Physics 539 | Spring Semester 1997 | Chris Quigg

April 3, 1997

Recommended Reading:

On tests of lepton-number conservation, see the minireview, "Tests of Conservation Laws," in the 1996 *Review of Particle Physics* [Particle Data Group, *Phys. Rev. D***54**, 1 (1996)], pp. 59–63.

On neutrino mass and mixing, see the minireviews, "Neutrinos" (pp. 275–280) and "Searches for Massive Neutrinos and Lepton Mixing" (pp. 287–301), also in the 1996 *RPP.*

On neutrino oscillations in matter, see T. K. Kuo and J. Pantaleone, *Rev. Mod. Phys.* **61**, 937 (1989).

Cultural Reading:

F. Boehm and P. Vogel, *Physics of Massive Neutrinos,* second edition (Cambridge University Press, Cambridge, 1992)

B. Kayser, F. Gibrat-Débu, and F. Perrier, *The Physics of Massive Neutrinos* (World Scientific, Singapore, 1989).

S. M. Bilenky and S. T. Petcov, "Massive neutrinos and neutrino oscillations," *Rev. Mod. Phys.* **59**, 671 (1987).

T. J. Bowles and V. N. Gavrin, "The Status of the Solar Neutrino Problem," *Ann. Rev. Nucl. Part. Sci.* **43**, 117 (1993).

J. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, 1989).

M. Moe and P. Vogel, "Double Beta Decay," *Ann. Rev. Nucl. Part. Sci.* **44**, 247 (1994).

Escapist Literature for Those Afflicted by Generals:

Ettore Majorana vanished without a trace from a ferry between Sicily and the Italian mainland. Leonardo Sciascia has written a fictional account that Majorana's colleagues denounced as scurrilous fantasy. Firestone Library has the Italian original, *La Scomparsa di Majorana,* as well as an English translation, which is in *The Moro Affair and the Mystery of Majorana.*

Problems (due April 10, 1997)

32. Because the neutrino-nucleon cross section increases with energy, the Earth is opaque to ultrahigh-energy neutrinos. From what you know of the measured cross section for charged-current νN interactions [Particle Data Group, *Phys. Rev. D***45**, S1 (1992); see p. III.82], compute the "water equivalent" interaction length, $\mathcal{L}_{\rm int} = 1/[\sigma_{\nu N}(E_\nu)N_A]$, where N_A is Avogadro's number. If the diameter of the Earth is 11 kilotonnes/cm^2, *i.e.*, 1.1×10^{10} cmwe (centimeters of water equivalent), at what energy does the charged-current interaction length become smaller than one Earth diameter? How does the W-boson modify your conclusions? [A. DeRújula, S. L. Glashow, R. R. Wilson, and G. Charpak, *Phys. Rep.* **99**, 341 (1983), §8; R. Gandhi, C. Quigg, M. H. Reno, and I. Sarcevic, *Astroparticle Physics* **5,** 81 (1996).]

33. Compute the rate for the dominant decay rate of the top quark, $t \to bW^+$, neglecting the mass of the b quark. You will need to sum over the possible polarizations of the W-boson. Show that it takes the form
$$\Gamma(t \to bW^+) = \frac{G_F m_t^3}{8\pi\sqrt{2}}|V_{tb}|^2 \left(1 - \frac{M_W^2}{m_t^2}\right)^2 \left(1 + \frac{2M_W^2}{m_t^2}\right),$$
where V_{tb} measures the strength of the $t \to bW^+$ coupling. What fraction of the decays produce W-bosons with longitudinal polarization? [I. Bigi, Yu. L. Dokshitzer, V. Khoze, J. Kühn, and P. Zerwas, *Phys. Lett.* **B181**, 157 (1986).]

34. Consider the one-loop modifications to Coulomb scattering in the limit of low momentum transfer $-q^2 \ll m^2$. Beginning from Equation (8.2.31) in *Gauge Theories*, show that the amplitude is modified by a factor

$$\left[1 - \frac{\alpha_R}{15\pi}\frac{q^2}{m^2} + \mathrm{O}(\alpha_R^2)\right] .$$

Show that this corresponds, in position space, to an additional interaction of the form

$$\frac{4}{15}\frac{\alpha_R^2}{m^2}\delta^3(\mathbf{x}) ,$$

and estimate the first-order shift in the energy levels of the hydrogen atom. [E. A. Uehling, *Phys. Rev.* **48**, 55 (1935); for an early application, see R. Serber, *ibid.*, p. 49.]

35. The model of M.-Y. Han and Y. Nambu [*Phys. Rev.* **149B**, 1006 (1965)] is an integer-charge alternative to the fractional-charge quark model, with charges assigned as

	Flavor		
Color	u	d	s
Red	0	−1	−1
Green	1	0	0
Blue	1	0	0

(a) Show that below the threshold for color liberation, the ratio

$$R \equiv \sigma(e^+e^- \to \text{ hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$$

is $R = 2$, as in the fractional-charge model, and that $R = 4$ if color can be liberated.

(b) Consider the reaction $\gamma\gamma \to$ hadrons, viewed as $\gamma\gamma \to q\bar{q}$. Show that with fractionally charged quarks,

$$\sigma(\gamma\gamma \to \text{ hadrons}) \propto \sum_i e_i^4 = \tfrac{2}{3},$$

and that in the Han-Nambu model,

$$\sigma(\gamma\gamma \to \text{ hadrons}) \propto \sum_i e_i^4 = \begin{cases} 2 \text{ below color threshold} \\ 4 \text{ above color threshold} \end{cases}$$

[References: F. E. Close, *An Introduction to Quarks and Partons,* (Academic Press, New York, 1979), Chapter 8; H. J. Lipkin, *Nucl. Phys.* **B155**, 104 (1979); M. S. Chanowitz, "Color and Experimental Physics," in *Particles and Fields–1975,* edited by H. J. Lubatti and P. M. Mockett (University of Washington, Seattle, 1975), p. 448.]

36. Show that the $n \times n$ unitary matrix U introduced to describe the mixing of quarks of different flavors [cf. equations (7.1.27–29) in *Gauge Theories*] can be parametrized in terms of $n(n-1)/2$ real mixing angles and $(n-1)(n-2)/2$ complex phases, after the freedom to redefine quark fields has been taken into account. For the specific case of three quark doublets ($n = 3$), show that U may be written in the form

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \cdot \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix},$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. Discuss the implications of the phase δ for CP invariance. [M. Kobayashi and T. Maskawa, *Prog. Theor. Phys. (Kyoto)* **49**, 652 (1973).]

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Update on parity violation in atomic physics. In Problem 27, I should have cited a second determination of the weak charge of $^{205}_{81}$Tl, derived from measurements of the parity nonconserving optical rotation near the $6P_{1/2} \rightarrow 6P_{3/2}$ magnetic dipole transiton in atomic thallium. The University of Oxford group [N. H. Edwards, S. J. Phipp, P. E. G. Baird, and S. Nakayama, *Phys. Rev. Lett.* **74**, 2654 (1995)] report $Q_W(^{205}\mathrm{Tl}) = -121.9 \pm 3.5 \pm 3.4$ (my interpretation).

The most incisive of the parity-violation experiments in atoms is the measurement of weak charge of $^{133}_{55}$Cs by a group from the University of Colorado [M. C. Noecker, B. P. Masterson, and C. E. Wiemann, *Phys. Rev. Lett.* **61**, 310 (1988), atomic structure calculations revised by S. A. Blundell, W. R. Johnson, and J. Sapirstein, *ibid.* **65**, 1411 (1990)]. Their value is $Q^W(^{133}_{55}\mathrm{Cs}) = -71.04 \pm 1.58 \pm 0.88$, where the first error is experimental and the second refers to the uncertainty in atomic theory.

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Typographical errors in the fifth paperbound printing of *Gauge Theories* (identified at the bottom of p. vi by `EFGHIJ-MA-89`): In equation (7.1.13), the coefficient of $x_W Q$ should be 4, not 2. This appears twice.

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