# Consortium for Electric Reliability Technology Solutions 

Distributed Energy Resources Integration

Industrial Application<br>of MicroGrids

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## Executive Summary

The following report summarizes the benefits of introducing Distributed Resources within an industrial site. To have a realistic example for this work it was necessary to design the site. The plant is assumed to be one that manufactures either paper or plastic film using a continuous process. This requires extensive induction machines that need to be tightly controlled to insure correct tension of the film during the process. The site has three main buildings: the factory with $50,000 \mathrm{ft}^{2}$ of floor space, the warehouse and the office with $57,000 \mathrm{ft}^{2}$ and $18,000 \mathrm{ft}^{2}$ of space respectfully. This site has an overall demand of 1 MW and is connected to the main grid through a series of transformers, overhead lines and cables. There is also another load in the immediate vicinity of the one under study which allows a platform for disturbances do to load changes and faults. The feeders from the 120 KV source to the 120 volt service deep within the plant was designed using standard engineering practices.

The first sections details the factory design, its electrical system, the loads and microturbines used. This detailed model is then used to look at issues, which arise when placing DR at a customer site. Steady state analysis is used to obtain voltage profiles, power flows and system losses for different scenarios. For example; with and without DR both grid connection and islanded, changes in load levels and loss of micro-turbines. In all cases the inclusion DR greatly improved voltage profiles, reduced reactive power flows and reduced losses.

Dynamics studies were also performed on this test site using the Electrical Magnetic Transient Program (EMTP) from Electrical Power Research Institute. The dynamic studies required detailed power electronic models and controls for each micro-source embedded in the site's electrical system. The results demonstrate the effectiveness of the local voltage controller during load changes and load sharing features when in island operation. Through simulation the key control concepts required for creation of MicroGrids are shown to be effective without fast communications.

## 1. Introduction

The following report summarizes the benefits of introducing Distributed Resources within an industrial site. A detailed description of the factory is followed by steady state analysis of the system. This industry has an overall demand of 1MW and is connected to the main grid with a series of transformers. Within the plant there are three main buildings: the factory, the warehouse and the offices. The grid supplies also another plant in the immediate vicinity of the one under study. Each parameter of the system is found from tables available from the literature. Steady state analysis is used to obtain voltage profiles and power flows for the system in the following scenarios: without units, with DR's with grid connection and with DR's in isolation mode, when the connection to the main grid fails.

## 2. Factory Design

The industrial site under study produces sheets of paper and can be found in any location of the North America. This company has a production line that is sensitive to quality of power at its feeders, and it is willing to explore the benefits of introducing DR's as a mean to improve the reliability of power delivery. Figure 1 shows the general overview of the plant, detailing the relative location and size of the main buildings: factory, warehouse and offices and it also provides the location of the transformers and main cables at different voltage levels. Figure 2 shows the electrical diagram of the plant with details of the buildings, load location and cable size and length. There also is the bus numeration used to model the factory and the overall active power demand is specified for each building. The dark boxes on the cables represent the switches that will trip in the event that the main connection with the grid fails and the system goes in island mode. Those switches shed part of the network that is not crucial to the plant survival.

Few miles away from the location of the factory there is a high voltage transmission line that feeds another nearby plant. The voltage is lowered from 120 KV to 13.8 KV and delivered to the industrial site through aerial lines and buried cables. Three transformers are located outside one of the walls of the factory. Voltages are lowered to 480 V and 240 V .

The factory has two floors, in which the company loads are equally distributed. At each floor there are five induction machines that carry a sheet of paper during its production process. To avoid damaging the sheet, it is critical to maintain a good control on the tension of the sheet. This is done by controlling the torque of the induction machines, operation that can be done as long as electric power is provided at the feeder. Three phase rectifying bridges are used to obtain the DC power to feed the ASD's that control the induction machines. Each floor has lights and an air conditioning system, composed
primarily of synchronous motors that drive the compressor for the unit. The second floor has also the machine room for the two elevators that are in the factory.

The warehouse is located in front of the factory and is reached by a cable at 480 V . In the warehouse there is the transformer responsible for lowering the voltage to 208 V , to provide both voltages in both warehouse and office.

The warehouse has a tall ceiling, with a single elevator. The air conditioning system is located at ground level, while the lights and elevator machine room are located on the ceiling of the warehouse.

The office has two floors where all the managing part of the company takes place. Computers and related accessories that represent a large quota of the office loads are evenly located at both floors, the same for the light and air conditioning loads. From the 208 V panel at the first floor of the office there is a cable that is responsible for delivering the power to all the perimeter lights, located around the property of the factory.


Figure 1. Site Plane


Figure 2 Single Line Diagram

### 2.1 Sensitive Loads

There are some loads inside the micro-grid that need to be supplied even if the connection to the main grid fails. These sensitive loads are located in the factory and in the office buildings. The induction motors inside the factory are sensitive loads and represent a large quota of the plant demand. There are three phase rectifying bridges that
feed sensitive DC loads that represent the ASD's that control the induction machines. In the office there are single phase rectifying bridges that are responsible for providing DC supply to the digital loads and computers which are considered sensitive loads.


Figure 3. Controlled Tension Paper Sheet
The motors move the rolls responsible for the tension and the movement of the sheet of paper. The control of the tension is crucial to obtain desired sheet thickness, and also to avoid its tear. If there is a voltage fluctuation at the machine terminals, the fluctuation induced in the torque of the motor may be enough to damage the sheet. It is very important that a high voltage quality is maintained at the terminals of the IM.

### 2.2 Non Sensitive Loads

During island mode, it is not convenient trying to supply all the loads of the plant, but rather only a subset of them. It is mandatory to maintain satisfactory quality for the electrical quantities at the sensitive loads. Non sensitive loads can be dropped to avoid the need to have DR's with overall ratings to cover all the plant loads. During island mode, 170 KW of loads are shed from the micro-grid. These loads are: all the elevators, all the warehouse loads and the perimetral lights and they are dropped by tripping the switches represented by black boxes in Figure 2. It is important to be able to shed non necessary loads during island mode because it lowers the total rating installed for the DR's. The ratings of the units are defined in the island operation mode, when their demand is at the peak. While installing DR in the industrial site it is important to try to achieve the following goals:
i) reduce the losses in the micro-grid
ii) holding down the ratings of the units while keeping voltage to desired values
iii) maintain ability to feed the plant during isolation even in the face of a loss of a microsource in the plant

### 2.3 Factory Summary

It is possible to break up the electrical diagram in three distinct groups: transformers, loads and branches. Each part of the network will be derived in the following section, where the components of the models will be described. The following Tables are very useful to directly compare different models and connections for the components of the network. The transformer data is summarized in Table 1, where all the voltages, ratings and connection types are listed.

| Transformer \# | V_primary <br> $[\mathrm{KV}]$ | V_secondary <br> $[\mathrm{KV}]$ | Actual Load <br> $[\mathrm{KW}]$ | Rating <br> $[\mathrm{KVA}]$ | Connection | Available <br> Voltages [V] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 120 | 13.8 | 1000 | 2000 | Delta-Wye-n | - |
| 2 | 13.8 | 0.480 | 560 | 1500 | Delta-Wye-n | 480,280 |
| 3 | 13.8 | 0.480 | 440 | 1500 | Delta-Wye-n | 480,280 |
| 4 | 0.480 | 0.240 | 70 | 200 | Delta-Delta-n | $240,208,120$ |
| 5 | 0.480 | 0.208 | 150 | 300 | Delta-Wye-n | 208,120 |

Table 1. Transformer Summary

Loads are summarized in Table 2, where utilization voltage, connection type and sizes are listed. The Table also shows which model is used to represent the load in the electrical network.

| Load Type | Voltage Level [V] | Connection | Sizes <br> [KW] | Power Factor | Model | Quantity |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Induction <br> Motors | 480 | 3 ph . Delta | 50 | 0.80 | Delta equiv. Impedance | 9 |
| Induction <br> Motor | 480 | 3 ph. Delta | 50 | - | Detailed IM | 1 |
| AC Systems | 280 | 3 ph . LN | 15,30, 40 | 0.95 | LN equiv. Impedance | 5 |
| Elevators | 480 | 3 ph . Wye-n | 30 | 0.97 | LN equiv. Impedance | 3 |
| Lights | 120 | Single ph. LN | 5, 10, 15, 30 | 0.98 | LN equiv. Impedance | 5 |
| Computers | 120 | Single ph. LN | 50 | 0.96 | LN equiv. Impedance | 2 |
| Factory Process | $\begin{gathered} 480,280,240 \\ 208,120 \end{gathered}$ | 3 ph. 1 ph. <br> LN | 20, $30 \ldots$ | $\ldots$ | $\ldots$ | 4 |
| Capacitors | 280 | 3 ph. LN | - | - | LN equivalent | 2 |

Table 2. Load Summary

Table 3 summarizes the properties of each of the branches responsible to distribute the power along the plant. The voltage level, ratings and cable type are listed along with the theoretic and allowable current per phase.

| Branch | Voltage [KV] | Actual Load [KW] | Worse pf | Actual <br> [KVA] | Rating <br> [KVA] | Loads Supplied | Current per phase [A] | Cable Type |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2>3$ | 13.8 | 1000 | 0.80 | 1250 | 2000 | All plant | 83.6 | AWG 6 |
| $3>4$ | 13.8 | 1000 | 0.80 | 1250 | 2000 | All plant | 83.6 | AWG 4 |
| $5>11^{\text {st }}$ Floor | 0.480 | 280 | 0.72 | 388 | 388 | IM, AC | 465 | MCM 750 |
| $5>2{ }^{\text {nd }}$ Floor | 0.480 | 280 | 0.72 | 388 | 388 | IM, AC | 465 | MCM 750 |
| $8>7$ | 0.480 | 120 | 0.80 | 150 | 160 | MP, <br> Elevators | 191 | AWG 3/0 |
| $7>1{ }^{\text {st }}$ Floor | 0.480 | 30 | - | - | 40 | MP | 47 | AWG 6 |
| $7>2^{\text {nd }}$ Floor | 0.480 | 90 | - | - | 130 | MP, <br> Elevators | 155 | AWG 2/0 |
| $9>10$ | 0.240 | 70 | - | - | 100 | MP, Lights | 238 | MCM 250 |
| $10>1{ }^{\text {st }}$ Floor | 0.240 | 35 | - | - | 50 | MP, Lights | 119 | AWG 1 |
| $10>2{ }^{\text {nd }}$ Floor | 0.240 | 35 | - | - | 50 | MP, Lights | 119 | AWG 1 |
| $8>11$ | 0.480 | 250 | 0.78 | 320 | 320 | Warehouse, Office | 382 | MCM 600 |
| $11>12$ | 0.480 | 70 | 0.85 | 82.3 | 82.3 | AC, Elevator | 99 | AWG 2 |
| $12>$ Ceiling | 0.480 | 30 | 0.80 | 37.5 | 37.5 | Elevator | 45 | AWG 8 |
| $14>$ Ceiling | 0.208 | 30 | 0.78 | 38.4 | 38.4 | Lights | 106 | AWG 2 |
| $11>13$ | 0.480 | 30 | - | - | 40 | AC | 48 | AWG 8 |
| $13>1^{\text {st }}$ Floor | 0.480 | 15 | - | - | 30 | AC | 36 | AWG 8 |
| $13>2{ }^{\text {nd }}$ Floor | 0.480 | 15 | - | - | 30 | AC | 36 | AWG 8 |
| $14>15$ | 0.208 | 120 | 0.90 | 133 | 150 | Office | 416 | MCM 600 |
| $15>1{ }^{\text {st }}$ Floor | 0.208 | 55 | - | - | 70 | Computers, lights | 194 | AWG 3/0 |
| $15>2{ }^{\text {nd }}$ Floor | 0.208 | 55 | - | - | 70 | Computers, lights | 194 | AWG 3/0 |
| $15>$ <br> Perimetral <br> Lights | 0.208 | 10 | 0.9 | 11.1 | 11.1 | Lights | 31 | AWG 8 |

Table 3. Cable Summary

With data from the previous tables it is possible to find the values for the electrical parameters. Next section will deal with obtaining these values. Table 4 summarizes the values in actual units for the electrical parameter of the transformer, reported on the high voltage side.

| Transformer \# | Primary Voltage [KV] | Ratings [KVA] | $\mathrm{R} \quad[\Omega]$ | X |
| :---: | :---: | :---: | :---: | :---: |
| T 1 | 120 | 2000 | 51.15 | 486 |
| T 2 | 13.8 | 1500 | 1.15 | 7.23 |
| T3 | 13.8 | 1500 | 1.15 | 7.23 |
| T4 | 0.48 | 200 | 0.0087 | 0.0357 |
| T5 | 0.48 | 300 | 0.0061 | 0.0261 |

Table 4. Transformer Data
Table 5 summarizes the values for the parameters of the load, the connection to the local feeder, the power level of the load and the utilization voltage.

| Load Type | Voltage [V] | P [KW] | R | X | Connection |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Induction Motor | 480 | 50 | 8.8474 | 6.6355 | L-L, 3 Ф, Delta |
| AC | 280 | 30 | 2.3614 | 0.7714 | L-n, 3 Ф, Wye |
| AC | 280 | 40 | 1.7701 | 0.5797 | L-n, 3 Ф, Wye |
| AC | 280 | 15 | 4.7227 | 1.5427 | L-n, 3 Ф, Wye |
| Elevators | 480 | 30 | 7.2282 | 1.8071 | L-n, 3 Ф,_Wye |
| Lights | 120 | 15 | 0.9231 | 0.1846 | L-n, $1 \Phi$ |
| Lights | 120 | 30 | 0.4609 | 0.0937 | L-n, 1 Ф |
| Lights | 120 | 5 | 2.7692 | 0.5538 | L-n, 1 Ф |
| Lights | 120 | 2 | 6.9231 | 1.3846 | L-n, 1 Ф |
| Computers | 120 | 50 | 0.2657 | 0.077 | L-n, 1 Ф |
| Factory Process | 280 | 30 | 6.9395 | 2.2669 | L-n, 3 Ф, Wye |
| Factory Process | 120 | 20 | 3.5455 | 1.1523 | L-n, 1 Ф |
| Capacitors | 480 | - | - | -1.536 | L-n, 3 Ф, Wye |
| Neighboring Plant | 120000 | 2000 | 6914.9 | 1404.1 | L-n, 1 Ф |

Table 5. Load Data

Table 6 shows the parameters used to represent the branches that deliver power to the network. There are two models, and a given cable is represented by either of them. The first model is a series of a resistance and impedance, the other specifies sequence parameters, here assuming that positive and negative sequence have the same value.

| Branch | Cable Size | Length [ft] | $R_{s}[\Omega]$ | $X_{s}{ }^{[\Omega]}$ | $R_{1}[\Omega]$ | $X_{1}{ }^{[\Omega]}$ | $R_{o}[\Omega]$ | $X_{o}{ }^{[\Omega]}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Substation > T1 | AWG 2 | 49700 |  |  | 8.541 | 11.003 | 11.741 | 20.719 |
| T1 $>$ Neighbor | AWG 2 | 9940 |  |  | 1.7082 | 2.2006 | 2.3482 | 4.1438 |
| $2>3$ | AWG 6 | 4970 |  |  | 2.16 | 1.088 | 2.59 | 2.357 |
| $3>4$ | AWG 4 | 4970 |  |  | 1.564 | 0.026 | 1.9 | 0.914 |
| $5>1{ }^{\text {st }}$ Floor | MCM 750 | 150 |  |  | 0.0027 | 0.0037 | 0.0724 | 0.0059 |
| $5>1{ }^{\text {st }}$ Floor | MCM 750 | 60 |  |  | 0.0011 | 0.0015 | 0.0290 | 0.0023 |
| $5>2{ }^{\text {nd }}$ Floor | MCM 750 | 180 |  |  | 0.0033 | 0.0044 | 0.0869 | 0.0070 |
| $8>7$ | AWG 3/0 | 50 |  |  | 0.0039 | 0.0013 | 0.0549 | 0.0024 |
| $7>11^{\text {st }}$ Floor | AWG 6 | 350 |  |  | 0.1761 | 0.0130 | 0.7507 | 0.0222 |
| $7>2{ }^{\text {nd }}$ Floor | AWG 2/0 | 380 |  |  | 0.0378 | 0.0106 | 0.4557 | 0.0191 |
| $9>10$ | MCM 250 | 50 |  |  | 0.0026 | 0.0013 | 0.0449 | 0.0023 |
| $10>1{ }^{\text {st }}$ Floor | AWG 1 | 350 |  |  | 0.0554 | 0.0109 | 0.4275 | 0.0180 |
| $10>2{ }^{\text {nd }}$ Floor | AWG 1 | 380 |  |  | 0.0601 | 0.0119 | 0.4641 | 0.0196 |
| $8>11$ | MCM 600 | 500 |  |  | 0.0114 | 0.0123 | 0.2757 | 0.0198 |
| $11>12$ | AWG 2 | 50 |  |  | 0.0099 | 0.0017 | 0.0703 | 0.0027 |
| $12>$ Ceiling | AWG 8 | 100 | 0.07275 | 0.00389 |  |  |  |  |
| $14>$ Ceiling | AWG 2 | 200 |  |  | 0.0397 | 0.0066 | 0.2813 | 0.0110 |
| $11>13$ | AWG 8 | 100 | 0.07275 | 0.00389 |  |  |  |  |
| $13>1{ }^{\text {st }}$ Floor | AWG 8 | 200 | 0.1455 | 0.0094 |  |  |  |  |
| $13>2{ }^{\text {nd }}$ Floor | AWG 8 | 230 | 0.1673 | 0.0108 |  |  |  |  |
| $14>15$ | MCM 600 | 100 |  |  | 0.0023 | 0.0025 | 0.0551 | 0.0040 |
| $15>1{ }^{\text {st }}$ Floor | AWG 3/0 | 150 |  |  | 0.0118 | 0.0040 | 0.1648 | 0.0073 |
| $15>2{ }^{\text {nd }}$ Floor | AWG 3/0 | 180 |  |  | 0.0142 | 0.0049 | 0.1977 | 0.0087 |
| $15>$ Perimetral Lights | AWG 8 | 200 | 0.1455 | 0.0094 |  |  |  |  |
| Perimetral Lights | AWG 8 | 100 | 0.07275 | 0.00468 |  |  |  |  |

Table 6. Cable Data

## 3. Components of the Models

In this section, the models of the system will be described and the parameters that represent them will be derived. The transformer data is found first starting from the terminal voltage levels, ratings and percent impedance. Cable data require some different set of tables to be identified. Each line or cable will be defined by its length, voltage level and current that has to withstand. Finally, loads will be considered: starting from model type, ratings and utilization voltage the parameters will be evaluated.

### 3.1 Transformers

There are 5 transformers in total in this system, four of them within the property of the industrial site. The transformers T2 and T3 are connected in parallel from the high side, and provide a normally open connection on the low side. This connection is closed during maintenance at either of these transformers, and allows to partially feed the loads that would otherwise be unpowered. Each transformer will be described and the electrical parameters evaluated.

## Transformer T1

This transformer, included in a substation cabin not too far from the plant, is connected to the high voltage aerial transmission line. The primary side busbar is at 120 KV , and the secondary busbar is at 13.8 KV . The secondary busbar is connected to the distribution aerial line of 13.8 KV . This transformer has to carry the full load of the plant, which is of 1MW. The size of this transformer is chosen to be 2 MW to meet this requirement and some possible expansion plans. This transformer is delta-wye with neutral solidly grounded on the secondary, and has a variable tap changer. From voltage levels and ratings, this transformer typically will have a percent impedance of $6.75 \%$ and a ratio $\frac{X}{R}=9.5$. With this data in hand it is possible to obtain the transformer data as:
$X=\frac{V^{2} Z \%}{\text { MVA }}=\frac{120000^{2} 0.0675}{2000000}=486 \mathrm{Ohms}$
$R=\frac{X}{9.5}=51.15 \mathrm{Ohms}$

## Transformer T2

This transformer is located on the back of the factory building, within the industrial plant compounds. The primary side at 13.8 KV is connected to a busbar that connects with the transformer T2 and with the underground cable coming from the distribution system. The secondary side, at 480 V , is connected to two cables each one feeding some of the loads of both floors in the factory. Essentially, loads are induction machines and AC systems. The
combined load is 560 KW , but redundancy requirements with transformer T 2 ask for a much higher capability. This translates in a 1.5MVA rating for transformer T1. This transformer is delta-wye connected and has a solidly grounded neutral at the secondary. The percent impedance of this transformer is $5.7 \%$ and $\frac{X}{R}=6.3$. The reactance and resistance of the transformer are:
$X=\frac{13800^{2} 0.057}{1500000}=7.23 \mathrm{Ohms}$
$\mathrm{R}=\frac{\mathrm{X}}{6.3}=1.15 \mathrm{Ohms}$

## Transformer T3

This transformer is twin of T2 for what concerns the electrical properties. This transformer is also located very close to T 1 , fed at its primary side by the distribution cable entering the plant. The secondary side busbar is connected to the remaining 480 V load inside the factory, to the cable that leads to the warehouse and to the primary side of the transformer T4. The resistance and reactance of this transformer are the same as of transformer T2:
$\mathrm{R}=7.23 \mathrm{Ohms}$
$\mathrm{X}=1.15 \mathrm{Ohms}$

## Transformer T4

This transformer is introduced to meet the demand of loads at voltage lower than 480 V , and it is located near the transformers T1 and T2. This transformer is connected at deltadelta with the central point a line to line grounded in the secondary side. This allows to obtain three different voltages at its terminals: 240 V as the line to line voltage, 208 V and 120 V as the line to neutral voltage. The load is a mix of rectifying bridges, lights and factory processes. Although the current load is of 70 KW , the rating for this transformer are 0.2 MVA to allow for further expansion. The percent impedance of this transformer is $3.1 \%$ and the ratio $\frac{X}{R}=4.1$, which yield:
$X=\frac{480^{2} 0.031}{200000}=0.0357 \mathrm{Ohms}$
$\mathrm{R}=\frac{\mathrm{X}}{4.1}=0.0087 \mathrm{Ohms}$

## Transformer T5

This transformer is located in the warehouse and is needed to feed loads at 208 V and 120 V in the warehouse and office buildings. The primary busbar is connected to the cable coming from the warehouse, while the secondary busbar is connected to two cables: the first reaches the ceiling of the warehouse, the other goes to a panel inside the office. The current load is 150 KW , and a transformer with rating of 0.3 MVA is chosen to fully meet the demand, as well as future plans for expansion. The percent impedance of this transformer is $3.4 \%$ and the ratio $\frac{X}{R}=4.3$, which yield:
$X=\frac{480^{2} 0.034}{300000}=0.0261 \mathrm{Ohms}$
$\mathrm{R}=\frac{X}{4.3}=0.0061 \mathrm{Ohms}$

### 3.2 Lines and Cables

This chapter describes each of the branches that transmit power: these branches are usually lines or cables. We have aerial lines at the 120 KV and 13.8 KV voltage levels, while cables are responsible for distributing power in one branch at 13.8 KV and in all the lower voltages ones. Each branch is described in detail, giving justification of the values for the electrical parameters used to describe it. Most of electrical parameters of the cables are found in tables existing in the literature. We report some significative tables for our study below. Table 7 shows the resistance and reactance for building wires in different conditions: we are interested in the case when three conductors are in the same duct and in presence of a non magnetic conduit.

Typical Resistance and Reactance Values for Building Wire and Cable, in Ohms per 100 ft , Line-to-Neutral, at Normal Operating Temperature

| $\begin{gathered} \text { Wire } \\ \text { Size } \\ \text { (ACM) } \end{gathered}$ | $\begin{aligned} & \text { Tempere- } \\ & \text { ture } \\ & \text { ( } \left.^{\circ} \mathrm{C}\right\} \end{aligned}$ | R. | $R_{\text {m }}$ | Magnetic or Interlocked Armor Cable Cooduit |  |  | Noomarretic Conduit |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\boldsymbol{\lambda}$ | $\boldsymbol{z}$ | $R / Z$ | $\boldsymbol{X}$ | $Z$ | $R / Z$ |
| Singlo-Condwctor Cable in Condui |  |  |  |  |  |  |  |  |  |
| 8 | 60 | 0.07275 | 0.07275 | 0.00585 | 0.0730 | 0.89 | 0.00488 | 0.0729 | 1.00 |
| 4 | 60 | 0.02928 | 0.02928 | 0.00525 | 0.0297 | 0.88 | 0.00420 | 0.0296 | 0.99 |
| 2 | 75 | 0.01947 | 0.01964 | 0.00591 | 0.0202 | 0.97 | 0.00392 | 0.0200 | 0.98 |
| 1 | 75 | 0.01530 | 0.01554 | 0.00515 | 0.0163 | 0.95 | 0.00412 | 0.0161 | 0.96 |
| 0 | 75 | 0.01218 | 0.01241 | 0.00510 | 0.0134 | 0.93 | 0.00408 | 0.0131 | 0.95 |
| 000 | 75 | 0.00768 | 11.00798 | 0.00480 | 0.0003 | 0.86 | 0.00384 | 0.0088 | 0.90 |
| 0000 | 75 | 0.00608 | 1).00639 | 0.00464 | 0.0079 | 0.81 | 0.00371 | 0.0074 | 0.88 |
| 250 | 75 | 0.00518 | 1. 00548 | 0.00461 | 0.0071 | 0.76 | 0.00368 | 0.0036 | 0.83 |
| 350 | 75 | 0.00368 | 11.00397 | 0.00456 | 0.0060 | 0.66 | 0.00365 | 0.0054 | 0.73 |
| 500 | 75 | 0.00257 | 11.00291 | 0.00432 | 0.0052 | 0.56 | 0.00346 | 0.0045 | 0.64 |
| 750 | 75 | 0.00172 | 11.00208 | 0.00417 | 0.0047 | 0.44 | 0.00334 | 0.0039 | 0.53 |
| 1000 | 75 | 0.00120 | 11.00170 | 0.00416 | 0.0045 | 0.38 | 0.00333 | 0.0037 | 0.45 |
| 1500 | 75 | 0.00088 | 0.00137 | 0.00408 | 0.0043 | 0.32 | 0.00328 | 0.0035 | 0.39 |
| Troo or ThrmeCondwest Cable in Condwis |  |  |  |  |  |  |  |  |  |
| 8 | 60 | 0.07275 | 0.07275 | 0.00541 | 0.0729 | 1.00 | 0.00389 | 0.0728 | 1.00 |
| 4 | 60 | 0.02928 | 0.02928 | 0.00404 | 0.0296 | 0.90 | 0.00349 | 0.0295 | 0.99 |
| 2 | 75 | 0.01947 | 0.01964 | 0.00378 | 0.0200 | 0.98 | 0.00326 | 0.0199 | 0.98 |
| 1 | 75 | 0.01530 | 0.01554 | 0.00397 | 0.0181 | 0.96 | 0.00342 | 0.0159 | 0.98 |
| 0 | 75 | 0.01218 | 0.01241 | 0.00393 | 0.0130 | 0.85 | 0.00339 | 0.0129 | 0.96 |
| 000 | 75 | 0.00788 | 0.00798 | 0.00370 | 0.0088 | 0.91 | 0.00319 | 0.0086 | 0.93 |
| 0000 | 75 | 0.00608 | 0.00639 | 0.00358 | 0.00731 | 0.87 | 0.00308 | 0.0071 | 0.90 |
| 250 | 75 | 0.00516 | 0.00546 | 0.00355 | 0.00851 | 0.84 | 0.00306 | 0.00626 | 0.87 |
| 350 | 75 | 0.00368 | 0.00397. | 0.00352 | 0.00531 | 0.75 | 0.00303 | 0.00499 | 0.78 |
| 500 | 75 | 0.00257 | 0.00291 | 0.00333 | 0.00442 | 0.66 | 0.00287 | 0.00409 | 0.71 |
| 750 | 75 | 0.00172 | 0.00288 | 0.00321 | 0.00383 | 0.54 | 0.00278 | 0.00347 | 0.60 |
| 1000 | 75 | 0.00129 | 0.00170 | 0.00320 | 0.00362 | 0.47 | 0.00271 | 0.00325 | 0.52 |
| 1500 | 75 | 0.00086 | 0.00137 | 0.00315 | 0.00342 | 0.40 | 0.00271 | 0.003104 | 0.45 |

For aluminum cables of the tame phymical sise, multiply the rasiatance by 1.64. (Soe NEC, article 9, table 8.)
This table is taken from IEEE JH 2112-1, Protection Fundamentale for Low-Voltage Electrical Distribution Systema in Commercial Builditaga. The letier aymbol uned in the table for kilocircular mila (MCM) has been deprecnted and replaced by kemil.

Table 7. Cables in Building

Table 8 shows the properties of different kind of insulation: the plant under study uses insulation RHW. Table 8 appears in [1] on page 60.


Table 8. Insulation Properties
Table 9 shows the current capability for each of the cable sizes, depending on the type of insulation used. This table refers to aerial cables and we are interested in the RHW isolation column and appears in [1] on page 197. Table 10 shows the current capability for cables in raceways or direct burial and is from [1] on page 196. Table 11 is used to find the electrical parameters of the cables when the size is known: we are interested only in the first row of data, the one relative to cables below 1 KV . Table 11 appears in [2] on page 674. The process to identify the parameters for a cable starts with finding the current that needs to be delivered and from there obtain the corresponding size of the cable that is capable of carrying it. Then the numerical values for electrical parameters are read from the tables.

# ALLOWABLE CURRENT CARRYING CAPACITIES 

(Ampacity)
Copper Insulated Conductors in Free Air* Single Conductors


Table 9. Current Capabilities - Aerial Cables

## ALLOWABLE CURRENT CARRYING CAPACITIES

(Ampacity)
Copper Insulated Conductors
in Raceway or Cable or Direct Burial*
Not More Than Three Conductors

| Size AWG <br> or <br> MCM | Rubber <br> Type $R$ <br> Type RW <br> Type RU <br> Type RUW <br> T(14-2) <br> Type RFR-RW <br> Thermo- <br> plastic <br> Type T <br> Type TW | Rubber <br> Type RHType RUH <br> (14-2)$\frac{\text { Type RH-RW }}{\text { Type RFWW }}$ | Paper <br> Thermo- <br> plastic <br> Asbestos <br> Type TA <br> Var-Cam <br> Type V <br> Asbestos <br> Var-Cam <br> Type AVB <br> MI Cable <br> Type RIFH*** | Asbestos <br> Var-Cam <br> Type AVA <br> Type AVL | Impregnated <br> Asbestos <br> Type AI <br> (14-8) <br> Type AIA | Asbestos <br> Type A <br> (14-8) <br> Type AA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AMPERES* PER CONDUCTOR |  |  |  |  |  |  |
| 14 | 15 | 15 | 25 | 30 | 30 | 30 |
| 12 | 20 | 20 | - 30 | 35 | 40 | 40 |
| 10 | 30 | 30 | 40 | 45 | 50 | 55 |
| 8 | 40 | 45 | 50 | 60 | 65 | 70 |
| 6 | 55 | 65 | 70 | 80 | 85 | 95 |
| 4 | 70 | 85 | 90 | 105 | 115 | 120 |
| 3 | 80 | 100 | 105 | 120 | 130 | 145 |
| 2 | 95 | 115 | 120 | 135 | 145 | 165 |
| 1 | 110 | 130 | 140 | 160 | 170 | 190 |
| $1 / 0$ | 125 | 150 | 155 | 190 | 200 | 225 |
| 270 | 145 | 175 | 185 | 215 | 230 | 250 |
| $3 / 0$ | 165 | 200 | 210 | 245 | 265 | 285 |
| 4/0 | 195 | 230 | 235 | 275 | 310 | 340 |
| 250 | 215 | 255 | 270 | 315 | 335 | $\cdots$ |
| 300 | 240 | 285 | 300 | 345 | 380 |  |
| 350 | 260 | 310 | 325 | 390 | 420 | $\cdots$ |
| 400 500 | 280 320 | 335 $\mathbf{3 8 0}$ | 360 405 | 420 | 450 500 |  |
| 600 | 355 | 420 | 455 | 525 | 545 |  |
| 700 | 385 | 460 | 490 | 560 | 600 |  |
| 750 | 400 | 475 | 500 | 580 | 620 |  |
| 800 | 410 | 490 | 515 | 600 | 640 |  |
| 900 | 435 | 520 | 555 | ... |  | .. |
| 1000 | 455 | 545 | 585 | 680 | 730 | $\ldots$ |
| 1250 | 495 | 590 | 645 |  | . . |  |
| 1500 | 520 | 625 | 700 | 785 | . . | . . . |
| 1750 | 545 | 650 | 735 |  | . . . | $\ldots$ |
| 2000 | 560 | 665 | 775 | 840 | - . - | - |
| CORRECTION FACTORS FOR ROOM TEMPERATURES OVER 30 C (86 F) |  |  |  |  |  |  |
| C F <br> 40  <br> 104  |  |  |  | . 94 |  |  |
| 45113 | . 71 | .82 | . 85 | . 94 | . 95 |  |
| 50122 | . 58 | . 75 | . 80 | . 87 | . 89 |  |
| 55131 | . 41 | . 67 | . 74 | . 83 | . 88 |  |
| 60140 |  | . 58 | . 67 | . 79 | . 83 | . 91 |
| 70.158 |  | .35 | . 52 | . 71 | . 76 | . 87 | latet issue of the NECS.

**The current carrying capacity for Type RHFI conductors for aizes 14,12 and 10 AWG shall be the same as designated for Type RH.

Table 10. Current Capability - Raceway and Buried

Table A-15 $\mathbf{6 0 - H z}$ characteristics of three-conductor belted paper-insulated cables


Ac resistance based upon $1000^{\circ}$, conductivity al $65^{\circ}{ }^{\circ}$ including $2^{\circ}$, allowance for stranding.
 applications.

- For dielectric constant - 3.7.
${ }^{4}$ Based upon all return current in the sheath: none in ground.
S See Fip. 7. pp. 67. of Ref. 1.
 wectur.

Source: [1].

Table 11. Sequence Parameters of Cables

## Grounding system

Transformer T1 is solidly grounded to earth on its secondary. Far away, with some earth resistance in between, there is the grounding of the transformers T2 and T3. Transformer T 4 is very close, and grounded at the midpoint at one of the delta sides of the secondary. The buildings are grounded throughout the area. It is therefore licit to assume that within the plant, connection to neutral is synonymous with connection to ground.

## Branch at 120 KV

This branch is meant to represent a transmission line that runs in the neighborhood. This branch starts from a transmission yard, 10 mi . away, connects with the substation for the industrial plant, and continues to a neighboring industrial park. The line is carried on a tower represented below in Figure 4


Figure 4. 120KV Tower

The actual loads of the transmission lines are 3MW combined, but the line can carry up to 12 MVA , due to a forecasted load growth in the area.

Each phase of the transmission line must be able to deliver 4MVA and therefore carry $\frac{4000}{\left(\frac{120}{\sqrt{3}}\right)}=57.7 \mathrm{~A}$. Assuming a current density of $2 \mathrm{~A} / \mathrm{mm}^{2}$, we need a $28.8 \mathrm{~mm}^{2}$ cross section wire. Since $1 \mathrm{Kcmil}=0.506 \mathrm{~mm}^{2}$, this requirement translates into 57.01 Kcmil for the cross-section. From Integrated Grounded System Analysis (IGS) software, it is
possible to obtain cable data from the geometrical disposition of the wires as of Figure 1 and from the cable size that meets the cross-section requirements (AWG size 2). IGS is a software that allows to find parameters for lines: the input parameters are voltage level, cable section, material and insulation, geometrical disposition of the three phases and medium between cables. IGS yields data in sequence domain, which is the preferred format to perform unbalanced analysis. For the 10 miles section the results are:
$\mathrm{R} 1=8.541$ Ohms $\quad \mathrm{X} 1=11.003$ Ohms
$\mathrm{R} 0=11.741 \mathrm{Ohms} \quad \mathrm{X} 0=20.719$ Ohms

For the 2 miles section the data is:

R1=1.7082 Ohms X1=2.2006 Ohms
$\mathrm{R} 0=2.3482$ Ohms $\mathrm{X} 0=4.1438$ Ohms

## Branch at 13.8KV

## Branch from Bus 2 to 3

It is an aerial line, using a single pole distribution line with 3 phase conductors placed at triangle. Neutral is also carried in the pole. The load is the whole plant, 1MW, and a conservative power factor of 0.8 brings the requirements for the apparent power to $\frac{1}{0.8}=1.25 \mathrm{MVA}$. The nature of the plant loads with induction machines and the requirement that the line must satisfy further plant expansions suggest that conservative MVA ratings of the transmission line over the three phases are 2MVA.

Each phase must be able to transfer $\frac{2000}{3}=666 \mathrm{KVA}$.
Current per each phase: $\frac{666}{\left(\frac{13.8}{\sqrt{3}}\right)}=83.6 \mathrm{~A}$

From Table 9 and remembering our choice for insulation, RHW, (see Table 8 for insulation properties) it is possible to see that AWG size 6 is the conductor that we need
to use here. From IGS software, after choosing the tower structure (shown in Figure 5) and selecting the cable size, the data relative to the one mile span of this aerial line is:
$\mathrm{R} 1=2.16$ Ohms $\quad \mathrm{X} 1=1.088$ Ohms
$\mathrm{R} 0=2.59$ Ohms $\quad \mathrm{X} 0=2.357$ Ohms


Figure 5. 13.8 KV Pole

## Branch from Bus 3 to 4

It is a buried cable line, in series to branch from bus 2 to 3 . The current carried by each conductor must be 83.6 A. From Table 10, it is possible to see that if we assume all three cables are buried in the same duct, then AWG size 4 is the conductor we need. From IGS software, using RFC3 cable configuration (shown in Figure 6), with a length of one mile, it is possible to find that the impedance is:
$\mathrm{Rs}=1.676 \mathrm{Ohm} \quad \mathrm{Xs}=0.322$
Rm=0.112 Ohm Xm=0.296


Figure 6. Geometrical Disposition of Buried Cable

The sequence values can be calculated as:
$\mathrm{Z} 1=\mathrm{Zs}-\mathrm{Zm}$ and $\mathrm{Z} 0=\mathrm{Zs}+2 * \mathrm{Zm}$
The values are as follows:
$\mathrm{R} 1=1.564 \mathrm{Ohms} \quad \mathrm{X} 1=0.026 \mathrm{Ohms}$
$\mathrm{R} 0=1.9 \quad$ Ohms $\mathrm{X} 0=0.914$ Ohms

## 480V System

## Branch from Bus 5 to First Floor 480V loads

Cable line in ducts: it is a four wire system with full sized neutral cable. It needs to supply all the 480 V loads in the factory building of the plant at the first floor. There are 5 motors and one AC system on each floor, the combined load is 280 KW with 250 KW alone of induction machines. During startups the power factor may impoverish considerably due to the inrush currents drawn by the machines. In the event that the power angle becomes as little as 0.72 , then the apparent power requirement for this part of the network is: $\frac{280}{0.72}=388$ KVA. Each phase must supply 129 KVA , and the current requirement per cable is: $\frac{129}{\left(\frac{0.48}{\sqrt{3}}\right)}=465 \mathrm{~A}$. From Table 10, considering RHW insulation, it is possible to see that we need a wire of size MCM 750. From Table 11 we obtain:
$\mathrm{R} 1=0.091 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 1=0.121 \mathrm{Ohm} / \mathrm{mile}$
$\mathrm{R} 0=2.40 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 0=0.194 \mathrm{Ohm} / \mathrm{mile}$

This line is composed of 5 parts: the first is 150 ft long, leaving from the transformer busbar and reaching the first induction machine. From there on, there are 4 consecutive equal branches of 60 ft length each. At the end of each of these branches there are the remaining 4 induction machines. For the first part, 150 ft long we have:

$$
\begin{array}{ll}
\mathrm{R} 1=0.091 * 150 / 4970=0.0027 \mathrm{Ohm} & \mathrm{X} 1=0.121 * 150 / 4970=0.0037 \mathrm{Ohm} \\
\mathrm{R} 0=2.40 * 150 / 4970=0.0724 \mathrm{Ohm} & \mathrm{X} 0=0.194 * 150 / 4970=0.0059 \mathrm{Ohm}
\end{array}
$$

For each of the sections 60 ft long we have:

$$
\begin{array}{ll}
\mathrm{R} 1=0.091 * 60 / 4970=0.0011 \mathrm{Ohm} & \mathrm{X} 1=0.121 * 60 / 4970=0.0015 \mathrm{Ohm} \\
\mathrm{R} 0=2.40 * 60 / 4970=0.0290 \mathrm{Ohm} & \mathrm{X} 0=0.194 * 60 / 4970=0.0023 \mathrm{Ohm}
\end{array}
$$

## Branch from Bus 5 to Second Floor 480V loads

This branch is identical in considerations as the preceding branch, with the only difference being that the part of the cable that spans from the transformer to the first motor has to reach one floor up. To keep this factor into consideration, an extra 30 ft of length have been added. Therefore, this branch is 180 ft long: the data for this part is:

$$
\begin{array}{ll}
\mathrm{R} 1=0.091 * 180 / 4970=0.0033 \mathrm{Ohm} & \mathrm{X} 1=0.121 * 180 / 4970=0.0044 \mathrm{Ohm} \\
\mathrm{R} 0=2.40 * 180 / 4970=0.0869 \mathrm{Ohm} & \mathrm{X} 0=0.194 * 180 / 4970=0.0070 \mathrm{Ohm}
\end{array}
$$

## Branch from Bus 8 to 7

This branch feeds the remaining 480V loads of the factory. In particular feeds the factory process and the elevators. This is a short cable that starts from the transformer busbar to reach the panel where the cable for first and second floor split. The total active power demand on this branch is 120 KW . The worse case power flow of 0.8 translates the
requirement in $\frac{120}{0.8}=150 \mathrm{KVA}$. To accommodate for expansion, the branch must be able to deliver 160 KVA . Each phase must carry 53 KVA which means that the current capability must be at least of $\frac{53}{\left(\frac{0.48}{\sqrt{3}}\right)}=191$ A. From Table 10 we see that a cable size
AWG 3/0 fits our needs when insulation RHW is chosen. From Table 11, we obtain the following data:
$\mathrm{R} 1=0.392 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 1=0.134 \mathrm{Ohm} / \mathrm{mile}$
$\mathrm{R} 0=5.46 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 0=0.241 \mathrm{Ohm} / \mathrm{mile}$

The length of this branch is of 50 ft , therefore the data is:
$\mathrm{R} 1=0.392 * 50 / 4970=0.0039 \mathrm{Ohm} \quad \mathrm{X} 1=0.134 * 50 / 4970=0.0013 \mathrm{Ohm}$
$\mathrm{R} 0=5.46 * 50 / 4970=0.0549 \mathrm{Ohm} \quad \mathrm{X} 0=0.241 * 50 / 4970=0.0024 \mathrm{Ohm}$

## Branch from bus 7 to First Floor 480V loads

This branch feeds the factory process at the first floor. The process has a mix of three and single phase rectifying bridges for a total of 30 KW . The company plans to add new loads on this feeder: the apparent power needed to be delivered is assumed to be 40KVA. Each phase must deliver 13 KVA and carry $\frac{13}{\left(\frac{0.48}{\sqrt{3}}\right)}=47$ A. From Table 10, we need cable size
AWG 6. From Table 11, we obtain the following data:
$\mathrm{R} 1=2.50 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 1=0.185 \mathrm{Ohm} / \mathrm{mile}$
$\mathrm{R} 0=10.66 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 0=0.315 \mathrm{Ohm} / \mathrm{mile}$

The length of this branch is of 350 ft , which means that the data is:

$$
\begin{array}{ll}
\mathrm{R} 1=2.50 * 350 / 4970=0.1761 \mathrm{Ohm} & \mathrm{X} 1=0.185 * 350 / 4970=0.0130 \mathrm{Ohm} \\
\mathrm{R} 0=10.66 * 350 / 4970=0.7507 \mathrm{Ohm} & \mathrm{X} 0=0.315 * 350 / 4970=0.0222 \mathrm{Ohm}
\end{array}
$$

## Branch from Bus 7 to Second Floor 480 V loads

This branch is 30 ft longer than the preceding because it needs to reach one floor up. This branch feeds the same amounts of loads as the first floor, but also includes the elevators, whose machinery is located on the top of the building. The total load for this floor is 90 KW . Assuming that the expansion program of the plant extends also at the second floor, then a safe value of apparent power that must be delivered is 130KVA. Each phase must deliver 43 KVA and carry $\frac{43}{\left(\frac{0.48}{\sqrt{3}}\right)}=155$ A. From Table 10, we need cable size
AWG $2 / 0$. From Table 11, we obtain the following data:
$\mathrm{R} 1=0.495 \mathrm{Ohm} / \mathrm{mile} \mathrm{X} 1=0.138 \mathrm{Ohm} / \mathrm{mile}$
$\mathrm{R} 0=5.96 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 0=0.250 \mathrm{Ohm} / \mathrm{mile}$

The overall cable length is of 380 ft , and the data is:

R1 $=0.495 * 380 / 4970=0.0378$ Ohm X1=0.138*380/4970=0.0106 Ohm
$\mathrm{R} 0=5.96 * 380 / 4970=0.4557$ Ohm $\quad \mathrm{X} 0=0.250 * 380 / 4970=0.0191$ Ohm

## Branch from Bus 8 to 11

This branch connects the secondary side busbar of the transformer T3 in the factory with the primary side busbar of the transformer in the warehouse. The cable must be able to carry the load for warehouse and offices: the current combined load is 250 KW . To accommodate for a power factor of 0.78 , we obtain $\frac{250}{0.78}=320 \mathrm{KVA}$. Each phase must deliver 106KVA and carry $\frac{106}{\left(\frac{0.48}{\sqrt{3}}\right)}=382$ A. From Table 10, we need cable size MCM
600. From Table 11, we obtain:

| $\mathrm{R} 1=0.113 \mathrm{Ohm} / \mathrm{mile}$ | $\mathrm{X} 1=0.122 \mathrm{Ohm} / \mathrm{mile}$ |
| :--- | :--- |
| $\mathrm{R} 0=2.74 \mathrm{Ohm} / \mathrm{mile}$ | $\mathrm{X} 0=0.197 \mathrm{Ohm} / \mathrm{mile}$ |

With the cable length of 500 , it is possible to obtain:
$\mathrm{R} 1=0.113 * 500 / 4970=0.0114 \mathrm{Ohm}$
$\mathrm{X} 1=0.122 * 500 / 4970=0.0123 \mathrm{Ohm}$
$\mathrm{R} 0=2.74 * 500 / 4970=0.2757 \mathrm{Ohm} \quad \mathrm{X} 0=0.197 * 500 / 4970=0.0198 \mathrm{Ohm}$

## Branch from Bus 11 to 12

This branch starts from the primary side busbar of the transformer in the warehouse and ends at the panel that connects with the cable that reaches the ceiling. We have three conductors in the same duct and the loads are the air conditioning and light systems of the warehouse, for a combined 70 KW value. With an expected worse power factor of 0.85 , we obtain $\frac{70}{0.85}=82.3 \mathrm{KVA}$. Each phase must deliver $\frac{82.3}{3}=27.4 \mathrm{KVA}$ and carry $\frac{27.4}{\left(\frac{0.48}{\sqrt{3}}\right)}=99$ A. From Table 10, we need cable size AWG 2. From Table 11, we obtain the following data:

R1 $=0.987 \mathrm{Ohm} / \mathrm{mile} \mathrm{X} 1=0.165 \mathrm{Ohm} / \mathrm{mile}$
$\mathrm{R} 0=6.99 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 0=0.273 \mathrm{Ohm} / \mathrm{mile}$

The length of this span is 50 ft , therefore the data is:
$\mathrm{R} 1=0.987 * 50 / 4970=0.0099 \mathrm{Ohm} \quad \mathrm{X} 1=0.0017 \mathrm{Ohm}$
$\mathrm{R} 0=0.0703 \mathrm{Ohm} \quad \mathrm{X} 0=0.0027 \mathrm{Ohm}$

## Branch from Bus 12 to Ceiling

This branch provides power to the elevator machinery that is located at the ceiling of the warehouse. The load for the elevator is 30 KW and with a worse power factor of 0.8 we obtain $\frac{30}{0.8}=37.5$ KVA. Each phase must supply 12.5 KVA and carry $\frac{12.5}{\left(\frac{0.48}{\sqrt{3}}\right)}=45 \mathrm{~A}$.
From Table 10, we need cable size AWG 8. From Table 7, when we have three conductors in the same duct we obtain:
$\mathrm{R}=0.07275$ Ohm $/ 100 \mathrm{ft} \quad \mathrm{X}=0.00389 \mathrm{Ohm} / 100 \mathrm{ft}$

Since the length of this branch is of 100 ft , the data is:
$\mathrm{R}=0.07275$ Ohm $\mathrm{X}=0.00389$ Ohm

## Branch from Bus 11 to 13

This branch connects the primary side busbar of the transformer in the warehouse to the panel in the office where cables for first and second floor split. This branch provides 480 V service to the AC system of the office. Total load is 30 KW but expansion needs require this cable to be able to deliver 40 KVA . Each phase must supply 13.3 KVA and carry $\frac{13.3}{\left(\frac{0.48}{\sqrt{3}}\right)}=48 \mathrm{~A}$. From Table 10, we need cable size AWG 8. From Table 7, looking at the table relevant to the three conductors belted we obtain the following data:
$\mathrm{R}=0.07275 \mathrm{Ohm} / 100 \mathrm{ft} \quad \mathrm{X}=0.00389 \mathrm{Ohm} / 100 \mathrm{ft}$

The length of this branch is of 100 ft , and the data is:
$\mathrm{R}=0.07275 \mathrm{Ohm} \quad \mathrm{X}=0.00389 \mathrm{Ohm}$

## Branch from Bus 13 to First Floor 480V Office Loads

This branch starts from the panel and reaches the AC system for the first floor.
Total load is 15 KW , expansion plans require the cable to be able to supply 30 KVA . Each phase must deliver 10KVA and carry $\frac{10}{\left(\frac{0.48}{\sqrt{3}}\right)}=36 \mathrm{~A}$. From Table 10, we need a cable size AWG 8. From Table 7, with three conductors in the same duct, we have:
$\mathrm{R}=0.07275 \mathrm{Ohm} / 100 \mathrm{ft} \quad \mathrm{X}=0.00468 \mathrm{Ohm} / 100 \mathrm{ft}$

This branch has to reach the office floors and is long 200 ft . The data is:
$\mathrm{R}=0.07275 * 2=0.1455 \mathrm{Ohm} \quad \mathrm{X}=0.0094 \mathrm{Ohm}$

## Branch from Bus 13 to Second Floor 480V Office Loads

This branch has identical requirements of the previous branch, but its length is 30 ft longer to be able to reach the second floor. Total length for this branch is 230 ft , and data is:
$\mathrm{R}=0.07275 * 2.3=0.1673 \mathrm{Ohm} \quad \mathrm{X}=0.0108 \mathrm{Ohm}$

## 240/208/120V System

## Branch from Bus 9 to 10

This branch starts from the secondary bus of the 240 V transformer and reaches the cabin where the cables that feed first and second floor split. The loads fed in this branch are light systems and other factory processes for a total of 70 KW . The conservative value for the overall apparent power requirement is 100 KVA . Each phase must deliver 33 KVA and carry $\frac{33}{\left(\frac{0.24}{\sqrt{3}}\right)}=238$ A. From Table 10, we need cable size MCM 250. From Table 11, we obtain the following data:

# R1=0.263 Ohm/mile $\quad \mathrm{X} 1=0.129$ Ohm/mile <br> $\mathrm{R} 0=4.46 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 0=0.224 \mathrm{Ohm} / \mathrm{mile}$ 

The length of the cable is of 50 ft , therefore the data is:
$\mathrm{R} 1=0.263 * 50 / 4970=0.0026 \mathrm{Ohm} \quad \mathrm{X} 1=0.129 * 50 / 4970=0.0013 \mathrm{Ohm}$
$\mathrm{R} 0=4.46 * 50 / 4970=0.0449 \mathrm{Ohm} \quad \mathrm{X} 0=0.224 * 50 / 4970=0.0023 \mathrm{Ohm}$

## Branch from Bus 10 to First Floor 240/208/120V Loads

The loads in this floor need voltage levels of 240, 120, 208V. For this purpose, each conductor is carried along in its own conduit. The first floor loads amount to 35 KW , and the apparent power requirement is set to 50 KVA . Each phase must deliver 16.6KVA and carry $\frac{16.6}{\left(\frac{0.24}{\sqrt{3}}\right)}=119$ A. From Table 10, we need cable size AWG 1. From Table 11, we obtain:
$\mathrm{R} 1=0.786 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 1=0.155 \mathrm{Ohm} / \mathrm{mile}$
$\mathrm{R} 0=6.07 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 0=0.256 \mathrm{Ohm} / \mathrm{mile}$

This branch must reach the whole floor, and its length is 350 ft . The data is:
$\mathrm{R} 1=0.786 * 350 / 4970=0.0554 \mathrm{Ohm}$
$\mathrm{R} 0=6.07 * 350 / 4970=0.4275 \mathrm{Ohm}$

$$
\begin{gathered}
\mathrm{X} 1=0.155 * 350 / 4970=0.0109 \mathrm{Ohm} \\
\mathrm{X} 0=0.256 * 350 / 4970=0.0180 \mathrm{Ohm}
\end{gathered}
$$

## Branch from Bus 10 to Second Floor 240/208/120V Loads

This branch is identical to the previous. The only change is in the length of the cable to allow reaching the second floor. The length of this branch is of 380 ft . The data is:
$\mathrm{R} 1=0.786 * 380 / 4970=0.0601 \mathrm{Ohm}$
$\mathrm{X} 1=0.155 * 380 / 4970=0.0119$ Ohm
$\mathrm{R} 0=6.07 * 380 / 4970=0.4641 \mathrm{Ohm}$

## Branch from Bus 14 to Warehouse Lights

This branch connects the low side busbar of the transformer in the warehouse to the ceiling of the warehouse, where the light system is located. The only load are the lights, and the voltage is 208 V . The load is meant to be connected on a single phase basis, therefore there will be one single conductor per conduit. The power needed for the lights is 30 KW and with a maximum power factor of 0.78 we obtain $\frac{30}{0.78}=38.4 \mathrm{KVA}$. Each phase must be able to supply 12.8 KVA and carry $\frac{12.8}{\left(\frac{0.208}{\sqrt{3}}\right)}=106$ A. From Table 10, we need a cable size AWG 2. From Table 11, we have:

| $\mathrm{R} 1=0.987 \mathrm{Ohm} / \mathrm{mile}$ | $\mathrm{X} 1=0.165 \mathrm{Ohm} / \mathrm{mile}$ |
| :--- | :--- |
| $\mathrm{R} 0=6.99 \mathrm{Ohm} / \mathrm{mile}$ | $\mathrm{X} 0=0.273 \mathrm{Ohm} / \mathrm{mile}$ |

This branch is long 200 ft , hence the data is:
$\begin{array}{ll}\mathrm{R} 1=0.987 * 200 / 4970=0.0397 \text { Ohm } & \mathrm{X} 1=0.165 * 200 / 4970=0.0066 \text { Ohm } \\ \mathrm{R} 0=6.99 * 200 / 4970=0.2813 \mathrm{Ohm} & \mathrm{X} 0=0.273 * 200 / 4970=0.0110 \mathrm{Ohm}\end{array}$

## Branch from Bus 14 to 15

This branch connects the secondary busbar of the transformer in the warehouse with the 208V panel in the office building. From this panel, cables will be directed to the two floors and the perimetral lights. The overall load for this branch is 120 KW and the estimated worse power factor of 0.9 translates the requirement for the apparent power to 133 KVA . The cable is sized to take care of future administrative needs of the office and rated for 150 KVA . Each phase must deliver 50KVA and carry $\frac{50}{\left(\frac{0.208}{\sqrt{3}}\right)}=416$ A. From Table10, we need cable size MCM 600. From Table 11, we obtain the following data:

```
\(\mathrm{R} 1=0.113 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 1=0.122 \mathrm{Ohm} / \mathrm{mile}\)
\(\mathrm{R} 0=2.74 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 0=0.197 \mathrm{Ohm} / \mathrm{mile}\)
```

The length of this branch spanning from the warehouse to the office is 100 ft , and the data is:
$\mathrm{R} 1=0.113 * 100 / 4970=0.0023 \mathrm{Ohm} \quad \mathrm{X} 1=0.122 * 100 / 4970=0.0025 \mathrm{Ohm}$
$\mathrm{R} 0=2.74 * 100 / 4970=0.0551 \mathrm{Ohm} \quad \mathrm{X} 0=0.197 * 100 / 4970=0.0040 \mathrm{Ohm}$

## Branch from Bus 15 to First Floor Office Loads 208V

This floor has a mix of lights and computer related loads. The overall demand is 55 KW and it is mainly at 120 V , which means that cables will run singularly inside each duct due to the single phase nature of the loads. With the extra quota considered, the apparent quota requirement for this floor is 70 KVA . Each phase must deliver 23.3 KVA and carry $\frac{23.3}{\left(\frac{0.208}{\sqrt{3}}\right)}=194$ A. From Table 10, we need cable size AWG 3/0. From Table 11, with one conductor per conduit, we have:
$\mathrm{R} 1=0.392 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 1=0.134 \mathrm{Ohm} / \mathrm{mile}$
$\mathrm{R} 0=5.46 \mathrm{Ohm} / \mathrm{mile} \quad \mathrm{X} 0=0.241 \mathrm{Ohm} / \mathrm{mile}$

The total length of this branch is 150 ft . Data is:
$\mathrm{R} 1=0.392 * 150 / 4970=0.0118 \mathrm{Ohm} \quad \mathrm{X} 1=0.134 * 150 / 4970=0.0040 \mathrm{Ohm}$
$\mathrm{R} 0=5.46 * 150 / 4970=0.1648 \mathrm{Ohm} \quad \mathrm{X} 0=0.241 * 150 / 4970=0.0073 \mathrm{Ohm}$

## Branch from Bus 15 to Second Floor Office Loads 208V

This branch is identical to the previous, with the only difference being that this cable is 30 ft longer because has to reach the second floor. The new length is 180 ft and data is:

| $\mathrm{R} 1=0.392 * 180 / 4970=0.0142 \mathrm{Ohm}$ | $\mathrm{X} 1=0.134 * 180 / 4970=0.0049 \mathrm{Ohm}$ |
| :--- | :--- |
| $\mathrm{R} 0=5.46 * 180 / 4970=0.1977 \mathrm{Ohm}$ | $\mathrm{X} 0=0.241 * 180 / 4970=0.0087$ Ohm |

## Branch from Bus 15 to Perimeter Lights

This branch provides power to the lights in the parking lots and around the perimeter of the industrial plant. Load is single phase, therefore the cable is run singularly for each conduit. The length is considered to be of 200 ft to the first light, then there are 4 subbranches of 100 ft each. At the end of each sub-branch there is a light. The power requirements for this cable are 10 KW , and with a power factor of 0.9 we have an apparent rating of $\frac{10}{0.9}=11.1 \mathrm{KVA}$. Each phase must deliver 3.7 KVA and carry $\frac{3.7}{\left(\frac{0.208}{\sqrt{3}}\right)}=31$ A . From Table 10, we need cable size AWG 8. From Table 7, with one conductor per conduit, we have:
$\mathrm{R}=0.07275 \mathrm{Ohm} / 100 \mathrm{ft} \quad \mathrm{X}=0.00468 \mathrm{Ohm} / 100 \mathrm{ft}$

For the first part, 200ft long, we have the data:
$\mathrm{R}=0.07275 * 2=0.1455 \mathrm{Ohm} \quad \mathrm{X}=0.0094 \mathrm{Ohm}$

For each of the sub-branches of 100 ft in length we have the data:
$\mathrm{R}=0.07275$ Ohm $\quad \mathrm{X}=0.00468$ Ohm

### 3.3 Loads

The following section describes the loads in detail: each load is defined by where it is located, what ratings it has and by the connections with the distribution system. Every load has its own model and a brief explanation describes how to obtain the electrical parameters for all loads in the plant.

## Induction Motors

There are 10 Induction Motors in the whole plant, all located in the factory. Each IM has 50 KW rating. Nine motors will be represented with a series equivalent impedance model, and the tenth will be fully modeled. This machine is the last one on the first floor and will be represented by an induction motor mechanically coupled with a load. The torque of this load will be modeled in detail to be able to capture dynamics.

To find the parameters of the induction machines, we use the guidelines of [3]. The data required to begin the calculations are terminal voltage, active power and the pole pairs of the machine. In this case we assume that each machine has four poles. Our goal is to obtain the six parameters that fully define the single phase equivalent circuit of an induction machine as represented in Figure 7.


Figure 7. Single Phase Diagram of an Induction Machine
Resistances $r_{1}$ and $r_{2}$ are respectively the stator and rotor winding resistance, while $s$ is the slip at which the machine is operating. Different slips correspond to different mechanical powers delivered to the shaft. A slip of zero corresponds to the shaft rotating at the speed of the magnetic field, while a slip of one corresponds to a still shaft. Reactances $x_{1}$ and $x_{2}$ represent the leakage part of the flux in the machine, respectively on the stator and on the rotor windings. Reactance $x_{m}$ represents the magnetizing inductance, responsible for creating the main magnetic field of the machine.

When expressed in per unit of the machine, due to design constraints, the leakage inductance is always about 0.2 in per unit, traditionally split in equal amounts on the rotor and stator circuits. Therefore:
$x_{1}=0.1 \mathrm{pu}$
$x_{2}=0.1 \mathrm{pu}$

To obtain the values for the resistance terms, it is necessary to calculate the pole pitch term, quantity that can be known once the ratings of the machine are converted to horsepower. Since $1 \mathrm{Hp}=768 \mathrm{~W}$, then our machines are rated for 65 Hp . Remembering that $\mathrm{P}=4$, then the pole pitch results from:
$\tau_{P}=0.095\left(\frac{H p}{\left(\frac{P}{2}\right)^{2}}\right)^{\frac{6}{23}}$

Now it is possible calculate the resistive terms as:

$$
\begin{aligned}
& r_{1}=0.0033\left(\tau_{P}\right)^{-1} \\
& r_{2}=0.004\left(\tau_{P}\right)^{-1}
\end{aligned}
$$

The numerical values for the resistances are expressed in pu of the machine ratings. Figure 8 shows the values of the resistances versus the machine size.


Figure 8. Per Unit Values of Resistance Versus Machine Size

Therefore we have that:
$r_{1}=0.0168 \mathrm{pu}$
$r_{2}=0.0203 \mathrm{pu}$

The value for the magnetizing inductance can be found with a similar process, involving the pole pitch previously calculated:

$$
x_{m}=10\left(\frac{\tau_{P}}{\left(\frac{P}{2}\right)}\right)^{\frac{1}{2}}
$$

Figure 9 shows the plot of magnetizing inductance versus the size of the machine, and it results that:

$$
x_{m}=3.13 \mathrm{pu}
$$



Figure 9. Per Unit Magnetizing Inductance Versus Machine Size

We need to convert these per unit values in actual units of Ohms, operation that we can do by defining the base of the machine as:
$V_{B}=480 \mathrm{~V}$
$\mathrm{S}_{\mathrm{B}}=50 \mathrm{KW}$
$\mathrm{Z}_{\mathrm{B}}=\frac{V_{B}^{2}}{S_{B}}=4.608 \Omega$

Notice that the base power is taken to be the three phase active power rating of the machine, rather than the apparent power. This convention is used following the guidelines that brought to the formulas to calculate the values of the parameters in per unit. It is possible to find the values of the parameters in actual units as:

$$
\begin{aligned}
& x_{1}=0.4608 \Omega \\
& x_{2}=0.4608 \Omega \\
& r_{1}=0.0773 \Omega \\
& r_{2}=0.0938 \Omega \\
& x_{m}=14.44 \Omega
\end{aligned}
$$

We now have five out of the six parameters that fully describe the induction machine as represented in Figure 7: we still miss the numerical value of the slip. This value depends on the operating point of the machine. The equivalent impedance of the circuit as seen from the terminals is given by:

$$
Z_{e q}=r_{1}+j x_{1}+\left[j x_{m} / /\left(\frac{r_{2}}{s}+j x_{2}\right)\right]=r_{1}+j x_{1}+\frac{j x_{m} \frac{r_{2}}{s}-x_{m} x_{2}}{\frac{r_{2}}{s}+j\left(x_{m}+x_{2}\right)}
$$

The power at the terminals is:

$$
P=\operatorname{Re}\left\{\frac{V^{2}}{Z_{e q}}\right\}
$$

Figure 10 shows the plot of the active power as a function of the slip: there are two solutions for the slip when a power of 50 KW is demanded at the terminals, but induction machine theory says that the only stable operating point is on the right side of the maximum reached by the characteristic. Figure 11 shows the magnification of this part of the curve, allowing for the identification of the value of the slip as the machine is outputting rated power.


Figure 10. Active Power at the Terminals Versus the Slip.


Figure 11. Identification of Slip at Rated Power

The value of the slip is: $s=0.023$. Now we can convert the circuit of Figure 7 to the equivalent circuit represented in Figure 12.


Figure 12. Equivalent Circuit of the Induction Machine.

In numerical terms it is possible to find the equivalent terms as:
$r_{e q}=\operatorname{Re}\left\{Z_{e q}\right\}=3.6391 \Omega$
$x_{e q}=\operatorname{Im}\left\{Z_{e q}\right\}=1.8812 \Omega$

The resulting power factor of the induction machine represented by these parameters is given by:

$$
\cos (\varphi)=\cos \left[\tan ^{-1}\left(\frac{x_{e q}}{r_{e q}}\right)\right]=0.888
$$

Now it is possible to find:
$\mathrm{Q}=\mathrm{P} \tan (\mathrm{phi})=50 \tan (\operatorname{arcos}(0.888))=25.9 \mathrm{KVAR}$

The machines are fed from a 480 V busbar, therefore the parallel conductance and susceptance is:

$$
G_{L N}=\frac{P}{V^{2}}=0.217 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}_{\mathrm{LN}}=\frac{Q}{V^{2}}=0.1124 \mathrm{Ohm}^{-1}
$$

To better reflect the IM design, we represent it with a delta configuration rather than a wye connection as it is now. Indeed, a delta configuration guarantees that no ground path is given from this terminal, which is what happens with IM.

The equivalent conductance and susceptance when connected in delta is:
$G_{D}=\frac{G_{L N}}{3}=0.0723 \mathrm{Ohm}^{-1}$
$B_{D}=\frac{B_{L N}}{3}=0.0375 \mathrm{Ohm}^{-1}$

The magnitude of the admittance is: $Y_{D}^{2}=G_{D}^{2}+B_{D}^{2}=0.0066 \mathrm{Ohm}^{-2}$. Then, the parallel resistance and reactance connected at delta are:
$R=\frac{G_{D}}{Y_{D}^{2}}=10.9 \mathrm{Ohm}, \quad \mathrm{X}=\frac{B_{D}}{Y_{D}^{2}}=5.64 \mathrm{Ohm}$

## AC Systems

We have three AC systems in the plant: one per each building. Size of the AC system is tailored to the needs of the particular environment. In the factory there are two units of 30 KW each, one per floor. There is a centralized AC system of 40 KW for the larger warehouse. Then there is a smaller system of two conditioners of 15 KW , one per each floor in the office building. The AC system is intended to be connected to the 480 V system, though using a 280 V supply, since the load is connected line to ground. The load is three phase, each of the phases with the same amount of load. The connection is wye, without a grounding for the center of the star. To represent the conditioners with a series equivalent impedance, we only need to specify a representative power angle. For this kind of load it is safe to assume a 0.95 power factor. The impedances for the different kind of AC systems are:

30KW in factory:
$Q=P \tan (\mathrm{pf})=30 \tan (\operatorname{acos}(0.95))=9.8 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.1302 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.0425 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.0188 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=6.9395 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=2.2669 \mathrm{Ohm}$

40KW in warehouse:
$Q=P \tan (\mathrm{pf})=40 \tan (\operatorname{acos}(0.95))=13.1 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.1736 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.0569 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.0334 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=5.202 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=1.7037 \mathrm{Ohm}$

15KW in office:
$Q=P \tan (\mathrm{pf})=15 \tan (\operatorname{acos}(0.95))=4.9 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.0651 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.0213 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.0047 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=13.879 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=4.5338 \mathrm{Ohm}$

## Elevators

Elevators are present in all the environments where lifting heavy materials from different floors is routine. The factory has two set of elevators, while the warehouse has only one. The size of the elevator is the same for all of them and is of 30 KW . Elevators always operate at 480 V and are synchronous machines that are responsible to lift the cabin as demanded. These machines are wye-connected with a floating center, that is, no grounding is provided at the center of the star. This is to reflect asynchronous machine design, with coils connected at wye, but with no direct path to ground. Therefore they can be represented by a series equivalent impedance model. Synchronous machines can be internally compensated to a nearly unitary power factor. Here, we assume a power angle of 0.97 for the elevators:
$Q=P \tan (\mathrm{pf})=30 \tan (\operatorname{acos}(0.97))=7.5 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.1302 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.0326 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.018 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=7.2282 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=1.8071 \mathrm{Ohm}$

## Lights

Each environment has its own light system: there are four different kind of lights. In the factory each floor has 15 KW , the warehouse has 30 KW and the office has 5 KW per floor. Then there are the perimetral lights all around the buildings and the parking lot. The external lights account for a load of 10 KW : external illumination is given by the sum of 5 lights of 2 KW each. Lights are very well internally compensated and have a power factor of 0.98 , their connection to the supply is always single phase, line to neutral. The level of voltage for the lights is always 120 V obtained with the line to neutral connection. The equivalent series circuit, supplied at 208 V is:

15KW in factory:
$Q=P \tan (\mathrm{pf})=15 \tan (\operatorname{acos}(0.98))=3 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.3467 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.013 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.1204 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=2.8802 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=0.1082 \mathrm{Ohm}$

30KW in warehouse:
$Q=P \tan (\mathrm{pf})=30 \tan (\operatorname{acos}(0.98))=6.1 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.6934 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.141 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.5007 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=1.3849 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=0.2816 \mathrm{Ohm}$

5KW in office:
$Q=P \tan (\mathrm{pf})=5 \tan (\operatorname{acos}(0.95))=1 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.1156 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.0231 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.0139 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=8.32 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=1.664 \mathrm{Ohm}$
$2 K W$ around the perimeter:
$Q=P \tan (\mathrm{pf})=10 \tan (\operatorname{acos}(0.95))=0.4 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.0462 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.0092 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.0022 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=20.8 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=4.16 \mathrm{Ohm}$

## Computers

Computers are only located in the office building, equally distributed on both floors. Each floor has a load of 50 KW related to computers, such as monitors, printers, faxmachines etc. Computers are connected to the 208 V line to neutral to obtain a utilization voltage of 120 V and are single phase loads. The computer loads are evenly distributed on the three phases. Assuming a power factor of 0.96 for the computers, then we have that the equivalent series impedance is:
$Q=P \tan (\mathrm{pf})=50 \tan (\operatorname{acos}(0.96))=14.5 \mathrm{KVAR}$

$$
\mathrm{G}=\frac{\mathrm{P}}{\mathrm{~V}^{2}}=1.1557 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{~V}^{2}}=0.3352 \mathrm{Ohm}^{-1}
$$

$$
Y^{2}=G^{2}+B^{2}=1.448 \mathrm{Ohm}^{-2}
$$

$R=\frac{G}{Y^{2}}=0.7982 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=0.2315 \mathrm{Ohm}$

## Factory Process

This is the load that represents the process that the factory is using. Mainly there are three phase and single phase rectifying bridges, to represent ASD's. This load amounts to 30 KW per floor at the voltage levels of 480 V and 280 V . It totals 20 KW per floor at the voltage levels of $240 \mathrm{~V}, 208 \mathrm{~V}, 120 \mathrm{~V}$. The simplest model to capture the behavior of the rectifying bridge is a R-L load. Assuming a power factor of 0.95 , then at the voltage level of 480 V (utilization voltage 280 V L-n) the impedance is:
$Q=P \tan (\mathrm{pf})=30 \tan (\operatorname{acos}(0.95))=9.8 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.1302 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.0425 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.0188 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=6.9395 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=2.2669 \mathrm{Ohm}$

At the voltage level of 208 V (utilization voltage 120 V L-n) the impedance is:
$Q=P \tan (\mathrm{pf})=20 \tan (\operatorname{acos}(0.95))=6.5 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=0.4623 \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=0.1502 \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=0.2363 \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=1.9565 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=0.6359 \mathrm{Ohm}$

## Capacitors

Capacitors are installed at the end of the 480 V branches that feed the induction machines. They are on the same bus where the last motor is located. They are wye connected to the ground. The goal is to compensate for the quota of reactive power adsorbed by the heavily inductive machinery. The compensation level is chosen to be 80 per cent. The total amount of reactive power adsorbed per each floor by the induction machines is:
$\mathrm{Q}=5 \mathrm{Q}$ _IM $=5 \mathrm{P} \tan (\mathrm{pf})=5 * 50 * \tan (\operatorname{arcos}(0.888))=129.5 \mathrm{KVAR}$

The capacitors must be able to compensate for:

Q_c $=0.8 \mathrm{Q}=103 \mathrm{KVAR}$

The amount of capacitive reactance needed is therefore:
$X_{-} c=\frac{V^{2}}{Q}=2.2246 \mathrm{Ohm}$

## Neighboring Plant

Not too far away, connected to the main 120KV grid, there is another industrial plant, with a 2MVA rating. The neighboring plant is introduced to capture the effects of short circuits in some electrical vicinity of the plant under study, in particular investigate how the sequence currents propagate from the neighbor plant to the industrial site under study. The power angle of the neighbor plant is 0.98 .
$Q=P \tan (\mathrm{pf})=2000 \tan (\operatorname{acos}(0.98))=400 \mathrm{KVAR}$
$\mathrm{G}=\frac{\mathrm{P}}{\mathrm{V}^{2}}=1.38 * 10^{-4} \mathrm{Ohm}^{-1}, \quad \mathrm{~B}=\frac{\mathrm{Q}}{\mathrm{V}^{2}}=2.82 * 10^{-5} \mathrm{Ohm}^{-1}$
$Y^{2}=G^{2}+B^{2}=2.0086 * 10^{-8} \mathrm{Ohm}^{-2}$
$R=\frac{G}{Y^{2}}=6914.9 \mathrm{Ohm}, \quad \mathrm{X}=\frac{\mathrm{B}}{\mathrm{Y}^{2}}=1404.1 \mathrm{Ohm}$

### 3.4 Micro-Sources

There are two basic classes of micro-source system; one is a DC source, such as fuel cells, photovoltaics, and battery storage, the other is a high frequency ac source such as the micro-turbine, which is then rectified. In both cases the DC source needs to be interfaced to the AC network using a voltage source inverter. The time constants of changes in power output for the micro-turbines and fuel cell range from 10 to 200 seconds. This slow response requires that the DC bus has adequate storage to insure fast response. For all systems studied it is assumed that there is a battery on the DC bus coupled to the AC system through an inverter as shown in Figure 13.


Figure 13. Interface Inverter System
As a minimum the inverter needs to control the flow of real and reactive power ( $\mathrm{P} \& \mathrm{Q}$ ) between the microsource and the power system. The $\mathrm{P} \& \mathrm{Q}$ are coupled with P predominantly dependent on the power angle, $\delta$, while Q is dependent on the magnitude of the converter's output voltage, V. It is also possible to independently control P and voltage E . The equations below indicate that for small values of $\delta_{P}$ and small difference in V and E , the real power P is proportional to $\delta_{P}$ and the reactive power Q depends on voltage difference.
$P=\frac{3 V E}{X} \sin \delta_{p}$
$Q=\frac{3 V}{X}\left(V-E \cos \delta_{p}\right)$
$\delta_{p}=\delta_{V}-\delta_{E}$

## Choice of Inductance

The inductance between the inverter terminals and the network must be carefully sized in order to be able to deliver the required amount of active and reactive power, without reaching instability. The condition of instability is reached whenever the angle $\delta_{P}$ between the two voltages approaches 90 degrees. It is common to design such inductor so that the ratings of the source can be delivered while never exceeding 30 degrees. This condition insures a desired property of all controllers: linearity. This is due to the fact that the $\sin$ function behaves almost linearly in the range $[0, \pi / 6]$.

If we assume that the inverter voltage magnitude, V can range from 0.6 to 1.2 pu , operating at $\delta_{P \text { max }}=30$ degrees and with the network voltage of 480 V , then we have that the active and reactive power injected into the network are as shown in Figure 14.


Figure 14. $P$ \& $Q$ Injected into the Network, Spanning $X$.

The above picture spans the values of the reactance from 0.1 to 0.5 pu. Given that the largest cluster of units has an overall rating of 300 KW , we have to insure that is it possible to deliver this amount of power with some margin. The value $\mathrm{X}=0.15 \mathrm{pu}$ seems to fit the needs. It is also useful to look at the deliverable powers when locking this desired value for the reactance to 0.15 pu and spanning the value of $\delta_{P}$, from 0 to 30 degrees.

Figure 15 shows the resulting map in the power plane. Each operating point (i.e. each pair of P and Q ) can be reached reading the corresponding voltage magnitude and angle difference that has to be applied. The points outside the map are not reachable without exceeding the voltage or angle constraints. The darker points in both Figures 14 and 15 represent the same points, indeed they are reached with $\mathrm{X}=0.15 \mathrm{pu}, \delta_{P}=30$ degrees and with $V$ spanning from 0.6 to 1.2 pu .


Figure 15. P \& Q Injected into the Network, Spanning $\delta_{P}$.

## Control Details

The inverters that supply power to an AC system in a distributed environment should have controls that are only based on information available locally at the inverter. In a system with many micro-sources, communication of information between units is impractical. Communication of information may be used to enhance system
performance, but must not be critical for system operation. Essentially, this implies that the inverter control should be based on terminal quantities.

It is essential to have good control of the power angle and the voltage level by means of the inverter. Control of the inverter's frequency dynamically controls the power angle, and the flow of the real power. To prevent overloading the inverter and the micro sources, it is important to ensure that load changes are taken up by the inverter in a predetermined manner, without communication.
From machine control theory [4], it results that it is much more stable to control the flux of the voltage, rather than the actual voltage. This continuos quantity is the time-integral of the inverter output voltage, often called the inverter flux vector $\psi_{V}$ :
$\psi_{V}(t)=\psi_{V}\left(t_{0}\right)+\int_{t_{0}}^{t} V d \tau$

The real power and the feeder voltage E are assumed to be the controlled quantities. Given set points for these quantities, $P_{o}$ and $E_{o}$, the inverter is controlled using the timeintegral of the d-q space vector. Input phase voltages are transformed to the stationary dq reference frame. The resulting $\mathrm{d}-\mathrm{q}$ components are time integrated, resulting in the flux vector, $\bar{\Psi}_{e}$ for the AC system voltage. The control system for the inverter is given in Figure 16. The two variables that are controlled directly by the inverter are the magnitude and phase of the flux $\bar{\Psi}_{v}$. The vector $\bar{\Psi}_{v}$, is controlled so as to have a specified magnitude and a specified position relative to the AC system flux vector $\bar{\Psi}_{e}$. This control forms the innermost control loop, and is very fast. The AC system voltage space vector is obtained from instantaneous voltage measurements and is available locally.


Figure 16. Detailed Inverter Control Scheme

The ideal model uses stiff sinusoidal voltage sources to represent the voltage generated by the inverter. This allows for faster computer simulation, since all the details of the matrix of switches and its firing scheme based on the choice of the switching vector does not need to be implemented. Multimachine cases can be studied easily in this case. Voltage flux magnitude is converted to voltage magnitude with a multiplication by $\omega_{o}$, while the angle difference is converted to angle of inverter source by adding it with the AC side angle, $\delta_{E}$.

In this case we have that the three phases voltages are:
$V_{a}=|V| \cos \left(\omega t+\delta_{v}\right)$
$V_{b}=|V| \cos \left(\omega t+\delta_{v}-\frac{2 \pi}{3}\right)$
$V_{c}=|V| \cos \left(\omega t+\delta_{v}+\frac{2 \pi}{3}\right)$

## Controls for Island Operation

During islanding operation, the unit has always the task to sustain the voltage, but also to provide the extra active power that the grid cannot give anymore. Requirements of the power electronics interface are:

- Provide fixed power and local voltage regulation
- Provide for fast load tracking using storage
- Incorporate "frequency droop" methods to insure load sharing between micro-sources in island operation without communications

The basic principle that lets the machines communicate without an explicit network that links them, is to allow the frequency at the inverter's terminal to change as a function of power demand. When two points in the network are operating at different frequencies there is an increase of active power delivery from the place at higher frequency to the location at lower frequency. As this happens, the two frequencies tend to drift towards a common central value and the new steady state is reached at a lower frequency than the system had when grid was connected. The basic equation that allows the droop to work is:

$$
\begin{equation*}
\omega_{i}^{*}(t)=\omega_{o}-m_{i}\left(P_{c, i}-P_{i}(t)\right) \tag{Eq. 1}
\end{equation*}
$$

Figure 17 shows the details for the droop governor. This governor [5] has two important characteristics: first it allows to maintain any desired value of power when the AC grid is connected, second, it slowly brings up the frequency near the customary $\omega_{0}$ value after the droop regulation has taken place.

While power dispatchment takes place in a time scale of fraction of seconds, the frequency restoration may take tens of seconds to reach its goal. The zero error condition in the integrator input block gives the steady state frequency that one can obtain. As the droop regulation may decrease the frequency of some fractions of Hz , the integrator block will make sure that at steady state the deviation from nominal frequency is very small.


Figure 17. Power with Frequency Droop
Traditionally, the value of $m$ was constant, and chosen in such a way that every machine will pick up the extra quota of power proportionally to its own rating. Larger machines will inject more power regardless their operating point before the islanding. Our control behaves differently, since new power is given in such a way to cancel the power flow on some key branches of the network. This means that local demand is met by local generation, and there will be no flow of power in the transformers and in the cable that connects the factory with the warehouse and office. This goal is pursued to lower transmission losses in the system.

As a consequence of this choice, the coefficient $m$ results dependent on the local setpoint before islanding, $P_{0 i}$, and on the new setpoint, to be reached after the grid has failed, $P_{1 i}$ :

$$
\begin{equation*}
m_{i}=\frac{\omega_{O}-\omega_{\min }}{P_{0, i}-P_{1, i}} \tag{Eq. 2}
\end{equation*}
$$

Figure 18 gives the characteristic of the droop regulation when only two machines are present. Machine 2 operates at higher output, than unit 1 . As the system enters island mode, the frequency will reduce. With the full load of the system, the new frequency will be $\omega_{\min }$, while if with a lighter load, the new frequency will be somewhere in between $\omega_{0}$ and $\omega_{\text {min }}$. The slower frequency restoration loop will rigidly translate the characteristics upwards, until $\omega_{1} \approx \omega_{o}$.


Figure 18. P- $\omega$ Droop Characteristic
It is very important that the power stays constant at the new dispatched level during the frequency restoration phase. The droop power control results in a single new steady state point with a lower frequency $\omega_{1}$ at $t=t_{1}$ along with a set of different power levels $P_{i}\left(t_{1}\right)$ for each power source. A control loop is used to uniformly bring the island systems frequency near to $\omega_{0}$ while holding the power levels of each source fixed at $P_{i}\left(t_{1}\right)$. The rate of changes of frequency for each source must also be held equal to insure fixed power angles between sources. This basic condition implies:
$\frac{d \omega_{i}}{d t}=\frac{d \omega_{j}}{d t} \quad \forall i, j$
From Eq. 1, assuming $P_{i}(t)$ is constant for $t \geq t_{1}$, the rate of change of speed is:
$\frac{d \omega_{i}}{d t}=-m_{i} \frac{d P_{c, i}}{d t}$
Eq. 4
Assuming again that $P_{i}(t)$ is constant for $t \geq t_{1}$, then for $t \geq t_{1}$ it is possible to write:
$\omega_{i}\left(t_{1}\right)-\omega_{i}(t)=m_{i}\left(P_{c, i}\left(t_{1}\right)-P_{c, i}(t)\right)$
Eq. 5
Since the frequency restoration loop is much slower than the droop controller, it can be assumed that:

$$
\begin{equation*}
P_{c, i}\left(t_{1}\right) \approx P_{0, i} \tag{Eq. 6}
\end{equation*}
$$

From the integral block of Figure 9 we can write:

$$
\begin{equation*}
\frac{d P_{c, i}}{d t}=k^{\prime \prime}\left[\left(\omega_{o}-\omega_{i}\right)+k_{i}^{\prime}\left(P_{0, i}-P_{c, i}\right)\right] \tag{Eq. 7}
\end{equation*}
$$

Substitution of Eq. 4,5,6 and 7 yields:

$$
\frac{d \omega_{i}}{d t}=-m_{i} k^{\prime \prime}{ }_{i}\left[\left(\omega_{o}-\omega_{i}\right)+\frac{k_{i}^{\prime}}{m_{i}}\left(\omega_{i}-\omega_{1}\right)\right]
$$

We want the r.h.s. of Eq. 8 to be equal for all sources or the terms $m_{i} k^{\prime \prime}{ }_{i}$ and $k_{i}^{\prime} / m_{i}$ must be equal for all sources. This implies that gains $k_{i}^{\prime}$ and $k^{\prime \prime}{ }_{i}$ are dependent on the set-point $P_{0, i}$ and $P_{1, i}$ of each system, Figure 18. If a machine changes its set-point, it doesn't need to communicate to the other machines for this procedure to be successful.

## 4. System Analysis

The analysis of the performance of the micro-sources in the industrial site is aimed at both steady state and dynamic aspects. Transient analysis is fundamental for understanding the dynamics of the events after a perturbation in the system, and steady state analysis is important for quantifying operating points with performance indexes such as overall losses, voltage levels and the ratings needed by the units to operate at this point. The tools used to carry the steady state and transient analysis will be described first, and then the results from both analysis will be displayed and commented.

## Steady State Analysis

Steady state analysis is the fundamental tool to begin the study of the system. Voltage profiles and power flows can give insights on where to install the DR's and what ratings these units should have. The units will need to have a combined rating to be able to supply all the loads while in island mode. These loads are all of the plant loads besides the non critical loads. During grid connection they are required to provide part of the active power demand and regulate voltages.

The power flow problem is formulated by assigning a number to each of the buses of the system: there are a total of 43 buses. There is a set of significative branches that are indicative of the system behavior: they are mainly the cables that distribute the power along the plant, avoiding the details of the sub-sections that feed separate loads. The system overall losses are evaluated from the transformer T1 down to the rest of the distribution in the industrial site. The losses in the 120 KV network are not included in the number reported with the results. There are four locations where DR are installed and each one has more than one unit. We have clusters of micro-sources because of the high power demand of the plant versus the small typical rating of the units. In this report we assume that we are operating with micro-sources that have 75 KW rating of active power. The size of clusters are arranged as shown in Table 12.

| Bus Number | Micro-Sources in Cluster | Total Available Power <br> $[\mathrm{KW}]$ |
| :---: | :---: | :---: |
| 16 | 4 | 300 |
| 22 | 4 | 300 |
| 8 | 3 | 225 |
| 11 | 2 | 150 |

Table 12. Available Power at Each Cluster Location.

The large clusters of 300 KW (bus 16 and 22) are located at the first and second floor of the factory and help feeding the induction machine power demand. The cluster in bus 8 is located at the secondary of the transformer T2, inside the factory. The two units in bus 11 are located in the warehouse, on the high side of the transformer located in the same building.

Distributed Resources can control the active power injection and the local bus voltage magnitude, and the decision on the setpoints can affect other quantities, such as network overall losses and unit ratings in terms of KVA. The total available KW must be enough to power the critical load in island mode, with some level of redundancy.

## Analysis Tool

The mathematical tool used to perform the steady state analysis of the electrical system is the power flow. It is a very know technique in power system analysis [6], and a brief outline of its basic principle will be given here.

For each bus of the network we can associate 4 parameters that are:

- Voltage magnitude
- Angle of the voltage
- Active power injected into this bus from outside the network
- Reactive power injected into this bus from outside the network.

If we have a network with $n$ buses, then we have $4 n$ independent variables that fully describe it.

Kirchoff's laws demand that the powers injected into a bus always sum up to zero. Therefore, by summing the power injected from the network and from outside the network on each bus it is possible to write two real equations per bus, respectively one for the active and reactive power. With an $n$ buses system, we would have $2 n$ overall equations: below the two equations relative to a generic bus are reported:

$$
\begin{aligned}
& \sum P_{n e t, i}+P_{e x t, i}=0
\end{aligned}
$$

$$
\begin{aligned}
& \sum Q_{n e t, i}+Q_{e x t, i}=0 \\
& \text { where : } \left.\sum Q_{\text {net }, i}=\operatorname{Im}\left\{\sum_{\substack{j \text { ranging over } \\
\text { all adjacent buses }}}^{\underset{\mathrm{V}}{\mathrm{i}}} \stackrel{\left.\stackrel{\mathrm{r}}{V_{j}}\right)}{\mathrm{r}}\right) Y_{i j}^{\mathrm{r}}\right\}
\end{aligned}
$$

If for each bus of the network we specify two of the describing parameters (i.e. $\mathrm{V}, \delta$, $P_{e x t}, Q_{\text {ext }}$, then the system can be solved with a unique solution. Indeed, in a system of $n$ buses we would have $4 n-2 n=2 n$ unknowns, related by $2 n$ equations. Buses are usually divided in three main categories:

- Slack Generation Bus: V and angle $\delta$ are specified
- Load Buses: external P and Q (negative) injections are specified
- Remaining Generation Buses: P injected and V are specified

Only one generator can be classified as slack to fulfill the requirement of a reference for the angles. Indeed, all the angles are measured relative to this reference value. There can be as many load or generation buses as one desires. The system of equations is solved using Newton-Raphson iterative method implemented using MATLAB [7].

## Transient Analysis

Transient analysis is important because it gives the response of the system during changes from one steady state to another. Events of great importance are the units going on service, starting injecting power and regulating voltage. Another important event that has to be studied is when the connection to the grid fails and the units have to provide the full amount of power to the critical loads. The micro-sources are interfaced with the network by means of a power electronics inverter. In general, there is a DC voltage source which must be converted to an AC voltage or current source at the required frequency, magnitude and phase angle. In most cases the conversion will be performed
using a voltage source converter with a possibility of phase width modulation to provide fast control of voltage magnitude. This creates a very different situation when compared to synchronous generators. Fundamental frequency in a converter is created using an internal clock which does not change as the system is loaded.

Power electronic interfaces introduces new control issues and new possibilities. A system with clusters of micro-generators and storage could be designed to operate in both an island mode and as a satellite system connected to the power grid. In such systems load swings become a major issue. A step load will affect the system. In the case of a power electronics based source the DC bus voltage will decrease in response to the added load. Micro-turbines and fuel-cells have regulators which use the decrease in DC voltage to increase their power output. Regulator response could be as slow as one to two minutes requiring some form of storage during this period.

Basic system problems include the control of the power feeder from the grid, speed of response of the micro-source, load sharing and tracking among the distributed resources, reactive power flow and power factor control and steady state and transient stability.

To obtain time domain traces of electrical quantities for the industrial system under study we use the EMTP software - ElectroMagnetic Transient Program [8]. The EMTP is able to model the details of the network, along with the details of the controller for the microsources. The voltage vector flux control as described in Section 3.4 is implemented, along with the power-frequency droop and frequency restoration blocks.

## System Without Micro-sources

This case is used as a benchmark to see what is the load flow and the voltage profile in the current configuration of the industrial site, when no micro-sources are installed. The voltages fall quite low in some point of the network and losses are quite high. Table 13 summarizes the results for this case. Notice that the voltages of the buses with number larger than 29 have a magnitude that is lower than 0.94 per unit. That is a rather low value for some loads to operate at, and would leave very little room for voltage sags.

| Bus Number | Voltage [pu] |
| :---: | :---: |
|  |  |
| 1 | 0.9978 |
| 2 | 0.9944 |
| 3 | 0.9825 |
| 4 | 0.9747 |
| 5 | 0.9726 |
| 7 | 0.9703 |
| 8 | 0.9724 |
| 9 | 0.9709 |
| 10 | 0.9666 |
| 11 | 0.9572 |
| 12 | 0.9542 |
| 13 | 0.9482 |
| 14 | 0.9546 |
| 15 | 0.9469 |
| 16 | 0.9689 |
| 22 | 0.9681 |
| 28 | 0.9480 |
| 29 | 0.9551 |
| 30 | 0.9236 |
| 31 | 0.9201 |
| 32 | 0.9452 |
| 33 | 0.9391 |
| 34 | 0.9378 |
| 35 | 0.9282 |
| 36 | 0.9316 |
| 37 | 0.9285 |


| Branch |  | S k->m [KW +j KVAR] | Branch Losses [ $\mathrm{KW}+\mathrm{j}$ KVAR] |
| :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |
| 5 | 16 | $263.0509+$ j 35.3397 | $0.87197+$ j 1.1951 |
| 5 | 22 | $262.8038+\mathrm{j} 35.50649$ | $1.0645+\mathrm{j} 1.4191$ |
| 8 | 7 | $111.2814+\mathrm{j} 31.92453$ | $0.23984+\mathrm{j} 0.079947$ |
| 9 | 10 | $62.6376+$ j 12.7065 | $0.26045+\mathrm{j} 0.13001$ |
| 8 | 11 | $226.4448+$ j 66.45747 | $2.9157+\mathrm{j} 3.1454$ |
| 11 | 12 | $63.6997+$ j 18.679 | $0.20679+$ j 0.035586 |
| 11 | 13 | $26.9941+\mathrm{j} 8.66442$ | $0.27701+\mathrm{j} 0.014824$ |
| 14 | 33 | $26.5869+$ j 5.37867 | $0.74089+\mathrm{j} 0.12321$ |
| 14 | 15 | $106.0658+\mathrm{j} 29.80935$ | $0.70864+\mathrm{j} 0.76991$ |

Total losses $=36.9433 \mathrm{KW}$

| Source | $\mathrm{P} \quad[\mathrm{KW}]$ | $\mathrm{Q} \quad[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Flow from T1 | 946.7871 | 204.3681 | - |
| DR at Bus 16 | - | - | - |
| DR at Bus 22 | - | - | - |
| DR at Bus 8 | - | - | - |
| DR at Bus 11 | - | - | - |

Table 13. Results Without the Micro-sources

### 4.1 Micro-Source Power setpoints.

In this section we are examining the case when the control regulates the power injected by the unit and the voltage at the local point of connection with the feeder to a desired amount. To this end, the active power injected and the feeder voltage are measured and passed back to the control loop. Figure 19 shows the diagram of this control: the measures of power injected by the unit and voltage E are given to the governor that regulates them to desired values. These values can be chosen independently from the setpoints that a nearby cluster of unit may have.


Figure 19. Diagram of Independent Power Setpoints.
In this case we have no means of controlling the power that comes from the grid: we can indirectly changing it by choosing different setpoints for the power of the unit, but we can't plan ahead the amount of power that is drawn from the AC system. With this system of regulating the power, it is impossible to sign a contract with the utility agreeing on a determined amount of power to be taken: the units can only control their power output, without any notion of the power from the grid. The choice of power dispatched by the units when the grid is connected is arbitrary and dependent on the choices of the industry.

The system can operate in island mode because of the power-frequency droop control, that will use the droop in frequency subsequent to the grid failure to regulate power among the units. The frequency will be restored in a short time frame after the regulation has taken place. The value of power injected by the unit during island mode can be chosen on beforehand, based on considerations about losses. Figure 20 shows the power frequency characteristics of two units, with the system transferring to island. First, there is the regulation of power as the frequency drops ( A and $\mathrm{A}^{\prime}$ ) and then the frequency is slowly restored to the nominal value while maintaining the power at the new regulated point ( B and B ').


Figure 20. Droop Characteristic with Frequency Recovery.
The units are operating with a desired point $P_{0, i}$ that is locally chosen. The new operating point for the power when in island, $P_{1, i}$, is chosen as to minimize the system losses. This point is chosen before the islanding occurs, and is only an approximation to losses minimization, the reason for this is because the load burden on the system at the time of the islanding is not know when the $P_{1, i}$ setpoints are chosen, before the islanding occurs. To be able to make a choice of power command, we suppose that the system is fully loaded. During transferring to island the non sensitive loads will be dropped by the system, while all of the sensitive loads are connected. The micro-sources will have to inject all the requested power, since there is no more a connection with the grid.

Therefore, the sum of the power injected by the units must equal to the total load, plus the system losses. These losses can be somewhat controlled by intelligently picking up the desired amount of power generated by the units. To minimize losses it is important to minimize the transfer of power from different feeders on the isolated micro-grid, and to this end, we need to generate on the feeder the full amount of power required by the local loads, i.e. the loads hanging off the feeder.

This justifies also the choice of number of units per cluster, as of Table 12. There are 4 units on each of the plant's floor where the induction machines are located to make sure that we could meet all the power request from the machines. The other clusters don't need to be so largely sized since the loads hanging off in the immediate vicinity are not so large. The warehouse and the office have small loads when in island mode due to the fact that there are many non sensitive loads in this part of the plant that are being shed. That is why the cluster in the office is only of two units. The reason why the cluster on bus 8 has three units is to allow for redundancy in the system in case of a loss of one of the micro-sources anywhere in the isolated grid.

## Connected to the grid

## Steady State Studies

In this case, we have four clusters of units at the buses $8,11,16$ and 22 . Each one is injecting power and the network provides the remaining quota of power to meet the rest of the plant's needs. The units are operating at about half of their nominal active power output. Figure 21 shows the general view of the factory plant with the unit locations. All loads are connected to the feeders and the connection with the grid is active.


Figure 21. Factory Diagram, showing Cluster Locations.
The units are not regulating at 1.00 per unit voltage because that would require too many VARs. The overall VA ratings of the units appear to be reduced by allowing the voltage in the warehouse to be regulated at 0.98 pu , while maintaining the requested voltage in all other clusters at 0.99 pu .

If we requested all the units to regulate voltage at 0.99 pu we would notice that the unit on Bus 11 needs lots of reactive power to hold its voltage. By allowing the regulated voltage on Bus 11 to be just 1 percent in pu lower, that is 0.98 pu , then the needs for reactive power are dramatically reduced. Table 14 summarizes the steady state results for this case. The losses are dramatically reduced and the voltage profile greatly improved when compared to Table 13 . The voltage profile is also improved by locally controlling the value of the voltages at the buses where the units are installed: now only two buses, namely 30 and 31 are below or around 0.94 pu.

For each of the units there is the corresponding injection of active and reactive power, plus a third column, with the KVA rating needed to obtain this very point of operation. The column with the rating is important because shows what are the requirements for the micro-source as the unit operates at a certain operating point: it gives us a yardstick to measure how demanding in terms of rating each point is.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | 0.9983 |
| 2 | 0.9990 |
| 3 | 0.9941 |
| 4 | 0.9901 |
| 5 | 0.9908 |
| 7 | 0.9878 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9769 |
| 13 | 0.9707 |
| 14 | 0.9773 |
| 15 | 0.9694 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9652 |
| 29 | 0.9724 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9676 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9503 |
| 36 | 0.9537 |
| 37 | 0.9506 |


| Branch |  | S k->m [KW +j KVAR] | Branch Losses [ $\mathrm{KW}+\mathrm{j}$ KVAR] |
| :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |
| 5 | 16 | 123.9277 - j 40.7859 | $0.20304+\mathrm{j} 0.27829$ |
| 5 | 22 | 123.9871 - j 51.18487 | $0.26246+\mathrm{j} 0.34989$ |
| 8 | 7 | $115.3511+\mathrm{j} 33.09207$ | $0.24861+\mathrm{j} 0.082871$ |
| 9 | 10 | $64.9283+$ j 13.1712 | $0.26997+\mathrm{j} 0.13476$ |
| 8 | 11 | $155.5926+\mathrm{j} 41.54142$ | $1.3098+$ j 1.413 |
| 11 | 12 | $66.7642+\mathrm{j} 19.5777$ | $0.21673+\mathrm{j} 0.037299$ |
| 11 | 13 | $28.2928+$ j 9.08125 | $0.29033+\mathrm{j} 0.015537$ |
| 14 | 33 | $27.866+$ j 5.63743 | $0.77654+\mathrm{j} 0.12914$ |
| 14 | 15 | $111.1685+\mathrm{j} 31.24344$ | $0.74273+\mathrm{j} 0.80695$ |

Total losses $=19.3535 \mathrm{KW}$

| Source | $\mathrm{P}[\mathrm{KW}]$ | $\mathrm{Q}[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: |
| Flow from T1 | 489.3545 | -113.3788 | - |
| DR at Bus 16 | 150.0 | 76.7 | 168.4 |
| DR at Bus 22 | 150.0 | 87.2 | 173.5 |
| DR at Bus 8 | 100.0 | 115.4 | 152.7 |
| DR at Bus 11 | 80.0 | 26.2 | 84.2 |

Table 14. Steady state with full load.

## Reduced load: loss of computers

This case is aimed to see what is the new operating point of the system when all the computer loads from the office building are turned off. The controller of the units regulates the power of the units so that the injected amount stays constant as loads are turned on or off. This implies that the remaining request of power is all met by the grid. In this case, we have that the grid will reduce the amount of power injected due to the computers being turned off.

Figure 22 shows the factory diagram for the case when the grid is connected and the computers are missing. The remaining part of the load is there and the units are operating at the same setpoint of the previous test.


Figure 22. Factory Diagram, without Computers, and with Grid.
Table 15 shows the steady state results of the system under the reduced load. Since the computers are located in the office building the voltages at both floors (bus 36 and 37) are improved and since the overall load is lower and smaller currents are required to supply them, also the total losses of the system are reduced when compared to Table 14.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | 0.9983 |
| 2 | 0.9982 |
| 3 | 0.9939 |
| 4 | 0.9907 |
| 5 | 0.9910 |
| 7 | 0.9878 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9769 |
| 13 | 0.9707 |
| 14 | 0.9792 |
| 15 | 0.9780 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9652 |
| 29 | 0.9724 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9676 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9521 |
| 36 | 0.9766 |
| 37 | 0.9763 |


| Branch |  | S k->m [KW +j KVAR] | Branch Losses [ $\mathrm{KW}+\mathrm{j}$ KVAR] |
| :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |
| 5 | 16 | 123.9164 - j 26.97529 | $0.19176+\mathrm{j} 0.26283$ |
| 5 | 22 | 123.9716 - j 39.57502 | $0.24692+\mathrm{j} 0.32917$ |
| 8 | 7 | $115.3511+\mathrm{j} 33.09207$ | $0.24861+\mathrm{j} 0.082871$ |
| 9 | 10 | $64.9283+\mathrm{j} 13.1712$ | $0.26997+\mathrm{j} 0.13476$ |
| 8 | 11 | $62.72301+\mathrm{j} 127.3353$ | $1.0176+\mathrm{j} 1.0978$ |
| 11 | 12 | $66.7642+$ j 19.5777 | $0.21673+\mathrm{j} 0.037299$ |
| 11 | 13 | $28.2928+$ j 9.08125 | $0.29033+\mathrm{j} 0.015537$ |
| 14 | 33 | $27.9752+\mathrm{j} 5.65951$ | $0.77958+\mathrm{j} 0.12965$ |
| 14 | 15 | $18.6525+\mathrm{j} 3.68552$ | $0.020057+\mathrm{j} 0.021791$ |

Total losses $=14.3163 \mathrm{KW}$

| Source | $\mathrm{P}[\mathrm{KW}]$ | $\mathrm{Q}[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: |
| Flow from T1 | 394.1965 | -34.08861 | - |
| DR at Bus 16 | 150.0 | 63.3 | 162.8 |
| DR at Bus 22 | 150.0 | 75.6 | 167.9 |
| DR at Bus 8 | 100.0 | 143.4 | 174.8 |
| DR at Bus 11 | 80.0 | -93.1 | 122.7 |

Table 15. Grid connection, without the computer

## Reduced load: loss of an Induction Machine

This test is aimed to study the system under reduced load, and is a very similar scenario to the one carried on in the previous pages: here one induction machine is turned off starting from the full load case. The machine, of 50 KW rating, is on bus 21 and it is the last machine of the first floor of the factory.


Figure 23. Factory Diagram, without an Induction Machine, and with Grid.
Figure 23 shows the factory diagram, with the grid connection, the cluster of units and the induction machine at the bus 21 that is lost: the rest of the load is still connected.

The steady state operating point is summarized in Table 16, where it is shown that the cluster of units do not change their operating point in terms of active power injection or regulated voltage magnitude.

It is possible to see that the voltages in proximity to the missing induction machine are improved, and the total losses of the system are reduced. The single machine load is lower than all the computers, so although losses are lower than in Table 14, they are larger than in Table 15.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | 0.9983 |
| 2 | 0.9985 |
| 3 | 0.9940 |
| 4 | 0.9904 |
| 5 | 0.9907 |
| 7 | 0.9878 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9769 |
| 13 | 0.9707 |
| 14 | 0.9773 |
| 15 | 0.9694 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9652 |
| 29 | 0.9724 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9676 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9503 |
| 36 | 0.9537 |
| 37 | 0.9506 |


| Branch |  | S k->m [KW +j KVAR] | Branch Losses [ $\mathrm{KW}+\mathrm{j}$ KVAR] |
| :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |
| 5 | 16 | 75.2482 - j 8.58169 | $0.068428+\mathrm{j} 0.093789$ |
| 5 | 22 | 123.9912 - j 53.84949 | $0.26659+\mathrm{j} 0.35539$ |
| 8 | 7 | $115.3511+\mathrm{j} 33.09207$ | $0.24861+\mathrm{j} 0.082871$ |
| 9 | 10 | $64.9283+$ j 13.1712 | $0.26997+\mathrm{j} 0.13476$ |
| 8 | 11 | $155.5926+\mathrm{j} 41.54142$ | $1.3098+$ j 1.413 |
| 11 | 12 | $66.7642+$ j 19.5777 | $0.21673+\mathrm{j} 0.037299$ |
| 11 | 13 | $28.2928+$ j 9.08125 | $0.29033+\mathrm{j} 0.015537$ |
| 14 | 33 | $27.866+$ j 5.63743 | $0.77654+\mathrm{j} 0.12914$ |
| 14 | 15 | $111.1685+\mathrm{j} 31.24344$ | $0.74273+\mathrm{j} 0.80695$ |

Total losses $=17.7884 \mathrm{KW}$

| Source | $\mathrm{P} \quad[\mathrm{KW}]$ | $\mathrm{Q} \quad[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: | :---: |
| Flow from T1 | 439.2489 | -69.69356 | - |
| DR at Bus 16 | 150.0 | 18.6 | 151.1 |
| DR at Bus 22 | 150.0 | 89.9 | 174.8 |
| DR at Bus 8 | 100.0 | 97.7 | 139.8 |
| DR at Bus 11 | 80.0 | 26.2 | 84.1 |

Table 16. Grid connection, without an induction machine

## Details of Dynamic Study

The units are originally off-line and turned on-line one cluster at a time. All the units within the same cluster are supposed to be turned on at the same time. At $t=0$, all the units are off line, they inject zero active and reactive power in the network. At $\mathrm{t}=1 \mathrm{sec}$ the
cluster at Bus 16 is brought on line, regulating injected power and voltage magnitude at the desired values. Then 3 seconds apart from each other the clusters at Bus 22, 11 and 8 are brought on line. In Figure 24 it is possible to see that the powers match the steady state desired points as of Table 14. Figure 25 shows the reactive power, while Figure 26 shows the voltage profiles: in both these Figures it is still possible to notice the correspondence with the data of Table 14. In Figure 25 it is possible to notice the large demand in reactive power for units on the Bus 16 . This is the first cluster to be turned on, and has to regulate voltage without any other neighboring clusters helping it. It turns out that this high reactive demand does not surpass the rating of the unit because of the corresponding low active power request: the cluster is regulating to 150 KW out of 300 KW available, leaving lots of VA available to be used for the voltage regulation.

In Figure 26, the voltage profiles are shown in two different scales: the upper plot from 0 to 1 pu , while the second scale is magnified around the values that the voltages assume. The upper plot shows the relative small magnitude of the voltage excursion during the regulation, while the enlarged plot gives a better idea on how the voltages settle to the requested values as the units are connected to the micro-grid. Notice that the voltage profiles for $t \leq 1$ match the steady state voltage of Table 13, when no units are present.


Figure 24. Micro-Sources Active Power Injection, Startup


Figure 25. Micro-Sources Reactive Power Injection, Startup.


Figure 26. Regulated Buses Voltages, Startup.
Figure 27 shows the power provided by the grid while the units are turned on one at a time. The power provided from the grid almost halves, backing off from almost 1 MW to near 500 KW , when all the units are in service and regulating power according to our choice.


Figure 27. Power from the Grid, Startup.

## Reduced load: loss of computers

The case of loss of computers was studied under the transient analysis and the system response is displayed in the following figures. Figure 28 shows the active power injected by the units after the computers are turned on: there is a small transient around the event, but the regulated power after the computer loss is fixed to the same amount it had before, equal to the desired value.


Figure 28. Active power, with grid, computers turning off.

The reactive power injection is shown in Figure 29, where it is possible to see that in order to maintain the desired voltages at the local point of connection with the feeders the units have to inject different amounts of reactive power than they did with the full load. Since the computers, amounting to about 100 KW , are located near the cluster of microsources at bus 11 , then it is this bus more than any other that sees the voltage coming up as the loads is turned off. As a consequence of this, the reactive power injected by this cluster is of inductive nature, that is, if the Q injection was zero, the voltage would be higher than the requested amount.

Figure 30 shows the voltages during the event: all of them rise as a consequence of the lighter load in the system, but the regulator ensures that they are brought back to their desired values they had before, with the full load.


Figure 29Reactive power with computers turning off


Figure 30. Voltage with grid, computers turning off.

Figure 31 shows the active power that is provided by the grid, through the transformer T1: as the load diminishes, the grid has to inject less power, since the local distributed generation does not change its requested output power.


Figure 31. Power injected by the grid, when computers turn off.

## Reduced load: loss of an Induction Machine

The dynamic behavior is shown in the following Figures. Figure 32 shows the active power injection of the units as the IM at bus 21 is turned off: after a short transient, the units return to their previous desired amount of output power. The deviation of power from the steady state values are reduced compared to the case when the computers where turned off (Figure 28), due to the fact that one IM is roughly half of all the computer loads.

Figure 33 shows the reactive power injections, aimed to maintain a fixed voltage at the bus on the feeder. The overall reduced load implies a smaller reactive injection of reactive power to support the voltage. The cluster at bus 16, being the one electrically nearest to the dropped induction machine, is the one that is most affected by the event. Notice that although the reactive amount needed for the regulation is greatly reduced, the source still has to inject capacitive power, meaning that if Q was zero the voltage would be lower than the requested value.


Figure 32. Active power, with grid, IM turned off.


Figure 33. Reactive power, with grid, IM turned off.
Figure 34 shows the amount of active power that comes from the grid: since the units regulate to constant output power and the load is reduced by an induction machine, the overall power needed from the AC system is reduced. Notice that the reduction here is smaller than in Figure 31, where we were turning off a larger load, i.e. both floors of computer loads in the office building.


Figure 34. Power injected by the grid, when one IM is turned off.

## Island Mode

Whenever the connection with the main grid fails, the micro-sources offer the possibility of operating in island mode. The plant needs to shed the non-sensitive loads and the units need to inject more active power. The cluster of units at the bus 8 is chosen to represent the slack bus, while the power of the remaining clusters regulate so that the units generate only the power needed locally. Table 17 summarizes the results: the flows of power in some of the distributing cables is reduced to almost zero and the losses are also reduced. The setpoints for the units in steady state is aimed to zero out the power in the following branches:

Branch from Bus 5 to 16
Branch from Bus 5 to 22
Branch from Bus 4 to 8
Branch from Bus 8 to 11
Figure 35 shows the island mode diagram, where the power must all be provided locally. The non sensitive loads (NSL) are dropped as the main connection with the grid fails: the branches where we specify the power flow to be zero during islanding operation are also shown.


Figure 35. Island Mode, with Zero Power Flow in some Branches.
The reduction in the overall system losses is partially due to the choice of the power setpoints for the micro-sources that minimizes cable losses, but also due to the fact that there are fewer loads in the network without the non sensitive loads, and therefore an overall reduced current magnitude in the cables. Figure 36 shows the droop characteristic used in this example: each of the four clusters has its own regulating curve.


Figure 36. Power-Frequency Droop used in this Study.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | - |
| 2 | - |
| 3 | - |
| 4 | 0.9900 |
| 5 | 0.9900 |
| 7 | 0.9889 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9800 |
| 13 | 0.9707 |
| 14 | 0.9779 |
| 15 | 0.9706 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9662 |
| 29 | 0.9836 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9800 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9779 |
| 36 | 0.9549 |
| 37 | 0.9518 |


| $\backslash$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Branch |  | S k->m [KW +j KVAR] | Branch Losses [KW +j KVAR] |
| Bus_k | Bus_m |  |  |
| 5 | 16 | $0.0003+$ j 0.00023 | 0. +j 0 . |
| 5 | 22 | $0.0003+\mathrm{j} 0.00025$ | 0. +j 0 . |
| 8 | 7 | $57.9639+$ j 18.7483 | $0.06407+\mathrm{j} 0.021357$ |
| 9 | 10 | $64.9283+$ j 13.1712 | $0.26997+\mathrm{j} 0.13476$ |
| 8 | 11 | $0.0080594+\mathrm{j} 15.7805$ | $0.00002+\mathrm{j} 0.8805$ |
| 11 | 12 | $0 .+\mathrm{j} 0$. | 0. +j 0 . |
| 11 | 13 | $28.2928+$ j 9.08125 | $0.29033+$ j 0.015537 |
| 14 | 33 | 0. +j 0 . | $0 .+\mathrm{j} 0$. |
| 14 | 15 | 102.3857 + j 29.47302 | $0.63146+\mathrm{j} 0.68607$ |

## Slack coefficient $\mathrm{g}=1.0013$

Total losses $=10.4934 \mathrm{KW}$

| Source | $\mathrm{P} \quad[\mathrm{KW}]$ | $\mathrm{Q}[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ | \# of Units <br> in Cluster | Available <br> KW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Flow from T1 | - | - | - | - | - |
| DR at Bus 16 | 274.0 | 25.5 | 275.1 | 4 | 300 |
| DR at Bus 22 | 274.0 | 25.5 | 275.1 | 4 | 300 |
| DR at Bus 8 | 122.5 | 218.2 | 250.2 | 3 | 225 |
| DR at Bus 11 | 132.0 | -144.9 | 196.0 | 2 | 150 |

Table 17. Island Mode, Full Load.

## Steady state island mode analysis tool

To obtain the data of Table 17, the classical formulation of the power flow problem had to be revisited to allow for the droop characteristic as explained in section 3.4 to be implemented. As of Figure 18, the missing power from the grid must be taken in by all the units contemporarily as they all ramp up their output power. We introduce here the concept of slack coefficient $g$ such that the power injections of each machine is given by: $P_{e x t, i}=P_{0, i}+g\left(P_{1, i}-P_{0, i}\right)$

The coefficient g is the same for the whole network. If $\mathrm{g}=0$, then we are connected to the grid and output power is the one requested at $\omega=\omega_{0}$. If $g=1$, then the output power of the unit is the one requested at $\omega=\omega_{\text {min }}$ (see Figure 18). Notice that $g$ can be smaller or larger than one: it is smaller than one when some of the loads are missing (see Tables 18 and 19), it is larger than one when some of the generation is missing (see Table 20, 21 and 22), and it is about one when all the loads and generations are in (see Table 17). This last case is the one that insures a zero flow of power in the transformers T 1 and T 2 , which can be seen in Table 17 by the fact that the power flows on the branches 5-16 and $5-22$ is about zero. The island setpoint for the power is also chosen to minimize the flow of power on the cable that connects the factory with the warehouse, namely branch 8-11, whose active power flow is about zero.

The new formulation of the power flow must be corrected in the following way from the classic case described in Section 4.:

- Slack Generation Bus: V and angle $\delta$ are specified plus the $P$ injected
- Load Buses: external P and Q (negative) injections are specified
- Remaining Generation Buses: P injected and V are specified

The only difference here is in the fact that the slack bus has a specified power, unlike the classical formulation. Remember that injected power from the units is determined once the slack coefficient $g$ is known. That is: we are introducing a new variable, $g$, then we need to add one extra constraint to the equation system. To summarize, for each bus we can write the following two real equations:

$$
\begin{aligned}
& \sum P_{n e t, i}+P_{e x t, i}=0 \quad \text { that is: } \quad \sum P_{n e t, i}+P_{0, i}+g\left(P_{1, i}-P_{0, i}\right)=0 \quad \text { for the generation buses } \\
& \sum Q_{n e t, i}+Q_{e x t, i}=0
\end{aligned}
$$

We then have $2 n$ constraining equations. We have $4 n+1$ quantities that uniquely define the system: 4 each bus, as the classic approach, plus the slack coefficient $g$. We have $2 n+1$ specified quantities: 2 per each bus, like in the classic approach, plus the active power injection for the slack bus. In total we have $4 n+1-(2 n+1)=2 n$ unknown quantities, with $2 n$ equations, which represent the solvable system for the modified power flow problem.

## Reduced load: loss of computers

Figure 37 shows the factory diagram when we are operating in island mode and when we are disconnecting the non sensitive loads and the computers from the office.


Figure 37. Factory Diagram, Island Mode without Computers.
Table 18 shows the steady state results when all the units are in place, but all of the computer loads are missing. Since we are in island mode, it is understood that the non sensitive loads are also missing. There is need for less power to be injected compared to the case just described, and since all the units ramp up power contemporarily through the slack coefficient $g$, then we have to expect $g$ to be smaller than one.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | - |
| 2 | - |
| 3 | - |
| 4 | 0.9901 |
| 5 | 0.9903 |
| 7 | 0.9889 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9800 |
| 13 | 0.9707 |
| 14 | 0.9798 |
| 15 | 0.9792 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9662 |
| 29 | 0.9836 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9800 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9798 |
| 36 | 0.9778 |
| 37 | 0.9775 |


| Branch |  | S k->m [KW +j KVAR] | Branch Losses [ $\mathrm{KW}+\mathrm{j}$ KVAR] |
| :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |
| 5 | 16 | 34.5404 - j 8.97799 | $0.015209+\mathrm{j} 0.020846$ |
| 5 | 22 | 34.5449 - j 12.267 | $0.019624+\mathrm{j} 0.026161$ |
| 8 | 7 | $57.9639+$ j 18.7483 | $0.06407+\mathrm{j} 0.021357$ |
| 9 | 10 | $64.9283+$ j 13.1712 | $0.26997+\mathrm{j} 0.13476$ |
| 8 | 11 | $-75.87001+$ j 258.461 | $3.6646+\mathrm{j} 3.9533$ |
| 11 | 12 | $0 . \quad+\mathrm{j} 0$. | $0 .+\mathrm{j} 0$. |
| 11 | 13 | $28.2928+$ j 9.08125 | $0.29033+\mathrm{j} 0.015537$ |
| 14 | 33 | $0 . \quad+\mathrm{j} 0$. | $0 . \quad+\mathrm{j} 0$. |
| 14 | 15 | $9.578+\mathrm{j} 1.9224$ | $0.0052881+\mathrm{j} 0.0057453$ |

Slack Coefficient $\mathrm{g}=0.7193$
Total losses $=9.9731 \mathrm{KW}$

| Source | $\mathrm{P}[\mathrm{KW}]$ | $\mathrm{Q}[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: |
| Flow from T1 | - | - | - |
| DR at Bus 16 | 239.2 | 45.6 | 243.5 |
| DR at Bus 22 | 239.2 | 47.9 | 243.9 |
| DR at Bus 8 | 116.2 | 269.5 | 293.5 |
| DR at Bus 11 | 117.4 | -263.5 | 288.4 |

Table 18. Island mode, without all the Computers.

## Reduced load: loss of an Induction Machine

Figure 38 shows the factory diagram in island mode, without an induction machine: all non sensitive loads are already taken out as the system transferred to island. The machine
that is disconnected from the rest of the network is the last one on the first floor, located on bus 21 . The nearest cluster is on bus 16 and is regulating its own power according to the droop characteristic, shared with all the other units in the isolated micro-grid.


Figure 38. Factory Diagram, Islanding, with loss of an IM.
Table 19 shows the island case results when the induction machine at the Bus 21 is not connected and all the non sensitive loads are taken out. Like in the case of the computer missing, we have that the overall load is lower, associated with lower overall losses and with a slack coefficient smaller than one. Notice also that anytime that $g$ is different from one we can't meet anymore the desired property of zeroing the active power flow in some of the key branches.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | - |
| 2 | - |
| 3 | - |
| 4 | 0.9900 |
| 5 | 0.9900 |
| 7 | 0.9889 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9800 |
| 13 | 0.9707 |
| 14 | 0.9779 |
| 15 | 0.9706 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9662 |
| 29 | 0.9836 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9800 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9779 |
| 36 | 0.9549 |
| 37 | 0.9518 |


| Branch |  | S k->m [KW +j KVAR] | Branch Losses [ $\mathrm{KW}+\mathrm{j}$ KVAR] |
| :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |
| 5 | 16 | $-30.262+\mathrm{j} 19.4665$ | $0.015471+\mathrm{j} 0.021204$ |
| 5 | 22 | 18.276 - j 15.9106 | $0.0085796+\mathrm{j} 0.011437$ |
| 8 | 7 | $57.9639+$ j 18.7483 | $0.06407+\mathrm{j} 0.021357$ |
| 9 | 10 | $64.9283+$ j 13.1712 | $0.26997+\mathrm{j} 0.13476$ |
| 8 | 11 | $8.171931+$ j 178.499 | $1.6126+\mathrm{j} 1.7396$ |
| 11 | 12 | $0 . \quad+\mathrm{j} 0$. | 0. +j 0. |
| 11 | 13 | $28.2928+$ j 9.08125 | $0.29033+\mathrm{j} 0.015537$ |
| 14 | 33 | $0 . \quad+\mathrm{j} 0$. | 0. +j 0 . |
| 14 | 15 | $102.3857+$ j 29.47302 | $0.63146+\mathrm{j} 0.68607$ |

$$
\text { Slack Coefficient } \mathrm{g}=0.8505
$$

Total losses $=10.145 \mathrm{KW}$

| Source | $\mathrm{P} \quad[\mathrm{KW}]$ | $\mathrm{Q} \quad[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: | :---: |
| Flow from T1 | - | - | - |
| DR at Bus 16 | 255.5 | -22.6 | 256.4 |
| DR at Bus 22 | 255.5 | 45.5 | 259.5 |
| DR at Bus 8 | 119.1 | 214.5 | 245.3 |
| DR at Bus 11 | 124.2 | -147.8 | 193.1 |

Table 19. Island mode, without the Induction Machine on Bus 21.

## Loss of a Micro-source

Figure 39 shows the factory diagram when a unit in cluster 16 is suddenly lost and the available power to that bus is 225 KW , versus the 300 KW that was before when all the
four units were operating. All the remaining loads, besides the non sensitive loads, are connected to the micro grid.


Figure 39. Factory Diagram, Loss of a Micro-source, Islanding.
Table 20 shows the case when a unit in the cluster at bus 16 is suddenly lost. The available power is no longer 300 KW , but rather 225 KW . This amount is more than enough to provide the required power when grid is still connected, but in the island mode the other units have to inject more power to meet this missing amount: indeed the slack coefficient is larger than one. Notice that now the cluster in Bus 22 is operating near its maximum allowed amount. Table 21 shows the case when a unit in the cluster at bus 22 is lost: this case is very similar to the loss of the unit on bus 16 . Table 22 shows the case when a unit at bus 11 is lost. When a unit in cluster 8 is lost, nothing will change. This is due to the fact that there is lots of availability of power on this bus: Table 20 shows that even by losing a unit, cluster at Bus 8 could still inject up to 150 KW , enough to meet its request of 122.5 KW .

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | - |
| 2 | - |
| 3 | - |
| 4 | 0.9900 |
| 5 | 0.9901 |
| 7 | 0.9889 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9800 |
| 13 | 0.9707 |
| 14 | 0.9779 |
| 15 | 0.9706 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9662 |
| 29 | 0.9836 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9800 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9779 |
| 36 | 0.9549 |
| 37 | 0.9518 |


| Branch |  | S k->m | [KW +j KVAR] | Branch Losses | [KW +j KVAR] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |  |  |
| 5 | 16 | 48.39 | - j 30.2577 | 0.038913 | + j 0.053334 |
| 5 | 22 | -25.67 | + j 23.4901 | 0.017691 | + j 0.023584 |
| 8 | 7 | 57.9 | $9+\mathrm{j} 18.7483$ | 0.06407 | + j 0.021357 |
| 9 | 10 | 64.9 | $3+\mathrm{j} 13.1712$ | 0.26997 | + j 0.13476 |
| 8 | 11 | -15.91 | 9 + j 201.2759 | 2.0589 | + j 2.2211 |
| 11 | 12 | 0. | + j 0. | 0. | + j 0. |
| 11 | 13 | 28.2 | $8+\mathrm{j} 9.08125$ | 0.29033 | $+\mathrm{j} 0.015537$ |
| 14 | 33 | 0. | $+\mathrm{j} 0$. | 0. | + j 0. |
| 14 | 15 | 102.3 | 7 + j 29.47302 | 0.63146 | + j 0.68607 |

## Slack coefficient $\mathrm{g}=1.3223$

Total losses $=10.8680 \mathrm{KW}$

| Source | $\mathrm{P} \quad[\mathrm{KW}]$ | Q | $[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: | :---: |
| Flow from T1 | - | - | - |  |
| DR at Bus 16 | 225.0 | 66.0 | 234.5 |  |
| DR at Bus 22 | 299.4 | 12.2 | 299.6 |  |
| DR at Bus 8 | 129.8 | 226.7 | 261.22 |  |
| DR at Bus 11 | 148.8 | -160.1 | 218.5 |  |

Table 20. Loss of Unit in Cluster at Bus 16

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | - |
| 2 | - |
| 3 | - |
| 4 | 0.9900 |
| 5 | 0.9901 |
| 7 | 0.9889 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9800 |
| 13 | 0.9707 |
| 14 | 0.9779 |
| 15 | 0.9706 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9662 |
| 29 | 0.9836 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9800 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9779 |
| 36 | 0.9549 |
| 37 | 0.9518 |


| Branch |  | S k->m | [KW +j KVAR] | Branch Losses | [KW +j KVAR] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |  |  |
| 5 | 16 | -25.6 | $2+\mathrm{j} 24.3745$ | 0.014976 | j 0.020527 |
| 5 | 22 | 48.4 | - j 31.5305 | 0.048745 | j 0.064982 |
| 8 | 7 | 57.9 | + j 18.7483 | 0.06407 | 0.021357 |
| 9 | 10 | 64.9 | + j 13.1712 | 0.26997 | j 0.13476 |
| 8 | 11 | -15.91 | + j 201.2774 | 2.0589 | j 2.2211 |
| 11 | 12 | 0. | + j 0. | 0. | 0. |
| 11 | 13 | 28.2 | + j 9.08125 | 0.29033 | 0.015537 |
| 14 | 33 | 0. | + j 0. | 0. |  |
| 14 | 15 | 102.38 | + j 29.47302 | 0.63146 | j 0.68607 |

Slack coefficient $g=1.3223$
Total losses $=10.8751 \mathrm{KW}$

| Source | P [KW] | Q [KVAR] | S [KVA] |
| :---: | :---: | :---: | :---: |
| Flow from T1 | - | - | - |
| DR at Bus 16 | 299.4 | 11.3 | 299.61 |
| DR at Bus 22 | 225.0 | 67.2 | 234.8 |
| DR at Bus 8 | 129.8 | 226.3 | 260.9 |
| DR at Bus 11 | 148.8 | -160.1 | 218.6 |

Table 21. Loss of Unit in Cluster at Bus 22

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | - |
| 2 | - |
| 3 | - |
| 4 | 0.9899 |
| 5 | 0.9898 |
| 7 | 0.9889 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9800 |
| 13 | 0.9707 |
| 14 | 0.9779 |
| 15 | 0.9706 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9662 |
| 29 | 0.9836 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9800 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9779 |
| 36 | 0.9549 |
| 37 | 0.9518 |


| Branch |  | S k->m | [KW + j KVAR] | Branch Losses | [KW + j KVAR] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |  |  |
| 5 | 16 | -26. | + j 6.7991 | 0.008813 | + j 0.01208 |
| 5 | 22 | -26.2 | $1+\mathrm{j} 9.31282$ | 0.011368 | j 0.015155 |
| 8 | 7 | 57.9 | + j 18.7483 | 0.06407 | j 0.021357 |
| 9 | 10 | 64.9 | + j 13.1712 | 0.26997 | j 0.13476 |
| 8 | 11 | 56.83 | + j 132.8277 | 1.0542 | j 1.1373 |
| 11 | 12 | 0. | + j 0. | 0. | $+\mathrm{j} 0$. |
| 11 | 13 | 28.2 | + j 9.08125 | 0.29033 | j 0.015537 |
| 14 | 33 | 0. | $+\mathrm{j} 0$. | 0. | $+\mathrm{j} 0$. |
| 14 | 15 | 102.3 | + j 29.47302 | 0.63146 | j 0.68607 |

## Slack Coefficient g $=1.2099$

Total losses $=9.8370 \mathrm{KW}$

| Source | $\mathrm{P} \quad[\mathrm{KW}]$ | Q | $[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: | :---: |
| Flow from T1 | - | - | - |  |
| DR at Bus 16 | 298.2 | 27.4 | 299.4 |  |
| DR at Bus 22 | 298.2 | 28.9 | 299.6 |  |
| DR at Bus 8 | 127.2 | 181.2 | 221.4 |  |
| DR at Bus 11 | 75.0 | -92.7 | 119.2 |  |

Table 22. Loss of Unit in Cluster at Bus 11

## Steady State Summary

The steady state results are summarized in the following Tables. Table 23 shows the losses of the system on the results described so far. Losses are greatly reduced by the introduction of the micro-sources on the factory floors. It is possible to notice yet another step in losses reduction when operating in island mode, since branch flow is reduced. The flow of power from the network is reduced of all the amount injected locally by the micro-sources. All the sources are operating at constant power setting when connected to the grid, so each load change is adjusted with different amount of power taken from the grid. When the system operates in island mode the settings of the units depend on the load condition of the plant: with full load the units operate as to zero out the power in some branches of the system, as of Table 24. This Table shows the power flowing in some branches during all conditions: the large flows corresponding to the case when no units are installed, while flow may become negative during islanding because power is moving from one part of another of the network, against the normal flow when provided by the grid.

|  |  | Total Losses [KW] | Flow from <br> T1 [KW] | DR at Bus 16 [KW] | DR at Bus 22 [KW] | DR at Bus 8 [KW] | DR at Bus 11 [KW] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No Units | Full Load | 36.94 | 946.78 | - | - | - | - |
| Connection | Full Load | 19.35 | 489.35 | 150.0 | 150.0 | 100.0 | 80.0 |
| to the | No Computers | 14.31 | 394.19 | 150.0 | 150.0 | 100.0 | 80.0 |
| Grid | No IM on $1^{\text {st }}$ Floor | 17.78 | 439.24 | 150.0 | 150.0 | 100.0 | 80.0 |
| Island | Full Load | 10.49 | - | 274.0 | 274.0 | 122.5 | 132.0 |
|  | No Computers | 9.97 | - | 239.2 | 239.2 | 116.2 | 117.4 |
| Mode | No IM on $1^{\text {st }}$ Floor | 10.14 | - | 255.5 | 225.5 | 119.1 | 124.2 |
|  | Loss of Unit on Bus 16 | 10.86 | - | 225.0 | 299.4 | 129.8 | 148.8 |

Table 23. Summary for Steady State Power Injections.

|  |  | Branch Flow from Bus $\qquad$ | Branch Flow from $\text { Bus } 5 \text { to } 22 \text { [KW] }$ | Branch Flow from $\text { Bus } 4 \text { to } 8 \text { [KW] }$ | Branch Flow from Bus 8 to 11 [KW] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No Units | Full Load | 263.05 | 262.8 | 401.2 | 226.44 |
| Connection | Full Load | 123.92 | 123.98 | 235.85 | 155.59 |
| to the | No Computers | 123.91 | 123.97 | 142.99 | 62.72 |
| Grid | No IM on $1^{\text {st }}$ Floor | 75.24 | 123.99 | 235.33 | 155.59 |
| Island | Full Load | 0.0 | 0.0 | 0.0 | 0.0 |
|  | No Computers | 34.54 | 34.54 | -69.19 | -75.87 |
| Mode | No IM on $1^{\text {st }}$ Floor | -30.26 | 18.27 | 11.95 | 8.17 |
|  | Loss of Unit on Bus 16 | 48.39 | -25.67 | -22.83 | -15.91 |

Table 24. Summary for Steady State Branch Flows.

## Details of Dynamic Study.

It is also important to examine the transients when we make the transfer to island mode. Now critical loads only are supplied by the micro-sources, since all the non critical loads are shed when the grid connection fails. Figure 40 shows the active power injections: the transient takes place between the steady state operating points described by Table 14 and 17. Figure 41 shows the reactive power injection, while Figure 42 shows the voltage regulation during transfer to island mode: the lower plot represents the voltages in a 0 to 1 pu scale, while the upper plot is a magnification of the voltages around the 1 pu value. The full scale plot allows us to see that the voltage profile is almost unaltered during islanding, while the magnification gives us insight on the actual dynamics that take place. Both plots are in the same time scale.


Figure 40. Active Power Injection, Transfer to Island.


Figure 41. Reactive Power Injection, Transfer to Island.


Figure 42. Regulated Bus Voltages, Transfer to Island.
Figure 43 shows the local frequencies at the buses where the micro-sources are connected with the feeders. The frequency of the micro-grid assumes the same values across the islanded network, following the regulating characteristics of the controllers. Figure 43 shows the overlap of the frequencies at the point of connection with the local network at each cluster, and they all look as if they were one single trace. As the grid fails, the frequency is free to swing: a slower loop in the controller will restore the frequency to a value that is very close to the nominal value.


Figure 43. Frequency Restoration, Transfer to Island.

## Reduced load: loss of computers

Transient analysis shows us time domain traces of the system as it transfers to island mode with a reduced load: not only the non sensitive loads are dropped, but also the computers are turned off. Figure 44 shows the active power that each of the cluster of units inject into the system: a comparison with Figure 31 shows that now the output power is slightly smaller than with the case with the computers.


Figure 44. Active power, transfer to island, without computers.
Figure 45 shows the reactive power that the units have to provide when the system transfers to island mode without the computers. This reactive quota changes to maintain the regulated voltage fixed to the desired values at steady state. Since the computers are located in electrical proximity to cluster of bus 11, then it is this bus that shows the largest difference from Figure 41, when the computers are on. Notice that in Figure 41 once the system becomes isolated the cluster of bus 11 needed to inject inductive reactive power to hold down the voltage. Now, in Figure 45 it has to provide more inductive power to maintain the voltage at the same value. Figure 46 shows the regulated voltage as a consequence of the islanding event. After a short transient, the voltage is brought back to the desired value.


Figure 45. Reactive power, transfer to island, without computers.


Figure 46. Regulated voltage, transfer to island, without computers.
Figure 47 shows the frequencies measured at the feeders where the cluster of microsources are connected. The frequencies at bus $8,11,16$ and 22 are overlapped, even if it looks like there is only one single trace. The system frequency is slowly restored to near the nominal value after having dropped almost instantaneously to the regulating value.


Figure 47. Network frequency, transfer to island, without computers.

## Reduced load: loss of an Induction Machine

The transient analysis will show us how the system will behave after the event that transfers the system to island, without the last induction machine on the first floor of the factory, on bus 21 . Figure 48 shows the active power output of the micro-sources as they rearrange their output power due to the islanding.


Figure 48. Active power, transfer to island, without an IM

Figure 49 shows the reactive power needed to maintain the requested voltage at the buses where the units are connected to the system. Since the induction machine that is turned off is on the first floor, we see that the reactive power injection from the units at the bus 16 is lower than the one saw in Figure 41, where only the non critical loads where turned off. Figure 50 shows the network frequency: notice that the lowest frequency ever reached is higher than the one already seen in Figure 43, due to the fact that the overall power demand is lower in this case.


Figure 49. Reactive power, transfer to island, without an IM


Figure 50. Network frequency, transfer to island, without an IM

## Loss of a Micro-source

It is possible to simulate a unit missing from a cluster by simply setting an upper limit to the active power that the cluster can overall inject into the network. For this purpose, we will look at the results obtained during the transfer to island with full load, and with the cluster at the Bus 16 that can't exceed 0.75 pu of its nominal active power output. Since the overall rating of the cluster is 300 KW then, under this limiting condition we have that the units on this bus will not be able to inject more than 225 KW in steady state. This is because before we had 4 units of 75 KW each in the cluster, as of Table 12, and after the failure of one of them, we only have 3 .

To understand the way we enforce the limit, it is useful to refer to Figure 17. The dynamics of the prime mover are controlled by the variable Pc, therefore we add a limiter block to that quantity to ensure that it will never exceed the amount of 0.8 pu . The active power of the micro-sources is shown in Figure 51. Transiently, the power output of the cluster at bus 16 exceeds the limit, settling to the value commanded by the droop characteristic. As recovery takes place and the variable Pc reaches its maximum allowed value then the output power adjusts according to the limit. All the other clusters increase their output power to compensate for the smaller injection from bus 16 .


Figure 51. Active Power from the Units, Island Mode, with Limit on Bus 16.

Figure 52 shows the control variable Pc: as the unit reaches its limit it is clamped to that value and held in steady state. The other units continue to ramp up according to the droop characteristic.


Figure 52. Control Variable Pc, Island Mode, with Limit on Bus 16.
The reactive power injection by the micro-sources is shown in Figure 53. This injection is the one required to hold the desired voltages at the buses where the units are installed. Figure 54 shows the regulated voltages during the transfer to island when the active power limit is enforced on the cluster at bus 16 .


Figure 53. Reactive Power from Units, Islanding with Limit on Bus 16.


Figure 54. Regulated Voltages, Islanding, with Limit on Bus 16.
The frequency recovery process is not altered by the limit coming into effect: Figure 55 shows the frequency during the islanding transient. This is the overlap of all the frequencies measured at the buses where the units are installed.


Figure 55. Frequency Recovery, Islanding, Limit on Bus 16.

### 4.2 Load Dispatch Strategy

In the sections that we have so far analyzed, the power output from the units is directly controlled by assigning a desired value to it. The controller would compare the measured injected power and generate the appropriate signal. It is desirable to be able to operate the units in such a way that the desired power chosen is not the one produced by the unit, but rather the power that flows in some key branch of the network. We saw in the islanded mode that the droop characteristics are targeted to obtain a zero power flow in the following branches:

- Branch from bus 5 to 16
- Branch form bus 5 to 22
- Branch from bus 4 to 8
- Branch from bus 8 to 11

When the slack coefficient $g$ was one, then the active power flows in the above branches was exactly zero. This operating point is reachable because the units can locally meet the demand of the neighboring loads, without the need to import power from other points of the isolated grid.

The main reason why we are interested in exploring the load dispatch strategy is because when regulating power on the branches to a constant value, then the power taken from the grid will remain unchanged whenever the load changes in the micro-grid. There are
cases when the utility is interested in having large customers to draw a constant amount of power from the grid, regardless of their changing local needs of power. This solution would solve the problem and the grid would see a constant power in the industrial site.

In this section we are exploring the possibility of controlling directly the active power that flows in these branches, not only during islanding, but in all conditions of operation. The first basic requirement for this load dispatch strategy to take place is to make a measure of the power flowing in those branches so that the control can compare it against the desired amount given on beforehand.

The control described in Figure 17 has to be revisited under the light of this new desired goal: the measured power is converted to the measure of the branch power and the desired unit output power is converted to the desired amount of power flow in the branch.


Figure 56. Frequency Droop with Load Dispatch.
Figure 56 shows the new diagram of the controller to keep into account of these changes. The most dramatic of all the changes is not the new measure and setpoint of power, but rather the inversion of the droop characteristic. Indeed, from Figure 57 it is possible to see that if the output power of the unit increases, then the power from the branch decreases, that is, we need to import less from other parts of the network to meet the local loads.


Source

Figure 57. Diagram of Load Dispatch.

We test this regulating technique by assigning to the desired flows of power in the lines the same values that they had when we were controlling the power injected by the units. The desired active power for the branches is shown in Table 25. The values are taken looking at the power flow in those branches in Table 14, where we were directly regulating the active power from the unit.

| Branch: starting Bus and ending Bus | Desired Power Setpoint [KW] |
| :---: | :---: |
| 5 | $->16$ | 124

Table 25. Desired Setpoints for Branches Power Flows.

It is possible to find a value for the branch 4 to 8 from Table 14 by adding up the power in the branches that stem from it, namely 8 to 7,9 to 10 and 8 to 11 . To this value, we need to subtract the amount of power already provided locally by the cluster at the bus 8 , which totals to 100 MW . The active losses in the transformer T4 are minimal, at the point that can be neglected.

## Connected to the grid

The factory diagram in Figure 58 summarizes the operation with load dispatchment: the measures of the line flows are fed back to the micro-source controls so that it is possible to regulate them. Here it is clear to see what cluster controls what line flow. In this test, it is understood that all loads are connected.


Figure 58. Factory Diagram, with Load Dispatch
Due to the choice of desired power in the regulated branches, with full load, the steady state results will be exactly identical to Table 14.

## Reduced Load: loss of $1^{\text {st }}$ Floor of computers

In this scenario, we are dropping one floor of computers when the system is operating in connection to the grid and with full load. Figure 59 shows the factory diagram when we are operating with load dispatch and removing one floor of computer from the load.


Figure 59. Factory Diagram with Load Dispatch and without the Computers.

Table 26 shows the steady state results for this mode of operation. The flow from the grid is unchanged, as well as all the flows in the key branches stayed unaltered. The units at the cluster on bus 11 are the ones who back off their output power in order to maintain the power flow on the selected branches constant.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | 0.9983 |
| 2 | 0.9990 |
| 3 | 0.9941 |
| 4 | 0.9901 |
| 5 | 0.9908 |
| 7 | 0.9878 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9769 |
| 13 | 0.9707 |
| 14 | 0.9783 |
| 15 | 0.9737 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9652 |
| 29 | 0.9724 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9676 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9512 |
| 36 | 0.9723 |



| Branch |  | S k->m [KW +j KVAR] | Branch Losses [ $\mathrm{KW}+\mathrm{j}$ KVAR] |
| :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |
| 5 | 16 | 123.9275 - j 40.61688 | $0.20288+\mathrm{j} 0.27807$ |
| 5 | 22 | 123.9869 - j 51.04278 | $0.26225+$ j 0.3496 |
| 8 | 7 | $115.3511+\mathrm{j} 33.09207$ | $0.24861+\mathrm{j} 0.082871$ |
| 9 | 10 | $64.9283+$ j 13.1712 | $0.26997+\mathrm{j} 0.13476$ |
| 8 | 11 | $155.4614+\mathrm{j} 42.57668$ | $1.2965+\mathrm{j} 1.3987$ |
| 11 | 12 | $66.7642+$ j 19.5777 | $0.21673+\mathrm{j} 0.037299$ |
| 11 | 13 | 28.2928 + j 9.08125 | $0.29033+\mathrm{j} 0.015537$ |
| 14 | 33 | $27.9207+\mathrm{j} 5.64849$ | $0.77806+\mathrm{j} 0.12939$ |
| 14 | 15 | $65.1025+\mathrm{j} 17.4261$ | $0.25249+\mathrm{j} 0.27432$ |

Total losses $=17.9245 \mathrm{KW}$

| Source | $\mathrm{P}[\mathrm{KW}]$ | $\mathrm{Q}[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: |
| Flow from T1 | 489.1934 | -112.3918 | - |
| DR at Bus 16 | 150.0 | 72.5 | 166.6 |
| DR at Bus 22 | 150.0 | 82.0 | 170.9 |
| DR at Bus 8 | 100.0 | 115.7 | 152.9 |
| DR at Bus 11 | 35.2 | 10.9 | 34.3 |

Table 26. Grid Connection, with Load Dispatch, without one floor of Computers

## Details of Dynamic Study.

The results from the transient analysis are shown in the Figures below. Starting with Figure 60, we can see the active power flow in the branches the units are turned on one at a time. As the units start to inject power in the network, the power drawn from the branches diminishes. Notice that the power in branch 8 diminishes also when unit at bus 11 is turned on, since this bus is behind branch 8 .


Figure 60. Branch Power during System Startup with Full Load
Figure 49 shows the active power injected by the micro-sources during startup with full load. The output power from the units is just a mere measurement here, and is not passed back to the controller, since it is the branch power that is regulated to a desired value. It is useful to compare it to Figure 24, where the startup with unit power regulation was shown: although the steady state values are identical, some of the dynamics differ from one Figure to the other.


Figure 61. Active Power from Units during Startup with Full Load
Figure 62 shows the reactive power injection from the units during startup. These injections are commanded by the controller to regulate the voltages to the desired setpoints. These results can be directly compared with Figure 25, when we were regulating the output active power of the sources. The reactive power plots are very similar. The regulated voltage profiles are shown in Figure 63 and can be compared to Figure 26. The two plots are remarkably similar to one another.


Figure 62. Reactive Power from Units during Startup with Full Load


Figure 63. Regulated Voltage during Startup with Full Load

Figure 64 shows the power from the grid during the startup with load dispatch: the power commands for the flows in the branches are the same we had in the startup with the independent power setpoint regulation. This explains why Figure 64 is qualitatively similar to Figure 27, although some small differences are noticeable in the dynamics.


Figure 64. Power from the Grid, Load Dispatch.

## Reduced Load: loss of $\mathbf{1}^{\text {st }}$ Floor of computers

The power flowing in the key branches is shown in Figure 65, where it is possible to see how the power stays constant in steady state, as regulated by the controller.
Instantaneously, the branches contribute to the new power request, and this drives the units to adjust their output power to cancel out any contribution from the grid in the steady state. Since the computes being dropped are located nearest the units at bus 11, then it is not surprise to see the measure of the power in its nearest branch have the largest deviations during the transients.


Figure 65. Branch Power with Grid, Turning off Computers
The nearest cluster of unit to the office is the one installed on bus 11 , therefore it will be this cluster who will need to back off its active power injection to keep constant the power flow in the branch after the load is dropped. Figure 66 shows the active power injection of the micro-sources, where it is possible to see that only the cluster at bus 11 has a steady state change in output power as a result of the computer dropping. These results are different from Figure 28, where we were dropping the computers with the units regulating their output power to a constant value.

Notice that in that case we were dropping the whole computer load from the system, that is both floors in the office building. With load dispatch we are only turning off one floor of computers, the first floor, because since the computers are a large load, if we were to drop both floors, the units at the bus 11 would have to behave as a load, that is adsorbing rather than injecting power, to maintain the flow in the branch unchanged.
As a result of this regulation, the amount of active power taken in from the grid stays unchanged in the steady state. Figure 67 shows the time domain trace of the power from the grid as the computers are turned off. The grid instantaneously backs off the power as the load disconnects, but then goes back to the previous steady state value as the microsources in the cluster at bus 11 reduce their power injection.


Figure 66. Micro-Sources Active Power, with Grid, turning off Computers


Figure 67. Power from the Grid, with Load Dispatch, turning off Computers
The reactive power is shown in Figure 68: this injection is needed to keep the voltage constant at the feeder buses. When compared to Figure 29, it is possible to notice that all the units have to inject a capacitive reactive power, while before the unit at the bus 11 where injecting inductive reactive power to hold down the voltage to the desired value.

The reasons for this behavior are twofold:

- first we are shedding only half of the load now, so some extra compensation is needed,
- second we are drawing the same amount of power from the branch 8 to 11 , therefore we are experiencing a higher drop in this line than we had before, where we would allow the power from this branch to diminish.

Figure 69 shows the regulated voltages, how they tend to rise as the computers are turned off, and how they are quickly brought back to desired value by the controller.


Figure 68. Micro-Sources Reactive Power, with Grid, turning off Computers


Figure 69. Regulated Voltage, with Grid, turning off Computers

## Island Mode

Figure 70 shows the factory diagram when we are adopting the load dispatch and when the system transfers to island. All the non sensitive loads (NSL) are dropped and the power for the remaining loads in the islanded network comes all from the micro-sources.


Figure 70. Factory Diagram, Island Mode with Load Dispatchment.
The case when the grid fails while operating in the load dispatch mode can be handled by changing the power-frequency droop characteristic. Figure 71 shows that the slopes have
opposite sign from the ones shown in Figure 18. At that time we were regulating the powers from the units, therefore we needed to have more power as the frequency was regulating to a lower level than nominal. Now we are regulating the power flows in the branches and we need less power as the frequency reduces. All the units regulate to zero power in the branches when operating in island mode.


Figure 71. P- $\omega$ Droop Characteristic, with Load Dispatch
The steady state results with full load are identical to Table 16, since both systems are operating to cancel out the power in the key branches.

## Reduced load: loss of $1^{\text {st }}$ Floor of computers

Figure 72 shows the factory diagram when we are operating with load dispatchment in isolation from the main grid. The first floor of computers have been disconnected, along with all non sensitive loads. The full computer load is about 100 KW , therefore by turning off only a floor we would end up with a power demand that is roughly 50 KW lower than it was before with full load.


Figure 72. Island Mode, Load Dispatch, no $1^{\text {st }}$ Floor Computers.
Table 27 shows the steady state results after the first floor of computers have been dropped and the island mode is in effect. The power is kept to zero in the key branches: Table 17 showed the zero flow with full load in island. From that scenario, here the computers have been shed, but the flow in the lines kept constant. That means that the units at the bus 11 have to diminish their output power of the amount of a floor of computers.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | - |
| 2 | - |
| 3 | - |
| 4 | 0.9900 |
| 5 | 0.9900 |
| 7 | 0.9889 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9800 |
| 13 | 0.9707 |
| 14 | 0.9789 |
| 15 | 0.9749 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9662 |
| 29 | 0.9836 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9800 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9789 |
| 36 | 0.9735 |
| 37 | 0.9560 |


| Branch |  | S k->m | [KW + j KVAR] | Branch Losses | [KW + j KVAR] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |  |  |
| 5 | 16 | 0.00 | + j 0.00015 | 0. | $+\mathrm{j} 0$. |
| 5 | 22 | 0.00 | + j 0.00015 |  | $+\mathrm{j} 0$. |
| 8 | 7 | 57.96 | + j 18.7483i | 0.26997 | $+\mathrm{j} 0.13476 \mathrm{i}$ |
| 9 | 10 | 64.92 | + j 13.1712i | 1.7584 | + j 1.8969i |
| 8 | 11 | 0.005 | $421+$ j 14.342 | 0.00001 | + j 0.4432 |
| 11 | 12 | 0. | $+\mathrm{j} 0$. | 0. | $+\mathrm{j} 0$. |
| 11 | 13 | 28.29 | + j 9.08121i | 0.29032 | j 0.015537i |
| 14 | 33 | 0. | $+\mathrm{j} 0$. | 0. | + j 0. |
| 14 | 15 | 56.17 | + j 15.6589i | 0.18881 | + j 0.20513i |

Total losses $=9.1958 \mathrm{KW}$

| Source | $\mathrm{P} \quad[\mathrm{KW}]$ | $\mathrm{Q} \quad[\mathrm{KVAR}]$ | $\mathrm{S}[\mathrm{KVA}]$ |
| :---: | :---: | :---: | :---: | :---: |
| Flow from T1 | - | - | - |
| DR at Bus 16 | 274.0 | 27.8 | 275.4 |
| DR at Bus 22 | 274.0 | 27.9 | 275.4 |
| DR at Bus 8 | 122.5 | 213.3 | 245.9 |
| DR at Bus 11 | 78.7 | -159.8 | 178.1 |

Table 27. Load Dispatch, Island Mode, without one Floor of Computers.

## Reduced load: loss of an induction machine

Figure 73 shows the factory diagram when we are operating in island mode, with load dispatchment and when one induction machine is disconnected. The machine in question is the one located on the Bus 21, and due to the islanding mode of operation, it is possible to see that also all non sensitive loads have been disconnected.


Figure 73. Island Mode, with Load Dispatchment, Loss of an IM.

Table 28 shows the power flow results for the transfer to island with load dispatch and with the load reduced of all non critical loads and of an induction machine.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | - |
| 2 | - |
| 3 | - |
| 4 | 0.9900 |
| 5 | 0.9900 |
| 7 | 0.9889 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9800 |
| 13 | 0.9707 |
| 14 | 0.9779 |
| 15 | 0.9706 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9662 |
| 29 | 0.9836 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9800 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9779 |
| 36 | 0.9549 |
| 37 | 0.9518 |


|  |  |  |  |
| :---: | :---: | :---: | :---: |
| Bus_k |  |  |  |
| 5 | Branch |  |  |

Total losses $=10.7853 \mathrm{KW}$

| Source | P [KW] | Q [KVAR] | S [KVA] |
| :---: | :---: | :---: | :---: |
| Flow from T1 | - | - | - |
| DR at Bus 16 | 225.4 | 6.9 | 225.5 |
| DR at Bus 22 | 274.0 | 33.2 | 276.0 |
| DR at Bus 8 | 122.5 | 207.6 | 241.1 |
| DR at Bus 11 | 132.0 | -143.3 | 194.8 |

Table 28. Load Dispatch, Island Mode, without one IM

## Loss of a Micro-Source

The loss of some power from the cluster of units can be simulated by enforcing a limit to the active power output that the unit can inject into the local grid. In this test we are
limiting the power of the cluster at the bus 16 to 0.75 pu of its nominal rating: the maximum active power that can be injected at this bus is 225 KW , corresponding to 3 units. Indeed, before the failure of one unit, we had four units in this cluster, as of Table 12, where all the clusters are defined.

Figure 74 shows the factory diagram when we are operating in island mode with load dispatch and we are losing a micro-source in the cluster at the bus 16 . All the non sensitive loads are turned off, while all sensitive loads are connected to the resulting isolated network.


Figure 74. Loss of a Micro-source, with Load Dispatch, Island Mode.

The steady state results when losing a micro-source in island mode and with load dispatchment are summarized in Table 29. It is possible to see that a new dispatching point is reached, with no cluster exceeding the amount allowed by its ratings.

| Bus Number | Voltage [pu] |
| :---: | :---: |
| 1 | - |
| 2 | - |
| 3 | - |
| 4 | 0.9901 |
| 5 | 0.9901 |
| 7 | 0.9889 |
| 8 | 0.9900 |
| 9 | 0.9885 |
| 10 | 0.9841 |
| 11 | 0.9800 |
| 12 | 0.9800 |
| 13 | 0.9707 |
| 14 | 0.9779 |
| 15 | 0.9706 |
| 16 | 0.9900 |
| 22 | 0.9900 |
| 28 | 0.9662 |
| 29 | 0.9836 |
| 30 | 0.9403 |
| 31 | 0.9368 |
| 32 | 0.9800 |
| 33 | 0.9614 |
| 34 | 0.9600 |
| 35 | 0.9779 |
| 36 | 0.9549 |
| 37 | 0.9518 |


| Branch |  | S k->m [KW +j KVAR] | Branch Losses [ $\mathrm{KW}+\mathrm{j}$ KVAR] |
| :---: | :---: | :---: | :---: |
| Bus_k | Bus_m |  |  |
| 5 | 16 | 48.7622 - j 27.763 | $0.037608+\mathrm{j} 0.051547$ |
| 5 | 22 | $-14.4829+$ j 17.4081 | $0.0074904+\mathrm{j} 0.0099855$ |
| 8 | 7 | $57.9639+$ j 18.7483 | $0.06407+\mathrm{j} 0.021357$ |
| 9 | 10 | $64.9283+\mathrm{j} 13.1712$ | $0.26997+\mathrm{j} 0.13476$ |
| 8 | 11 | $-12.23338+$ j 197.7858 | $1.9833+\mathrm{j} 2.1395$ |
| 11 | 12 | 0. +j 0 . | $0 .+\mathrm{j} 0$. |
| 11 | 13 | $28.2928+$ j 9.08125 | $0.29033+\mathrm{j} 0.015537$ |
| 14 | 33 | 0. +j 0 . | $0 .+\mathrm{j} 0$. |
| 14 | 15 | $102.3857+\mathrm{j} 29.47302$ | $0.63146+\mathrm{j} 0.68607$ |

Total losses $=10.0730 \mathrm{KW}$

| Source | P [KW] | Q [KVAR] | S [KVA] |
| :---: | :---: | :---: | :---: |
| Flow from T1 | - | - | - |
| DR at Bus 16 | 225.0 | 63.5 | 233.7 |
| DR at Bus 22 | 288.2 | 12.3 | 288.2 |
| DR at Bus 8 | 145.2 | 219.6 | 263.2 |
| DR at Bus 11 | 145.8 | -165.6 | 220.2 |

Table 29. Load Dispatch, Island Mode, Loss of a Unit in Bus 16.

## Steady State Summary

The steady state operating points for the operation with load dispatchment are summarized in the following Tables. Table 30 shows the losses in the different tests: these losses are very similar to the ones measured with the independent power setpoint regulation. The load dispatch strategy lowers the amount of losses, but not in a dramatic
way. The real difference can be noticed in the flow of power from the grid, that remains unaltered when the computers in the first floor of the office are turned off. The nearest unit (here on Bus 11), adjusts the output power so that the power from the incoming branch (and therefore from the grid), remains constant. During island mode we can see that the units arrange the power to values already seen with the independent setpoint strategy.

Table 31 shows the active power flow in the branches: the power is constant to the desired value when operating with the grid. In island mode, the power is always regulated to zero flow in the branches. This can be done as long as all the units can provide the locally requested power: if a unit fails and this condition can no longer be met, then flows will adjust accordingly to import power from other part of the micro-grid. As a consequence of this behavior, flows in the branches are different from zero.

|  |  | Total Losses [KW] | Flow from <br> T1 [KW] | DR at Bus 16 [KW] | DR at Bus 22 [KW] | DR at Bus 8 [KW] | DR at Bus 11 [KW] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No Units | Full Load | 36.94 | 946.78 | - | - | - | - |
| Connection to the Grid | Full Load | 19.35 | 489.35 | 150.0 | 150.0 | 100.0 | 80.0 |
|  | No 1 ${ }^{\text {st }}$ Floor Computers | 17.92 | 489.19 | 150.0 | 150.0 | 100.0 | 35.2 |
| Island <br> Mode | Full Load | 10.49 | - | 274.0 | 274.0 | 122.5 | 132.0 |
|  | No 1 ${ }^{\text {st }}$ Floor Computers | 9.19 | - | 274.0 | 274.0 | 122.5 | 78.7 |
|  | No IM on $1^{\text {st }}$ Floor | 10.07 | - | 255.4 | 274.0 | 122.5 | 132.0 |
|  | Loss of Unit on Bus 16 | 10.78 | - | 225.0 | 288.2 | 145.2 | 145.8 |

Table 30. Summary for Steady State Power Injections, Load Dispatch.

|  |  | Branch Flow from Bus 5 to 16 [KW] | Branch Flow from $\text { Bus } 5 \text { to } 22[\mathrm{KW}]$ | Branch Flow from $\text { Bus } 4 \text { to } 8[\mathrm{KW}]$ | Branch Flow from $\text { Bus } 8 \text { to } 11[\mathrm{KW}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No Units | Full Load | 263.05 | 262.8 | 401.2 | 226.44 |
| Connection to the Grid | Full Load | 123.92 | 123.98 | 235.85 | 155.59 |
|  | No 1 ${ }^{\text {st }}$ Floor Computers | 123.92 | 123.98 | 235.85 | 155.59 |
| Island <br> Mode | Full Load | 0.0 | 0.0 | 0.0 | 0.0 |
|  | No 1 ${ }^{\text {st }}$ Floor Computers | 0.0 | 0.0 | 0.0 | 0.0 |
|  | No IM on ${ }^{\text {st }}$ Floor | 0.0 | 0.0 | 0.0 | 0.0 |
|  | Loss of Unit on Bus 16 | 48.76 | -14.48 | -34.55 | -165.6 |

Table 31. Summary for Steady State Branch Flows, Load Dispatch.

## Details of Dynamic Study.

The dynamic results are shown in the following Figures. Figure 75 shows the active power flowing in the branches as the system transfers to island mode. All the flows go to zero, following the droop characteristic.


Figure 75. Branch Power, Transfer to Island, with Load Dispatch.
Figure 76 shows the active power injected by the micro-sources when the system goes in island mode. It is possible to compare with Figure 36 where the same transient occurred when regulating with the other droop.


Figure 76. Active Power from Units, Transfer to Island, with Load Dispatch.
Figure 77 shows the reactive power injection during redispatching to regulate the voltage at the feeder to a constant value. It is possible to compare with Figure 40 when the other droop was used. Figure 78 shows the voltage at the regulated buses during the transient after islanding. This Figure can be compared with Figure 42.


Figure 77. Reactive Power from Unit, Transfer to Island, with Load Dispatch.


Figure 78. Regulated Voltage, Transfer to Island, with Load Dispatch.
Figure 79 shows the frequency when islanding during load dispatch. This is the overlap of the measure of the frequencies on the buses where the units are connected to the network. The overlap shows one single trace, suggesting that the frequency drifts of the same amount everywhere in the isolated micro-grid.


Figure 79. Frequency Recovery, Transfer to Island, with Load Dispatch.

## Reduced load: loss of $1^{\text {st }}$ Floor of computers

Figure 80 shows the power in the key branches during the transient occurring after a grid failure. All the non critical loads are dropped along with the first floor computers in the office. We can see that the droop characteristic regulates the powers from the units as to reach a zero flow of power in all the branches. That was not happening in the case when the droop was regulating directly the power output of the micro-sources. In that case, the micro-sources regulated to the amount of power needed to cancel out flow in branches only when the non critical loads were off. That is, the droop didn't know of the missing
computers and therefore the flow in the branches was no longer zero. With load dispatch, the droop block does not need to know which loads are on or not during islanding, since it is directly regulating the flows of power in the branches themselves.


Figure 80. Branch Power, Islanding with Power Dispatch, no Computers
Figure 81 shows the active power output of the units. It can be compared with the results with the other droop, in Figure 44. The flows of power from the units are different in the two cases because in this case we reach zero flow of power in the branches. This case is very similar to Figure 76, with the islanding with full load. As a consequence of the computers turning off, only the cluster of units at bus 11 has to reduce its output amount to maintain the same zero flow in the branches.


Figure 81. Active Power from Units, Islanding, Load Dispatch, no Computers


Figure 82. Reactive Power from Units, Islanding with Power Dispatch, no Computers
Figure 82 shows the reactive power of the units needed to sustain the voltage at the required value during the transient. These results can be compared with Figure 55 representing the reactive power with the other droop.

Figure 83 shows the regulated voltage during islanding without the computers, and these results can be compared with Figure 46


Figure 83. Regulated Voltage, Islanding with Power Dispatch, no Computers


Figure 84. Frequency Recovery, Islanding with Power Dispatch, no Computers

Figure 84 shows the recovery of the frequency as the systems transfers to island mode, with load dispatching and without the first floor of computers. This plot, shows that the frequency drops down uniformly in the network before restoration takes place.

## Reduced load: loss of an induction machine

Figure 85 shows the power flowing in the key branches when we switch to island mode, we drop the non critical loads and also turn off one induction machine. In particular, this machine is the last one on the first floor, that is, on bus 21 . The control droop regulates the branch power to zero without any knowledge of the fact that one machine is taken off line.


Figure 85. Branch Power, Islanding with Power Dispatch, without one IM
Figure 86 shows the active power injected by the clusters when the induction machine on bus 21 is turned off in island mode. These results can be compared to Figure 48 where it is possible to notice the difference in the unit output power dispatched. Now it is only the cluster at bus 16 that notices a difference from the full load case, and backs off the output power corresponding to the IM load.

Figure 87 shows the reactive power to maintain desired voltages at the feeder terminals. These results can be compared with the other droop strategy shown in Figure 49


Figure 86. Active Power from Units, Islanding with Power Dispatch, without one IM


Figure 87. Reactive Power from Units, Islanding with Power Dispatch, without one IM.


Figure 88. Frequency, Islanding with Power Dispatch, without one IM.
Figure 88 shows the frequencies at the point where the clusters are installed during the islanding of the system without the last induction machine on the first floor of the factory.

## Loss of a Micro-Source

Looking at the block diagram of the control with load dispatch in Figure 56 it is possible to see that the control variable Pc loses its meaning of power from the first mover, as it had in Figure 17. To control the output power we need to resort to the formula that gives the active power injection across an inductor:

$$
P=3 \frac{V E}{X} \sin \left(\delta_{p}\right)
$$

We can limit the output power by setting a limit to the value that the angle difference can assume. The value of the angle is therefore given by:

$$
\delta_{p \max }=\arcsin \left(\frac{P_{\max } X}{3 V E}\right)
$$

The active power from the units is shown in Figure 89. It is possible to see that the active power in the bus 16 overshoots the limit during the transient, and then lowers its value to the maximum amount in steady state. The reason why we have the overshoot derives
from the formula used to calculate the power: the formula is exact in steady state, but is incorrect during dynamics, where derivative of the angle would appear, and solving for the angle difference would not be possible.


Figure 89. Active Power from Units, Islanding, Load Dispatch, with Limit on bus 16.

The power flowing in the key branches is shown in Figure 90, where we can see the consequences of the limit being reached at the bus 16 . Without the limit we were able to zero out the power in each branch during islanding. That was possible because there was enough power available locally to supply the loads, so that no power needed to be imported from other portions of the isolated grid.

The units at the bus 16 can't inject all the requested power, and some power needs to be imported from elsewhere: this is reflected by the positive value that this branch power , $P_{b 16}$, assumes in steady state. The other powers are negative, meaning that are exporting power to other parts of the network.


Figure 90. Branch Power, Islanding, with Load Dispatch and with Limit on Bus 16.

Figure 91 shows the reactive power injection to support the voltages to the desired setpoints. The reactive power is not limited, so it is always possible to reach the requested values for the voltages. Figure 92 shows the dynamics of the voltage magnitudes during the transfer to island with limit enforced.


Figure 91. Reactive Power from Units, Islanding, Load Dispatch, with Limit on Bus 16.


Figure 92. Regulated Voltage during Islanding, Load Dispatch, with Limit on Bus 16.

Figure 93 shows the overlap of the frequencies measured at the point of connection of each of the clusters. The frequency recovers to the nominal value regardless of the limit being reached.


Figure 93. Frequency Recovery, Islanding, Load Dispatch, with Limit on Bus 16.

## 5. References

[1] - Rome Cable Corporation, "Rome Cable Manual" Rome Cable Corporation, Rome NY 1957
[2] - Turan Gonen, "Electric Power Distribution System Engineering", McGraw Hill 1986
[3] - D.W. Novotny, "Scaling Laws for AC Electrical Machines". Tutorial Report 82-7, WEMPEC, Department of Electrical and Computer Engineering University of Wisconsin. Madison, WI
[4] - D.W. Novotny, T.A. Lipo, "Vector Control and Dynamics of AC Drives", Clarendon Press, Oxford, 1996
[5] - M.C. Chandorkar, D.M. Divan, B. Banerjee, "Control of Distributed UPS Systems", WEMPEC Report, 94-04, University of Wisconsin-Madison
[6] - A. Bergen, "Power Systems Analysis", 1986
[7] - MATLAB. (1984) Trademark of The MathWorks, Inc., Mass.
[8] - Electromagnetic Transients Program (1989). Workbook 4. EPRI EL-4151 Vol. 4, June

