Year	First Run	Last Run	Number Runs	N_{KTlive}
1989	3728	6148	1659	1.7688×10^{11}
1990	6749	8429	1169	2.1178×10^{11}
1991	9018	10141	609	1.4554×10^{11}
Total	-	-	3437	5.3647×10^{11}

Table 4.30: The run ranges analyzed, and the kaon flux into the target with the detector "live."

using the same multi-run veto as the analysis, a completely self-consistent count of $KTlive$'s for the data analyzed results. Of course, as with most things, it is necessary to bypass the default KOFIA code to get a good approximation to the right answer.

The resulting beam kaon flux is listed for 1989, 1990, and 1991 data in Table 4.30.

4.6 The $K^+ \to \mu^+\nu_\mu$ branching ratio and kaon stopping fraction, f_S

The $K^+ \to \mu^+\nu_\mu$ branching ratio is used for a stopping-flux normalization factor f_S. The most important factor is the fraction of $KTlive$'s that come to a stop in the target and decay as a K^+. One third of the $K_{\mu 2}(1)$ monitor triggers spanning the same run ranges as the $K^+ \to \pi^+\nu\overline{\nu}$ data are analyzed with the cuts outlined in Table 4.31. With a 63% branching ratio, $K^+ \to \mu^+\nu_\mu$ is easy to select. The trigger already has a severe range cut, requiring that the track penetrate to least to RS layer 19. The full online and offline delayed coincidence and timing cuts are used to insure decays at rest, the beam cuts remove π scats, and the full RS, BV, EC, IC and VC γ-veto cuts insure basic event cleanliness.[5]

The efficiency for a $K^+ \to \mu^+\nu_\mu$ decay at rest satisfying the $K_{\mu 2}(1)$ trigger and the DIPANG cut is measured with UMC, and summarized in Table 4.32. This efficiency depends on the average range stack scintillator thickness used in the simulation. The RS scintillator thickness was nominally specified as 3/4 inch (1.905 cm) per layer. In previous analyses, using the value 1.95 cm in UMC has been somewhat more popular, but it is unreasonable to assume that the "correct" value is known to better than

[5] Since the $K^+ \to \mu^+\nu_\mu$ branching ratio analysis does not veto events that may have penetrated to the barrel veto, the five BV sectors behind the apparent range stack stopping counter are not used in the offline photon veto.

Description	1989	1990	1991
	N_{passed}	N_{passed}	N_{passed}
Prescale	94929	107023	64430
L0DELC	67894	78071	43222
ISKCODE	54287	69150	38294
DIPANG	51850	66744	36946
PICER	49700	64414	35790
PISCAT_BW	43917	56733	31331
B4DEDX	43359	56012	30757
B4TDEDX	43324	55560	-
B4TFIT	42703	53122	-
PISCAT_B4	-	-	28969
PBGLASS	-	-	27963
PIHODO_B4	40085	49606	-
GAMVET	32968	39346	21650
DELC	28640	35254	18792
TIMCON	26347	33108	18471

Table 4.31: The $K^+ \to \mu^+\nu_\mu$ branching ratio analysis.

Cut	N_{passed} 1989		N_{passed} 1990		N_{passed} 1991	
	1.905 cm	1.95 cm	1.905 cm	1.95 cm	1.905 cm	1.95 cm
KT	95083	95197	95238	95310	95132	95066
Trigger	36634	34895	36725	35359	36480	35005
ISKCODE	35880	34166	35973	34675	35732	34304
DIPANG	35195	33681	35295	34191	35031	33738
$A^{\rm umc}_{\rm recon}$	0.979	0.979	0.980	0.981	0.979	0.980
$A^{\rm umc}_{K_{\mu2}}$	0.370	0.354	0.371	0.359	0.368	0.355
$A^{\rm umc}_{K_{\mu2}}$	0.362 ± 0.016		0.365 ± 0.012		0.362 ± 0.013	

Table 4.32: The UMC $K^+ \to \mu^+\nu_\mu$ analysis efficiency, for assumed range stack layer thicknesses 1.905 cm and 1.95 cm. The total efficiency of the analysis is $A^{\rm umc}_{K_{\mu2}}$. The reconstruction efficiency, $A^{\rm umc}_{\rm recon}$, for UMC data is the fraction of events passed by ISKCODE, and will be used in the branching ratio calculation below.

somewhere near this thickness.[6] For this analysis, both 1.905 cm and 1.95 cm are tried for the acceptance calculation; the average is used for the f_S measurement, and the difference is used for the (approximately 4%) error.[7] It is worth emphasizing that, by design, the $K^+ \to \pi^+\nu\overline{\nu}$ and $K^+ \to \pi^+X^0$ acceptances are essentially independent of small variations of this assumption; unfortunately, since our sensitivity is normalized to the $K^+ \to \mu^+\nu_\mu$ branching ratio, it has become dependent on a parameter that we do not know very well.

The $K^+ \to \mu^+\nu_\mu$ acceptance also requires a self-consistent measurement of the remaining cuts in the analysis in Table 4.31. The delayed coincidence and timing cut measurement ($A_{\rm DC}$) should still apply, but a new beam and photon veto acceptance ($A_{\rm rest}$) must be measured. In Table 4.15, the beam cut acceptance was evaluated after the target analysis which is not applied in the $K^+ \to \mu^+\nu_\mu$ branching ratio calculation. Since the losses of the beam cuts are strongly correlated with the target losses, a new measurement is required. This was done with the $K_{\mu2}(1)$ monitor sample used

[6] A brief study was undertaken varying the UMC RS scintillator thickness parameter, and comparing the $K^+ \to \mu^+\nu_\mu$ and $K^+ \to \pi^+\pi^0$ stopping layer distributions between data and KOFIA reconstructed UMC. The $K^+ \to \mu^+\nu_\mu$'s favoured about 1.97 cm, while the $K^+ \to \pi^+\pi^0$'s favoured about 1.91 cm.

[7] It is tempting to reject events that penetrate to the outer RS layer for the branching ratio analysis so that clean range and energy measurements are available for $K^+ \to \mu^+\nu_\mu$ selection, as was done in [18]. The variation of the $K^+ \to \mu^+\nu_\mu$ acceptance between the 1.905 cm and 1.95 cm RS scintillator assumptions increases to about 10% if such a selection is used. Since that selection is not needed to get a clean $K^+ \to \mu^+\nu_\mu$ sample, it is not used here.

Quantity	1989	1990	1991
$KTlive$	1.816×10^{11}	1.998×10^{11}	1.266×10^{11}
$\#\ K_{\mu 2}(1)$ triggers	3.749×10^{10}	4.211×10^{10}	2.551×10^{10}
Triggers analyzed	94920	107023	64430
$N^{\rm equiv}_{KTlive}$	459809	507912	319836

Table 4.33: The $KTlive$ calculation for the $K^+ \to \mu^+\nu_\mu$ branching ratio calculation.

for the $A_{K_{\mu 2}}$ above, but using only the remaining cuts from Table 4.31. The basic reconstruction efficiency $A_{\rm recon}$ should apply here, but it is partially included already in the $A^{\rm umc}_{K_{\mu 2}}$ factor. For example, when the UMC generated $K^+ \to \mu^+\nu_\mu$ sample is analyzed, some events will not be successfully fit by the drift chamber code, and this loss is counted in $A^{\rm umc}_{K_{\mu 2}}$; however, this loss is already counted in the $A_{\rm recon}$ measurement. This double-counting is removed using the $A^{\rm umc}_{\rm recon}$ factor from Table 4.32.

Finally, a consistent measurement of the effective number of $KTlive$'s used for the branching ratio measurement is needed. In principle, the $KTlive$ measurement from the $K^+ \to \pi^+\nu\overline{\nu}$ analysis can simply be corrected for the monitor trigger prescale factor, along with any additional offline prescale factor used in sample selection; in practice, the somewhat lax handling of monitor tapes makes it essential to re-measure this by counting the scaler records stored with the data during the branching ratio analysis. The effective number of $KTlive$'s is then

$$N^{\rm equiv}_{KTlive} = (\#\ KTlive)\frac{\#\ \text{triggers analyzed}}{\#\ \text{raw}\ K_{\mu 2}(1)\ \text{triggers}},$$

where the "$\#\ KTlive$" and "$\#$ raw $K_{\mu 2}(1)$ triggers" entries are counted from the scalers stored with the data in the branching ratio analysis, and "# triggers analyzed" is the number of events to enter the branching ratio analysis. This is summarized in Table 4.33. It is worth noting that the inferred $K_{\mu 2}(1)$ prescale factor from this count agrees well with the expected value (131072 from the online trigger prescale factor, times the factor of 3 used offline in all three years).

The $K^+ \to \mu^+\nu_\mu$ branching ratio is given by

$$B_{K_{\mu 2}} = \frac{(\#\ K_{\mu 2})_{\rm anal}}{f_S N^{\rm equiv}_{KTlive}} \frac{1}{A^{\rm umc}_{K_{\mu 2}}} \left(\frac{A^{\rm umc}_{\rm recon}}{A^{\rm data}_{\rm recon}}\right)_{K_{\mu 2}} \frac{1}{A_{\rm DC}} \frac{1}{A_{\rm rest}},$$

which is trivially solved for f_S:

$$f_S = \frac{(\#\ K_{\mu 2})_{\rm anal}}{B_{K_{\mu 2}} N^{\rm equiv}_{KTlive}} \frac{1}{A^{\rm umc}_{K_{\mu 2}}} \left(\frac{A^{\rm umc}_{\rm recon}}{A^{\rm data}_{\rm recon}}\right)_{K_{\mu 2}} \frac{1}{A_{\rm DC}} \frac{1}{A_{\rm rest}}.$$