An Examination of the Effects of Station Longitude Errors on Doppler Plus Range and Doppler Only Orbit Determination Solutions With an Emphasis on a *Viking* Mission Trajectory

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During the early Viking Mission accuracy analysis studies, it was discovered that station location errors may degrade the navigation more for doppler plus range solutions than for doppler only solutions. An explanation of this seemingly curious occurrence is given.

I. Introduction

Early in the *Viking* Mission accuracy analysis studies, a set of statistics describing the effect of station location errors on navigational accuracies were obtained which at first glance were hard to believe. These statistics, which were generated by a weighted least-squares batch solution filter operating on data supplied by the *Viking* trajectory and tracking station described in Table 1, are shown in Fig. 1 and involve the behavior of the semimajor axis (SMAA) of the error ellipse in the B-plane (described in Fig. 2) when various amounts of data were included in the solution. The standard deviation of SMAA given in Fig. 1 were computed by the Double Precision Orbit Determination Program (DPODP, Mod 5.2) consider option (Ref. 1). These consider standard deviations reflect the influence that both data noise and constant errors in particular parameters may have on the orbit determination solution.

The interesting feature of Fig. 1 is that, although initially the doppler plus range solution is superior to the doppler only solution, as more data is included the situation is reversed. This degradation of the solution by the addition of more information was very curious and required more of an explanation than just stating that it is a manifestation of an improperly modeled filter.

II. Verification of the DPODP Consider Option

When the results given in Fig. 1 were first acquired, one possible explanation was that the DPODP consider option was not working properly. A verification of the consider option for one parameter, p, may be obtained by first using the procedure outlined in Table 2 to determine the effect that a constant error in p may have on the solution.

Figure 3 contains errors in $\mathbf{B} \cdot \mathbf{R}$ obtained by following the above procedure for a station longitude error. The

longitude error was chosen for investigation because it is primarily responsible for the results shown in Fig. 1.

Also included in Fig. 3 are the computed standard deviations in $\mathbf{B} \cdot \mathbf{R}$ resulting from data noise alone, σ_d, and the consider standard deviation, σ_c, describing a station longitude error of 3 meters. If the consider option is working properly, the following equation will be satisfied:

$$\sigma_c^2 = \sigma_d^2 + [\Delta (\mathbf{B} \cdot \mathbf{R})]^2 \tag{1}$$

Substitution of the numbers contained in Fig. 3 into this equation did maintain the equality and it was therefore concluded that the consider option was working properly.

III. Spherical Spacecraft State Errors at Epoch

In order to explain the behavior of the standard deviation of the SMAA shown in Fig. 1, it is necessary to remove the effects of mapping nearly six months to encounter and examine the spacecraft state errors at epoch. Figure 4 contains the errors in the spacecraft state at epoch, in spherical coordinates, produced by a longitude error of 3 meters, when doppler only and doppler plus range data are included in the solution. It should be noted that the doppler only solutions may require a few days of data before stabilizing.

Starting with the results in Figs. 3 and 4, one may construct the following explanation of the results shown in Fig. 1:

(1) Initially the data noise is the dominant error source and the addition of range data reduces the effect of the data noise to such an extent that the doppler plus range solution is superior to the doppler only solution.

(2) As more data is included in the solution the effect of data noise is reduced and the station longitude error becomes the dominant error source.

(3) The station longitude error translates primarily into right ascension and range errors for doppler only solutions or right ascension, declination rate and right ascension rate errors for doppler plus range solutions.

(4) After a six-month mapping the velocity errors are magnified to such an extent that the doppler only solution will be superior to the doppler plus range solution.

IV. Analytical Explanation of Spacecraft Errors Produced by a Station Longitude Error

The problem has now been reduced from explaining the effects of station location errors at encounter to explaining these effects at epoch. To obtain such an explanation, it is convenient to use an analytic model of the observable.

For data arcs of a few days the spacecraft state errors produced by a station longitude error can be grossly predicted by the 6 parameter model. This model is described in Ref. 2 and uses the following equation to represent the range-rate or doppler observables:

$$\dot{\rho} = a + b \sin \omega t + c \cos \omega t + dt + e \omega t \sin \omega t + f \omega t \cos \omega t \tag{2}$$

where

$a = \dot{r}_0$

$b = r_{s\omega} \cos \delta_0$

$c = b\epsilon$

$d = \ddot{r}_{g0} + r_0 (\dot{\delta}_0^2 + \dot{\alpha}_0^2 \cos^2 \delta_0)$

$e = -r_s (\dot{\delta}_0 \sin \delta_0 - \epsilon \dot{\alpha}_0 \cos \delta_0)$

$f = -r_s (\dot{\alpha}_0 \cos \delta_0 + \epsilon \dot{\delta}_0 \sin \delta_0)$

$\dot{\rho}$ = topocentric range rate

$\ddot{r}_{g0} = -\mu \left\{ \dfrac{r_0}{r_{p0}^3} - r_e \left(\dfrac{1}{r_{p0}^3} - \dfrac{1}{r_{p0}^3} \right) \right.$

$\left. \times [\cos \delta_0 \cos \delta_{s0} \cos (\alpha_0 - \alpha_{s0}) + \sin \delta_0 \sin \delta_{s0}] \right\}$

= geocentric acceleration

$\epsilon = (\lambda - \lambda^*) - (\alpha - \alpha^*)$

μ = solar gravitational constant

r_e = Sun–Earth distance

$r_p = \{r^2 + r_e^2 - 2r r_e [\cos \delta \cos \delta_s$
$\times \cos (\alpha - \alpha_s) + \sin \delta \sin \delta_s]\}^{1/2}$

= Sun spacecraft distance

δ_s = declination of the Sun

α_s = right ascension of the Sun

x^* = a priori value of x

$\dot{x} = \dfrac{dx}{dt}$

$x_0 = x(t = 0)$

t = time past meridian crossing

For the *Viking* trajectory of Table 2, this representation retains its usefulness for data arcs of a few days in length.

As briefly described in the previous article[1] and more fully in Ref. 2, an error analysis using Eq. (2) proceeds by using the 6 coefficients $a \rightarrow f$ as correlated data points which described the information contained in the range-rate observable. These data points may then be used to obtain solutions and the associated covariances. In particular, an error in the station longitude will produce errors in the c, e, and f coefficients, which will be treated as "before-the-fit" residuals. The solution filter will then generate compensating errors in the spacecraft state to minimize the "after-the-fit" residuals in a least-squares sense. The results of following this procedure are also included in Fig. 4 for data arcs of two and four days and are in fairly good agreement with the DPODP values.

The physical process behind the results illustrated in Fig. 4 may be understood by examining Eq. (2). As mentioned previously, a longitude error will produce errors in the c, e, and f coefficients. Since the longitude and right ascension enter into these coefficients in the same way, the solution filter will want to make a compensating error in the right ascension. However, this change in the right ascension will produce a change in the d, or acceleration coefficient of Eq. (2), since the gravitation acceleration is a function of the spacecraft right ascension. This change in d must be accounted for by errors in the remaining components of the spacecraft state. For a doppler only solution, the error will appear in the range because the range occurs only in the d coefficient and therefore a change in the range affects this coefficient only. The range error, which will compensate for the change in d produced by the right ascension error, is given by the following equation:

[1]Ondrasik, V. J., and Rourke, K. H., "An Analytical Study of the Advantages Which Differenced Tracking Data May Offer for Ameliorating the Effects of Unknown Spacecraft Accelerations" (this volume).

$$\Delta r = \frac{\partial d / \partial \alpha}{\partial d / \partial r} \Delta \lambda$$

$$= \frac{0.489 \times 10^{-5}}{0.204 \times 10^{-13}} \times 0.524 \times 10^{-6} = 126 \, \text{km} \qquad (3)$$

Using the numerical values associated with the *Viking* trajectory of Table 1 yields the result shown in Eq. (3). This value is almost identical to the range error found by extrapolating the stable DPODP solutions of Fig. 4.

When the doppler data is supplemented by a range point, the range is essentially deleted from the solution and cannot be used to cancel the error in the acceleration coefficient produced by the right ascension error. Since the declination is strongly determined by the b coefficient, the acceleration coefficient error will be compensated for by errors in $\dot{\delta}$ and $\dot{\alpha}$. It is not possible to obtain a simple equation analogous to Eq. (3) to express these velocity errors because $\dot{\delta}$ and $\dot{\alpha}$ are also contained in the e and f coefficients.

V. Summary

The preceding section has shown that for fairly short data arcs a station longitude error will produce an error in the spacecraft's right ascension. This right ascension error will in turn generate an error in the spacecraft's geocentric acceleration. To minimize the effects of this acceleration error, compensating errors will be made in the range for doppler only solutions, and in $\dot{\delta}$ and $\dot{\alpha}$ for doppler plus range solutions. If these errors are mapped over a sufficiently long period of time, the velocity errors of the doppler plus range solution may assume a greater importance than the position errors of the doppler only solution. It is for this reason that station location errors may degrade doppler and range solutions more than doppler only solutions when an improperly modeled solution filter is used. This is the set of circumstances which lead to the seemingly strange *Viking* accuracy analysis results illustrated in Fig. 1.

References

1. Moyer, T. D., *Mathematical Formulation of the Double-Precision Orbit Determination Program (DPODP)*, Technical Report 32-1527, pp. 109–117. Jet Propulsion Laboratory, Pasadena, Calif., May 15, 1971.

2. Ondrasik, V. J., and Curkendall, D. W., "A First-Order Theory for Use in Investigating the Information Content Contained in a Few Days of Radio Tracking Data," in *The Deep Space Network Progress Report*, Technical Report 32-1526, Vol. III, pp. 77–93. Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1971.

Table 1. Description of the *Viking* trajectory

Geocentric coordinate[a]	Value
r = range	0.885×10^8 km
δ = declination	20.3 deg
α = right ascension	58.1 deg
$\dot{r}$ = range rate	15.3 km/s
$\dot{\delta}$ = declination rate	0.245×10^{-7} rad/s
$\dot{\alpha}$ = right ascension rate	0.890×10^{-7} rad/s
r_s = station distance off the spin axis	5.20×10^3 km
λ = station longitude	243 deg
[a]Epoch 1976 Jan 21. Encounter 1976 July 14.	

Table 2. Procedure for determining the errors in the solution produced by a constant error in a particular parameter

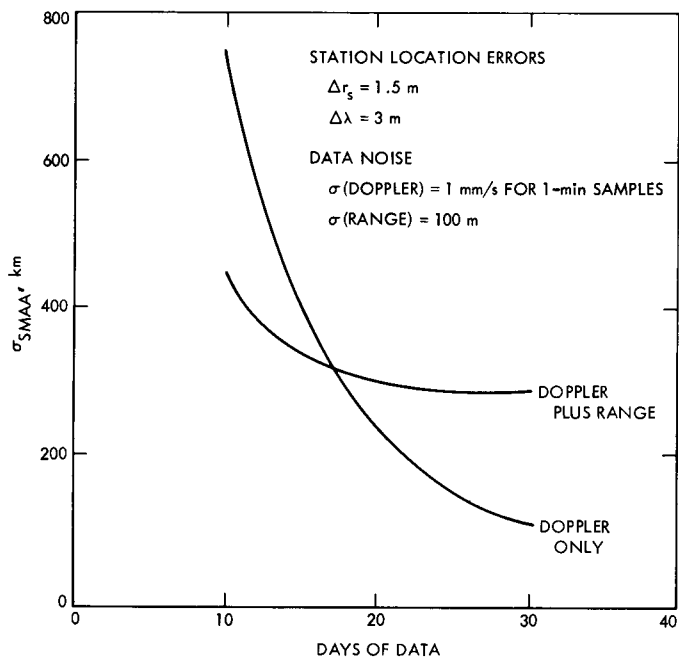| Step	Operation
1	Simulate the observed data using a particular value of the parameter, p_0.
2	Calculate the computed data using another value of the parameter, $p_c = p_0 + \Delta p$.
3	Form the (observed–computed) residuals.
4	Obtain the solution, with p not included in the solution set.

Fig. 1. Consider SMAA for doppler and doppler plus range solutions
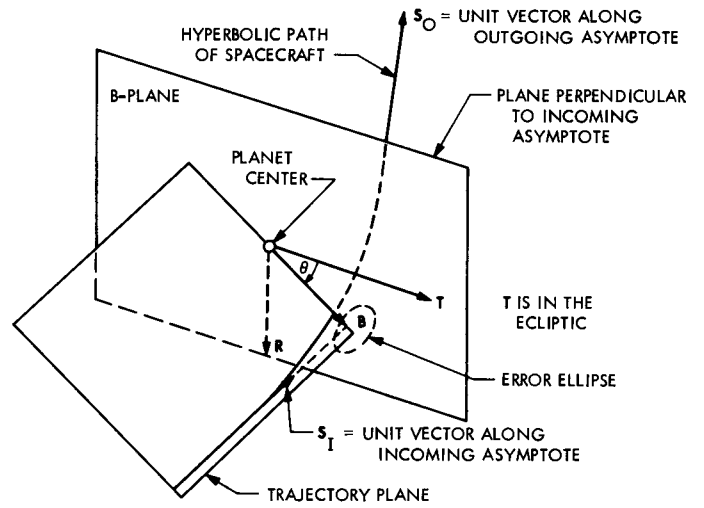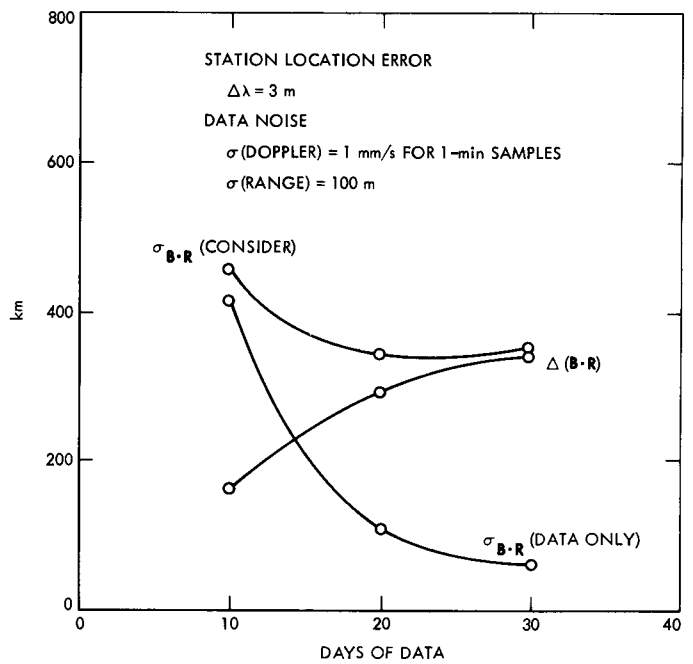

Fig. 2. B-plane and error ellipse

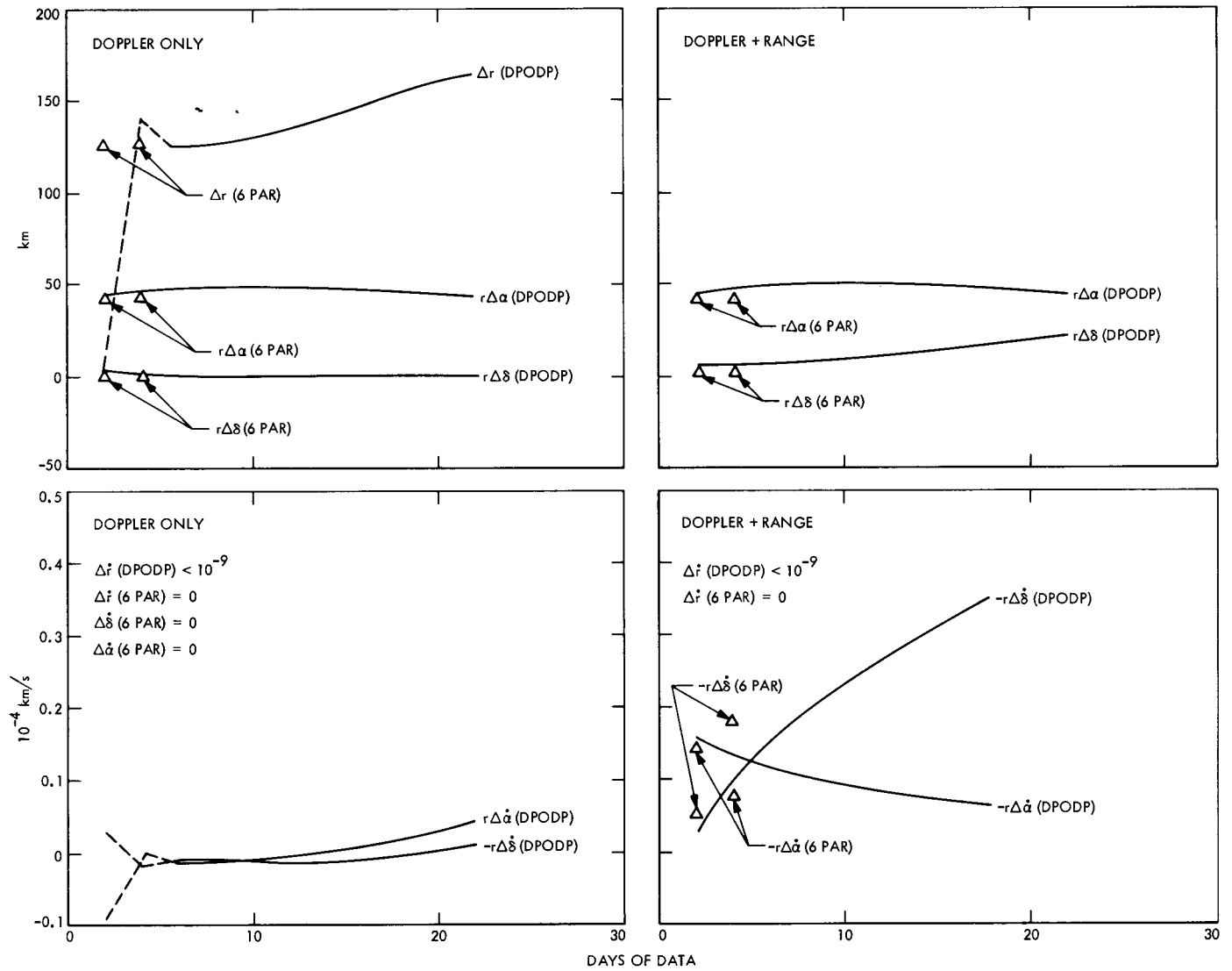Fig. 3. Errors in B·R produced by a station longitude error of 3 meters

Fig. 4. Spherical spacecraft state errors produced by a station longitude error of 3 meters