# Measurement of W-Boson Helicity Fractions in Top-Quark Decays Using $\cos \theta^{*}$ 

Shulamit Moed (Harvard)<br>R. Eusebi, D.Glenzinski, A.Hocker (Fermilab)

August 21, 2007

We present a measurement of the fraction of longitudinally $\left(f_{0}\right)$ and righthanded $\left(f_{+}\right)$polarized W bosons produced in top-quark decays through an analysis of the $\cos \theta^{*}$ distribution in $t \bar{t}$ candidate events using fully reconstructed lepton plus jets events. This analysis is based on a total integrated luminosity of $1.7 \mathrm{fb}^{-1}$. We find $f_{0}=0.57 \pm 0.11 \pm 0.04$ with $f_{+}$fixed to zero, and $f_{+}=-0.04 \pm 0.04 \pm 0.03$ with $f_{0}$ fixed to the Standard Model expectation at a top mass of $175 \mathrm{GeV} / c^{2}$. We then simultaneously fit for the longitudinal and the right-handed fractions and find $f_{0}=0.61 \pm 0.20 \pm 0.03$ and $f_{+}=-0.02 \pm 0.08 \pm 0.03$. Using a Bayesian method, we determine an upper limit on the fraction of right-handed W bosons of $f_{+}<0.07$ at $95 \% \mathrm{CL}$ assuming the Standard Model expectation for $f_{0}$; releasing this assumption we obtain a $95 \%$ CL exclusion area in the $\left(f_{+}, f_{0}\right)$ plane.

## Contents

1 Introduction 3
2 Measurement 4
3 Results from the Data 6
3.1 Exclusion limits . . . . . . . . . . . . . . . . . . . . . . . . . . . . 8

4 Conclusions 8

## 1 Introduction

An important consequence of the large mass of the top quark is that it decays in good approximation as a free quark. Its expected lifetime is $\approx 0.5 \times 10^{-24} \mathrm{~S}$ and it therefore decays about an order of magnitude faster than the time needed to form a bound state with other quarks. As a consequence, the spin information carried by the top quark is preserved in the decay products. The underlying dynamics can be studied with minimal impact from gluon radiation and binding effects of QCD.

Studies of decay angular distributions provide a direct probe of the $\mathrm{V}-\mathrm{A}$ nature of the Wtb coupling of the longitudinal and transverse polarized W bosons to the top quark. In the Standard Model, the fraction of decays to longitudinally polarized W bosons is determined by the masses of the involved particles and is expected to be

$$
\begin{equation*}
f_{0}=\frac{\Gamma\left(W_{0}\right)}{\Gamma\left(W_{0}\right)+\Gamma\left(W_{L}\right)+\Gamma\left(W_{R}\right)} \approx \frac{m_{t}^{2}}{2 m_{W}^{2}+m_{t}^{2}}=70.3 \pm 0.7 \% \tag{1}
\end{equation*}
$$

where the top-quark mass is $m_{t}=175 \pm 3 \mathrm{GeV} / \mathrm{c}^{2}$, the W -boson mass is $m_{W}=$ $80.4 \mathrm{GeV} / \mathrm{c}^{2}$ and the b-quark mass has been neglected. The fraction $f_{0}$ is enhanced due to the large coupling of the top quark to the Goldstone mode of the Higgs field. Fractions of left- and right-handed W bosons are denoted as $f_{-}$ and $f_{+}$respectively. In the Standard Model $f_{-}$is expected to be $\approx 30 \%$ and $f_{+}$ is suppressed by a factor $\left(m_{b} / m_{t}\right)^{2}$, yielding at next-to-leading order a value of $3.6 \times 10^{-4}$.

The angular distribution $\omega$ of the W-boson decay products with weak isospin $I_{3}=-1 / 2$ (charged lepton, down quark, or strange quark) in the rest frame of the W boson can be described by introducing the angle $\theta^{*}$ with respect to the W flight direction in the top-quark rest frame. The angular distribution has the general form

$$
\begin{equation*}
\omega\left(\theta^{*}\right)=f_{0} \cdot \omega_{0}\left(\theta^{*}\right)+f_{-} \cdot \omega_{-}\left(\theta^{*}\right)+f_{+} \cdot \omega_{+}\left(\theta^{*}\right), \tag{2}
\end{equation*}
$$

with

$$
\begin{align*}
& \omega_{0}\left(\theta^{*}\right)=\frac{3}{4}\left(1-\cos ^{2} \theta^{*}\right)  \tag{3}\\
& \omega_{-}\left(\theta^{*}\right)=\frac{3}{8}\left(1-\cos \theta^{*}\right)^{2}  \tag{4}\\
& \omega_{+}\left(\theta^{*}\right)=\frac{3}{8}\left(1+\cos \theta^{*}\right)^{2} \tag{5}
\end{align*}
$$

Figure 1 illustrates the definition of $\theta^{*}$ and shows angular distributions for longitudinal, right-handed and left-handed W boson decays as well as the distribution expected in the Standard Model.

Due to background, reconstruction and acceptance effects the measured distribution of $\cos \theta^{*}$ differs from $\omega\left(\cos \theta^{*}\right)$. However, the shape of the observed


Figure 1: The decay angle $\theta^{*}$ in top decays is defined as the angle between the down-type particle from the W decay and the top-quark direction in the W-boson rest frame (left). The right plot shows the angular distribution $\omega\left(\theta^{*}\right)$ for longitudinal, right- and left-handed W -boson decays as well as the distribution expected in the Standard Model.
$\cos \theta^{*}$ distribution depends on $f_{0}, f_{-}$and $f_{+}$and can therefore be used to extract these polarization fractions. The strategy of the measurement presented here is to fully reconstruct $t \bar{t}$ events and determine the $\cos \theta^{*}$ distribution, and then use a template method to determine the fractions $f_{0}, f_{-}$and $f_{+}$.

Measurement of $b \rightarrow s \gamma$ decay rates have indirectly limited the $\mathrm{V}+\mathrm{A}$ contribution in top-quark decays to less than a few percent. However, direct measurements of the $\mathrm{V}+\mathrm{A}$ contribution are still necessary because the limit from $b \rightarrow s \gamma$ assumes that electroweak penguin contributions are dominant. Direct measurements of the right-handed fraction did not find deviations from the Standard Model expectation.

## 2 Measurement

We select events with one lepton (e or $\mu$ ), missing transverse energy, and at least four reconstructed jets. One of the jets has to be b-tagged with a tight secondary vertex tagger. A cut on the scalar sum of the transverse momentum of all reconstructed objects in the event ( $H_{T}>250 \mathrm{GeV}$ ) has been applied to further reduce the background contribution. The dataset analyzed corresponds to a total integrated luminosity of $1.7 \mathrm{fb}^{-1}$. A total of 407 events satisfy the selection criteria and are used to extract the longitudinal and right-handed W boson helicity fractions. We estimate a contribution of background processes
mainly from $\mathrm{W}+$ jets production of 51.6 events. Top quarks are reconstructed using a kinematic fitter to find the best assignment of the top quark and its decay products to the reconstructed objects in the final state. All jet-parton assignments consistent with the b-tagging information are tried and nly the combination yielding the lowest fit chi-squared is used to reconstruct $\cos \theta^{*}$ from candidate events by boosting the lepton and the $W$ boson into the top quark rest frame.

The reconstructed $\omega\left(\theta^{*}\right)$ distributions for signal and background are parameterized $\left(\omega_{p a r}\left(\theta^{*}\right)\right)$ using a 3rd degree polynomial times two exponential functions of the following form:

$$
\begin{equation*}
\omega_{\text {par }}\left(\theta^{*}\right)=\left(\sum_{i=1,2,3} p_{i} \cdot \cos ^{i-1}\left(\theta^{*}\right)\right) \cdot e^{p_{4} \cdot \cos \left(\theta^{*}\right)} \cdot e^{p_{5} \cdot \cos ^{2}\left(\theta^{*}\right)} \tag{6}
\end{equation*}
$$

An unbinned extended likelihood fitter has been used to extract the uncorrected longitudinal and right-handed fractions, $F_{0}$, and $F_{+}$, from the data sample by comparing the $\cos \theta^{*}$ distribution for all selected events to parameterized MC templates for longitudinal, right-, and left-handed signal events, plus the parameterized background template. For a sample of $N_{s}$ selected events, the likelihood function is defined as:

$$
\begin{equation*}
\mathcal{L}=G\left(b \mid \mu_{b}, \sigma_{b}\right) \cdot P\left(s+b \mid \mu_{s}+\mu_{b}\right) \cdot \prod_{i=1}^{N_{s}}\left(f_{b} p_{b}\left(\cos \theta_{i}^{*}\right)+\left(1-f_{b}\right) p_{s}\left(\cos \theta_{i}^{*}\right)\right) \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{s}=F_{0} \cdot p_{0}+F_{+} \cdot p_{+}+\left(1-F_{0}-F_{+}\right) \cdot p_{-} \tag{8}
\end{equation*}
$$

The term $G\left(b \mid \mu_{b}, \sigma_{b}\right)$ is a Gaussian constraint on $b$, the mean number of expected background events in the sample. The Gaussian mean, $\mu_{b}$, and width, $\sigma_{b}$, are the a priori estimate of the background content of the lepton+jets sample mentioned above. The term $P\left(s+b \mid \mu_{s}+\mu_{b}\right)$ is the Poisson probability that the total number of signal + background events is $s+b$ given the mean expected number of signal + background events is $\mu_{s}+\mu_{b}$. Since we use the observed cross-section as an estimate of $\mu_{s}, \mu_{s}+\mu_{b}=N s$. The background fraction, $f_{b}$ is taken to be $b /(s+b)$. All probability density functions $p\left(\cos \theta^{*}\right)$ are required to satisfy the normalization condition $\int_{-1}^{+1} p\left(\cos \theta^{*}\right) d \cos \theta^{*}=1$.

Since the selecton and reconstruction efficiencies depend on the W boson helicity state, a correction is applied to obtain the true fractions. This is done by numerically inverting the equation

$$
\begin{equation*}
F_{i}=\frac{\sum_{j=1}^{3} f_{i} f_{j} A_{i j}}{\sum_{i, j=1}^{3} f_{i} f_{j} A_{i j}} \tag{9}
\end{equation*}
$$

to obtain the true fractions $f_{i}$ as a function of the fractions $F_{i}$ measured in the data. Here $i(j)$ refers to the polarization of the W that decays leptonically


Figure 2: The observed $\cos \theta^{*}$ distribution in the data (points) overlaid with the background (dotted), background + left-handed (dashed), and background + lefthanded + longitudinal (solid) distributions as determined by the 1D fit for $f_{0}$ (on the left) and for $f_{+}$(on the right).
(hadronically) and $A_{i j}$ is the acceptance determined from $t \bar{t}$ Monte Carlo samples generated with each of the $3^{2}$ possible helicity configurations for the two W bosons.

We perform three different fits to the data, always employing the constraint $f_{0}+f_{+}+f_{-}=1$. In the first two fits (the "1D" fits), $f_{+}\left(f_{0}\right)$ is fixed to its Standard Model expectation 0.0 (0.7) and we fit for $f_{0}\left(f_{+}\right)$. In the third fit (the " 2 D " fit), we allow both $f_{0}$ and $f_{+}$to float and fit for them simultaneously.

We evaluate systematic uncertainties by constructing ensembles of signal and background pseudoexperiments in which the systematic under study has been applied, and then fit them using the default signal and background templates. The resulting shifts in the mean longitudinal and right-handed fraction are used to quantify the systematic uncertainties. The absolute systematic uncertainties are $\pm 0.04( \pm 0.03)$ for the 1D $f_{0}\left(f_{+}\right)$fit, and $\pm 0.03$ for both fractions in the 2D fit. The dominant sources of uncertainty are the jet energy scale and uncertainties in the signal and background modeling.

## 3 Results from the Data

The $\cos \theta^{*}$ distribution observed in the data is shown in Figures 2 and 3. When fitting for $f_{0}$ with $f_{+}$fixed to zero, we find $f_{0}=0.57 \pm 0.11$ (stat) $\pm$ 0.04 (syst). When fitting for $f_{+}$with $f_{0}$ fixed to 0.7 (the SM expectation for $m_{t}=175 \mathrm{GeV} / c^{2}$ ), we find $f_{+}=-0.04 \pm 0.04$ (stat) $\pm 0.03$ (syst). When fitting for both simultaneously, we find $f_{0}=0.61 \pm 0.20$ (stat) $\pm 0.03$ (syst) and $f_{+}=-0.02 \pm 0.08$ (stat) $\pm 0.03$ (syst).

The likelihood curves for the fits are shown in Figures 4 and 5.


Figure 3: The observed $\cos \theta^{*}$ distribution in the data (points) overlaid with the background (dotted), background + left-handed (dashed), and background + lefthanded + longitudinal (solid) distributions as determined by the simultaneous fit for $f_{0}$ and for $f_{+}$.


Figure 4: Distributions of $-2 \Delta \ln \mathcal{L}$ for $f_{0}$ (left plot) and $f_{+}$(right plot) from the 1 D fits to the data.


Figure 5: Contours of constant $-\Delta \ln \mathcal{L}$ in the $\left(f_{+}, f_{0}\right)$ plane determined from the 2D fit to the data.

### 3.1 Exclusion limits

As our measurement of $f_{+}$is consistent with the SM expectation of zero, we proceed to set an upper limit on $f_{+}$following a Bayesian prescription. We multiply the $f_{+}$likelihood returned by the 1D fit by a prior constant between 0 and 1 to arrive at a posterior probability density distribution for $f_{+}$. We incorporate the effect of systematic uncertainties in the determination of the limit by modeling them as a Gaussian with mean zero and width equal to the total systematic uncertainty on $f_{+}$, and then convoluting the posterior probability density with this Gaussian. We then integrate this distribution out to the point where $95 \%$ of the area is captured; see Figure 6. We obtain the limit $f_{+}<0.07$ at $95 \%$ CL (assuming $f_{0}=0.7$ ) via this method.

We obtain a more general exclusion from the 2D fit by multiplying the likelihood curve of Figure 5 by a prior that is flat in the region $f_{0}>0, f_{+}>0$, $f_{0}+f_{+}<1$ and is zero everywhere else. This gives a posterior probability density surface in the $\left(f_{+}, f_{0}\right)$ plane for which we can find the contour of constant probability density that captures $95 \%$ of the volume under the surface. This is shown in Figure 7.

## 4 Conclusions

We use a sample of fully reconstructed $t \bar{t}$ events to determine the W-boson helicity fractions. Using an unbinned liklihood fit and fixing the right-handed fraction to zero we measure fraction of longitudinally polarized W -bosons to be

$$
\begin{equation*}
f_{0}=0.57 \pm 0.11 \text { (stat) } \pm 0.04 \text { (syst). } \tag{10}
\end{equation*}
$$

This is consistent with the Standard Model expectation of $f_{0}=0.70$. Using the same fit but fixing the longitudinal fraction to its Standard Model expectation


Figure 6: Posterior probability distribution for and $95 \%$ CL upper limit on $f_{+}$, assuming $f_{0}=0.7$.


Figure 7: The area of the ( $f_{+}, f_{0}$ ) plane excluded by this analysis at $95 \%$ CL (hatched region - do you have anti-aliasing switched off?).
we measure the fraction of right-handed polarized W-bosons to be

$$
\begin{align*}
& f_{+}=-0.04 \pm 0.04(\text { stat }) \pm 0.03 \text { (syst) }  \tag{11}\\
& f_{+}<0.07 \text { at } 95 \% \text { CL } \tag{12}
\end{align*}
$$

Fitting for the longitudinal and right-handed helicity fractions simultaneously, we obtain:

$$
\begin{align*}
f_{0} & =0.61 \pm 0.20(\text { stat }) \pm 0.03(\text { syst })  \tag{13}\\
f_{+} & =-0.02 \pm 0.08(\text { stat }) \pm 0.03(\text { syst }) \tag{14}
\end{align*}
$$

A measurement of $f_{+}$significantly different from zero would be evidence of new physics.

