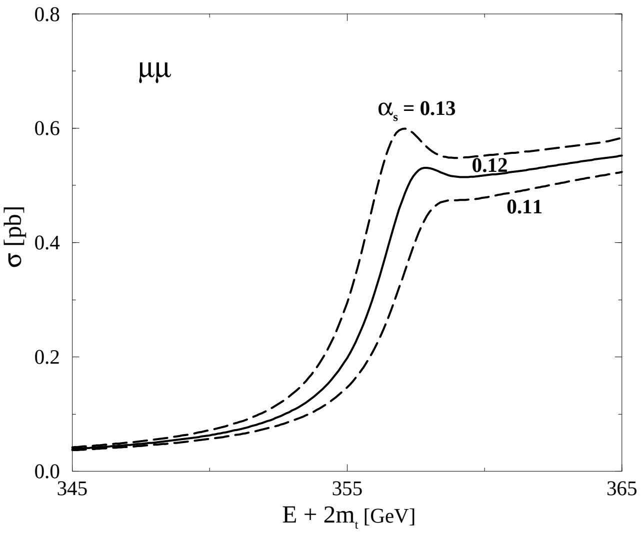

Fig. 3. Changing the value of the strong coupling constant $\alpha_s(M_Z)$ influences the threshold region. Large values lead to tighter binding and the peak shifts to lower values of $\sqrt{s} = E + 2m_t$. Both ISR and beam smearing have been included, and the top quark mass is taken to be 180 GeV.

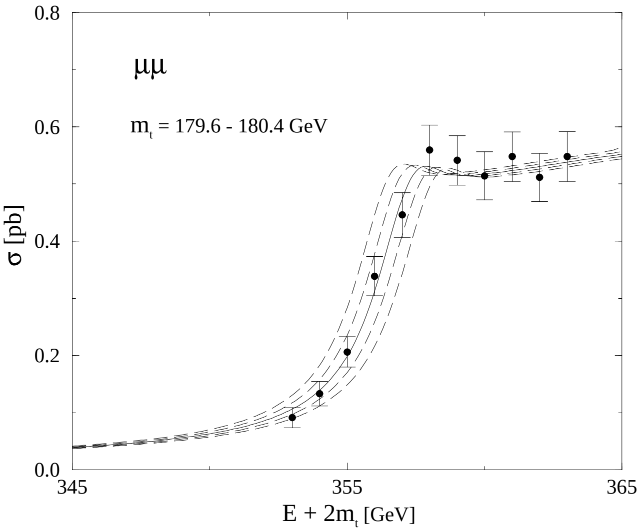

Fig. 4. Sample data obtained assuming a scan over the threshold region devoting 1 fb^{-1} luminosity to each data point. A detection efficiency of 29% has been assumed in obtaining the error bars. The threshold curves correspond to m_t in 0.2 GeV increments. Both ISR and beam smearing have been included, and the strong coupling $\alpha_s(M_Z)$ is taken to be 0.12.

Decreasing m_t and increasing $\alpha_s(M_Z)$ have similar effects on the threshold curve. To assess the precision of parameter determinations from measuring the total cross section, we take some hypothetical sample data as shown in Fig. 4, generated assuming 1 fb^{-1} integrated luminosity is used to measure the cross section in 1 GeV intervals and assuming the nominal values $m_t = 180$ GeV and $\alpha_s(M_Z) = 0.12$. Following Ref. [11] we assume a 29% detection efficiency to isolate the $W \to qq$ final state from background. The likelihood fit to the two parameters m_t and $\alpha_s(M_Z)$ is shown as the $\Delta\chi^2$ contour plot in Fig. 5. The outer curve is the 90% confidence level ($\Delta\chi^2 = 4.6$) for m_t and α_s jointly, while the inner curve is the 1σ confidence level ($\Delta\chi^2 = 2.3$).

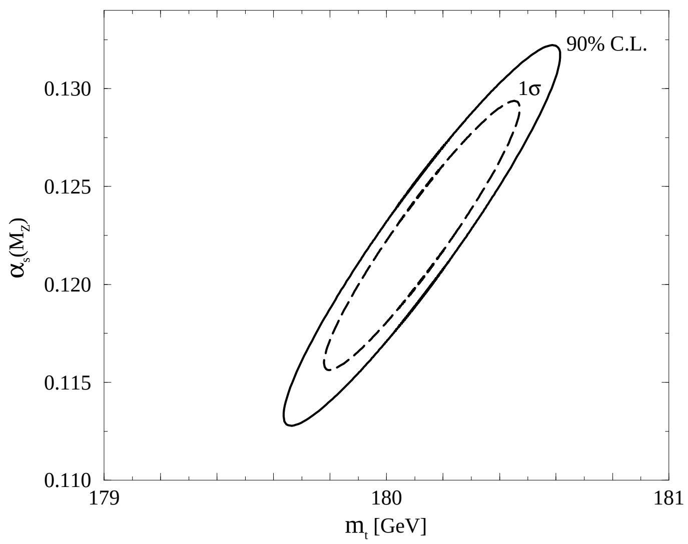

Fig. 5. The 1σ and 90% confidence limits for the sample data shown in Fig. 4.

A top quark mass around 180 GeV can be measured to about 300 MeV at 90% confidence level (the $\Delta\chi^2 = 2.7$ contour is not shown). This is roughly a factor of two better than the same measurement at an e^+e^- machine when realistic beam effects are included[11]. The improvement in the measurement of $\alpha_s(M_Z)$, however, is more modest. As at an e^+e^- collider, one will be able to make use of the top momentum measurement to improve on the parameter determinations from the total cross section[7,9,11]. A more complete treatment of the $t\bar{t}$ threshold measurement at a muon collider will be presented in Ref. [22].

6. Acknowledgements

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7. References

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