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SOME NOTES ON THE DETERMINATION OF THE STICK-FIXED NEUTRAL POINT FROM WIND-TUNNEL DATA

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SOME NOTES ON THE DETERMINATION OF THE STICK-FIXED NEUTRAL POINT FROM WIND-TUNNEL DATA

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SUMMARY

Two methods are presented for determining the horizontal location of the stick-fixed neutral point from wind-tunnel data. One method involves the solution of a mathematical equation; whereas the other method is a graphical solution for the same mathematical equation. A method is also included for determining the vertical variation of the neutral point. The combined horizontal and vertical variation of the neutral point completely describes the stick-fixed longitudinal stability of airplanes that have large allowable center-of-gravity shifts.

INTRODUCTION

The concept of the neutral point has been treated in references 1 to 4, and its usefulness in the analysis of static longitudinal stability, especially with regard to the effect of power, has been proved. The determination of the neutral point from flight data is discussed in reference 3; whereas reference 4 presents the methods used with wind-tunnel data.

The present report offers two simplified methods of determining the horizontal location of the neutral point from wind-tunnel data plotted as pitching-moment coefficient C_m against lift coefficient C_L for several stabilizer-setting tests with the elevator neutral; the method applies equally well to tests made with various elevator deflections with the stabilizer setting fixed. A method is presented for determining the vertical variation of the neutral point. The combined horizontal and vertical variation completely describes the stick-fixed longitudinal stability of airplanes that have large allowable center-of-gravity shifts.

The neutral point is defined as the location of the center of gravity of the airplane when the airplane is trimmed ($C_m = 0$) and when the stick-fixed stability, as measured by dC_m/dC_L about the center of gravity, is neutral $\left(\frac{dC_m}{dC_L} = 0\right)$. Data obtained from wind-tunnel tests are usually plotted as C_m against C_L for several stabilizer settings at a specified center-of-gravity location. The neutral point may readily be determined from these data provided the assumption is valid that the rate of change of the slope of the pitching-moment curve (about a given c.g. and at a given lift coefficient) is constant with stabilizer setting i_t. That this assumption is valid is proved in appendix A, in which the slope of the tail lift curve is assumed to be constant, a condition which usually holds up to the region near the stall of the tail surface. If the data are obtained for unstalled conditions of the tail - which can be attained by proper choice of stabilizer settings - the neutral-point determinations will be valid. The symbols used in this paper are defined as they occur in the text and are summarized in appendix B.

HORIZONTAL LOCATION OF NEUTRAL POINT

Method I

Consider the two arbitrary curves of C_m against C_L for different stabilizer settings shown in figure 1 and suppose that the neutral point of the airplane is to be determined at some lift coefficient $C_L = 1.2$. It is apparent that, at $C_L = 1.2$, the airplane is untrimmed ($C_m \neq 0$) for both stabilizer settings and that, as is general for power-on conditions, $\left(dC_m/dC_L\right)_x$ at $C_L = 1.2$ depends upon stabilizer setting. Even if C_m were zero, moreover, the value of $\left(dC_m/dC_L\right)_x$ would not indicate how far the center of gravity might be moved to obtain neutral stability because, when i_t is changed to retrim the airplane at a new center of gravity, the value of $\left(dC_m/dC_L\right)_x$ is changed.

The value of C_m/C_L at $C_L = 1.2$ does represent the distance the center of gravity may be moved parallel to the model reference line in order to balance C_m to zero.

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This movement also increases the stability by an amount approximately equal to C_m/C_L, because of the shift in center of gravity. For each stabilizer curve, therefore, the center of gravity for trim $(C_m = 0)$ and the stability about this new center of gravity may be determined.

About a new center-of-gravity location x_n such that

$$x_n = x - \frac{C_m}{C_L} \tag{1}$$

where x is the original center-of-gravity location in chords behind the leading edge of the mean aerodynamic chord, the pitching moment is trimmed $(C_m = 0)$, and the stability about this center of gravity is

$$\left(\frac{dC_m}{dC_L}\right)_{x_n} = \left(\frac{dC_m}{dC_L}\right)_x - \frac{C_m}{C_L} \tag{2}$$

where $\left(dC_m/dC_L\right)_x$ and C_m/C_L are values taken from the original data, as from figure 1. For neutral stability, therefore,

$$\left(\frac{dC_m}{dC_L}\right)_{x_n} = 0 \tag{3}$$

and

$$\left(\frac{dC_m}{dC_L}\right)_x = \frac{C_m}{C_L} \tag{4}$$

It is hence apparent that, if $\left(dC_m/dC_L\right)_x$ is plotted against C_m/C_L for two stabilizer settings at a given C_L (fig. 2), the location of the center of gravity for neutral stability is the point where $\left(dC_m/dC_L\right)_x$ is equal to C_m/C_L; that is, the neutral point is the point of intersection between a straight line connecting these two plotted points and a line having the equation $\left(dC_m/dC_L\right)_x = C_m/C_L$. In figure 2, the neutral point is given in chords forward or rearward of the center of gravity about which the data are given depending upon whether C_m/C_L is positive or negative at the point of intersection.

If more than two stabilizer curves are available and the values of $\left(dC_m/dC_L\right)_x$ against C_m/C_L do not form a straight line as in figure 2, a curve must be faired through the points to determine the intersection with the line $\left(dC_m/dC_L\right)_x = C_m/C_L$. In this case, the variation of tail lift with tail angle of attack is not linear. All the formulas presented herein, however, assume the usual condition that all points fall on a straight line.

It has been shown that the neutral point is the solution of a set of simultaneous equations, which are represented in figure 2 by the two straight lines. One line has $\left[C_{m_1}/C_L, \left(dC_m/dC_L\right)_1\right]$ as the coordinates of one point and $\left[C_{m_2}/C_L, \left(dC_m/dC_L\right)_2\right]$ as the coordinates of another point. (See fig. 1.) The equation of a line passing through these points is

$$\frac{\left(\frac{dC_m}{dC_L}\right)_x - \left(\frac{dC_m}{dC_L}\right)_1}{\left(\frac{dC_m}{dC_L}\right)_2 - \left(\frac{dC_m}{dC_L}\right)_1} = \frac{\frac{C_m}{C_L} - \frac{C_{m_1}}{C_L}}{\frac{C_{m_2}}{C_L} - \frac{C_{m_1}}{C_L}} \tag{5}$$

The equation for the other line is

$$\left(\frac{dC_m}{dC_L}\right)_x = \frac{C_m}{C_L} \tag{6}$$

Equations (5) and (6) are solved simultaneously to obtain an expression for C_m/C_L for neutral stability, which is the equation for the "static margin" specified in reference 4. Substituting the expression for C_m/C_L for neutral stability in equation (1) yields

$$x_o = x - \frac{\left(\frac{C_{m_1}}{C_L}\right)\left(\frac{dC_m}{dC_L}\right)_2 - \left(\frac{C_{m_2}}{C_L}\right)\left(\frac{dC_m}{dC_L}\right)_1}{\left[\left(\frac{dC_m}{dC_L}\right)_2 - \left(\frac{dC_m}{dC_L}\right)_1\right] + \left(\frac{C_{m_1}}{C_L} - \frac{C_{m_2}}{C_L}\right)} \tag{7}$$

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where

x_0 location of neutral point, chords behind leading edge of mean aerodynamic chord

C_{m_1} untrimmed pitching-moment coefficient at C_L for stabilizer setting 1 (measured from $C_m = 0$)

C_{m_2} untrimmed pitching-moment coefficient at C_L for stabilizer setting 2 (measured from $C_m = 0$)

$\left(\frac{dC_m}{dC_L}\right)_1$ slope of stabilizer curve 1 at C_L (measured from horizontal)

$\left(\frac{dC_m}{dC_L}\right)_2$ slope of stabilizer curve 2 at C_L (measured from horizontal)

x center-of-gravity location for which data are given, chords behind leading edge of mean aerodynamic chord

Method II

A graphical method of applying the same principles to find the neutral point may be designated method of intersection of tangents. It may be shown that, if the tangents to two or more stabilizer curves at a given lift coefficient are extended until they meet (fig. 3), the slope of the line drawn back through the origin of C_m and C_L from this point of intersection gives the location of the neutral point in chords forward or rearward of the center of gravity about which the data are computed.

If the stabilizer curves are parallel, as for power-off tests, the point of intersection would theoretically be at infinity and the slope of the stabilizer curve itself may be used to determine the neutral point; this procedure is customary for windmilling or propeller-off test results.

If a tangent can be drawn to any stabilizer curve passing through the origin $(C_m = 0, C_L = 0)$, the slope of this line is the distance of the neutral point in chords forward or rearward of the center of gravity about which the data are given at the lift coefficient of tangency (fig. 3). This method has been mentioned in reference 4 and is a special case of the method of intersection of tangents.

VERTICAL LOCATION OF NEUTRAL POINT

After the neutral points have been located along a horizontal line parallel to the thrust or reference line, the next step is the determination of the vertical variation of the neutral point.

If the moments about the center of gravity are transferred to a center of gravity y chords below the original center of gravity, C_m becomes

$$C_{m_b} = C_{m_a} + C_C y \qquad (8)$$

where subscript b denotes the pitching-moment coefficient about the lower center of gravity and subscript a, about the upper center of gravity. Then

$$\left(\frac{dC_m}{dC_L}\right)_b = \left(\frac{dC_m}{dC_L}\right)_a + \frac{dC_C}{dC_L} y \qquad (9)$$

where the chord-force coefficient

$$C_C = C_D \cos \alpha - C_L \sin \alpha$$

or approximately

$$C_C = C_D - C_L \frac{\alpha}{57.3}$$

and where C_D is the drag coefficient. Then

$$\frac{C_{m_b}}{C_L} = \frac{C_{m_a}}{C_L} + \frac{C_D}{C_L} y - \frac{\alpha}{57.3} y \qquad (10)$$

Because

$$\frac{dC_C}{dC_L} = \frac{dC_D}{dC_L} - \frac{\alpha}{57.3} - C_L \frac{d\alpha/dC_L}{57.3}$$

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$$\frac{dC_C}{dC_L} = \frac{dC_D}{dC_L} - \frac{\alpha}{57.3} - \left(\frac{\alpha - \alpha_{l_o}}{57.3}\right)$$

and

$$\frac{dC_C}{dC_L} = \frac{dC_D}{dC_L} - \left(\frac{2\alpha - \alpha_{l_o}}{57.3}\right)$$

Equation (9) then becomes

$$\left(\frac{dC_m}{dC_L}\right)_b = \left(\frac{dC_m}{dC_L}\right)_a + \frac{dC_D}{dC_L}\,y - \left(\frac{2\alpha - \alpha_{l_o}}{57.3}\right)y \tag{11}$$

If these values of dC_m/dC_L and C_m/C_L are substituted in equation (7), a new neutral point may be determined horizontally at a center of gravity y chords below the original center of gravity. Subtracting from this location of the neutral point the location of the original neutral point gives the horizontal change in neutral point Δx for a vertical center-of-gravity shift y. The expression may be shown to be

$$\frac{\Delta x}{y} = \frac{\left[\left(\frac{dC_m}{dC_L}\right)_2 - \left(\frac{dC_m}{dC_L}\right)_1\right]\left(\frac{C_D}{C_L} - \frac{\alpha}{57.3}\right) + \left(\frac{C_{m_1}}{C_L} - \frac{C_{m_2}}{C_L}\right)\left(\frac{dC_D}{dC_L} - \frac{2\alpha}{57.3} + \frac{\alpha_{l_o}}{57.3}\right)}{\left[\left(\frac{dC_m}{dC_L}\right)_2 - \left(\frac{dC_m}{dC_L}\right)_1\right] + \left(\frac{C_{m_1}}{C_L} - \frac{C_{m_2}}{C_L}\right)} \tag{12}$$

where

α angle of attack at given C_L, degrees

α_{l_o} angle of attack for zero lift, degrees

and where C_D/C_L and dC_D/dC_L are taken at the given C_L and are essentially independent of stabilizer setting. The directions in which Δx and y are measured are shown in figure 4.

Equation (12) may, of course, be avoided by transferring the data vertically (mathematically) to another center of gravity, plotting the results, determining neutral points along the new level, and thus establishing two points in the chart (fig. 4) through which the neutral-point line may be drawn.

If the method of intersection of tangents illustrated in figure 3 has been used to determine the neutral point, equation (12) may be simplified thus:

Let C_{m_P} and C_{L_P} be the ordinate and abscissa of the point of intersection of the tangents at any C_L (fig. 3). Formula (12) can be written

$$\frac{\Delta x}{y} = \frac{\left[\dfrac{\left(\dfrac{dC_m}{dC_L}\right)_2 - \left(\dfrac{dC_m}{dC_L}\right)_1}{\dfrac{C_{m_2}}{C_L} - \dfrac{C_{m_1}}{C_L}}\right] - \left(\dfrac{dC_D}{dC_L} - \dfrac{2\alpha}{57.3} + \dfrac{\alpha_{l_0}}{57.3}\right)}{\left[\dfrac{\left(\dfrac{dC_m}{dC_L}\right)_2 - \left(\dfrac{dC_m}{dC_L}\right)_1}{\dfrac{C_{m_2}}{C_L} - \dfrac{C_{m_1}}{C_L}}\right] - 1} \qquad (13)$$

From figure 4,

$$\left(\frac{dC_m}{dC_L}\right)_1 = \frac{C_{m_1} - C_{m_P}}{C_L - C_{L_P}}$$

and

$$\left(\frac{dC_m}{dC_L}\right)_2 = \frac{C_{m_2} - C_{m_P}}{C_L - C_{L_P}}$$

Subtracting gives

$$\left(\frac{dC_m}{dC_L}\right)_2 - \left(\frac{dC_m}{dC_L}\right)_1 = \frac{C_{m_2} - C_{m_1}}{C_L - C_{L_P}}$$

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and

$$K = \frac{\left(\frac{dC_m}{dC_L}\right)_2 - \left(\frac{dC_m}{dC_L}\right)_1}{\frac{C_{m_2}}{C_L} - \frac{C_{m_1}}{C_L}}$$

$$= \frac{\frac{C_{m_2} - C_{m_1}}{C_L - C_{L_p}}}{\frac{C_{m_2} - C_{m_1}}{C_L}}$$

$$= \frac{C_L}{C_L - C_{L_p}} \tag{14}$$

If equation (14) is then substituted in equation (13), equation (13) becomes

$$\frac{\Delta x}{y} = \frac{\frac{dC_D}{dC_L} - \frac{2\alpha}{57.3} + \frac{\alpha l_o}{57.3} - K\left(\frac{C_D}{C_L} - \frac{\alpha}{57.3}\right)}{1 + K} \tag{15}$$

It is seen that only one slope dC_D/dC_L must be determined graphically to find the neutral-point variation with vertical movement of the center of gravity.

When the power effect is small or, in any case, when the C_m-curves are parallel, the point of intersection is at infinity and $C_{L_p} = -\infty$. Because K goes to zero, equation (15) simplifies to

$$\frac{\Delta x}{y} = \frac{dC_D}{dC_L} - \frac{2\alpha}{57.3} + \frac{\alpha l_o}{57.3} \tag{16}$$

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APPENDIX A

The variation of the slope of the pitching-moment curve may be shown to be constant with stabilizer setting provided the slope of the tail lift curve is constant. The proof follows:

If the pitching-moment coefficient of the airplane about its center of gravity is given in terms of the pitching-moment coefficient with tail off and the pitching-moment coefficient contributed by the tail,

$$C_m = C_{m_o} + \Delta C_m$$

or

$$C_m = C_{m_o} - V\alpha_t \frac{dC_L}{d\alpha_t} \frac{q_t}{q_o}$$

which may be rewritten as

$$C_m = C_{m_o} - V\ (\alpha - \epsilon)\ \frac{dC_L}{d\alpha_t} \frac{q_t}{q_o} - Vi_t \frac{dC_L}{d\alpha_t} \frac{q_t}{q_o} \qquad (17)$$

where

C_{m_o} pitching-moment coefficient, tail off

ΔC_m pitching-moment coefficient contributed by tail

V tail volume $\left(\frac{S_t}{S} \cdot \frac{l_t}{\bar{c}}\right)$

S_t horizontal tail area

S wing area

l_t tail arm

$\bar{c}$ mean aerodynamic chord

α_t angle of attack of horizontal tail with respect to relative wind at tail, degrees

ϵ angle of downwash at tail with respect to undisturbed stream, degrees

$\dfrac{dC_{L_t}}{d\alpha_t}$ slope of tail lift curve, per degree

q_t/q_o dynamic pressure at tail with respect to free-stream dynamic pressure

Equation (17) may be differentiated with respect to C_L to give

$$\frac{dC_m}{dC_L} = \left(\frac{dC_m}{dC_L}\right)_o - V(\alpha - \epsilon)\,\frac{dC_L}{d\alpha_t}\,\frac{d\frac{q_t}{q_o}}{dC_L} - V\,\frac{dC_L}{d\alpha_t}\,\frac{q_t}{q_o}\left(\frac{d\alpha}{dC_L} - \frac{d\,\epsilon}{dC_L}\right) - Vi_t\,\frac{dC_L}{d\alpha_t}\,\frac{d\frac{q_t}{q_o}}{dC_L} \qquad (18)$$

At a given C_L, all values in equation (18) are fixed except the value of i_t. The expression at a given C_L then becomes

$$\frac{dC_m}{dC_L} = C_o - C_1 - C_2 - C_3\,i_t$$

or

$$\frac{dC_m}{dC_L} = C_4 - C_3\,i_t \qquad (19)$$

where the C's are constants. Differentiating equation (19) with respect to i_t gives

$$\frac{d\left(\frac{dC_m}{dC_L}\right)}{di_t} = -C_3 \qquad (20)$$

which indicates that the rate of change of slope dC_m/dC_L is constant with change in i_t if the slope of the tail lift curve is constant.

It may also be noted, by a similar procedure, that

$$\frac{dC_m}{di_t} = -C_5 \tag{21}$$

It is then apparent that a plot of dC_m/dC_L against C_m or against C_m/C_L will be a straight line at a given C_L for various values of i_t.

APPENDIX B

SYMBOLS

C_m pitching-moment coefficient

C_L lift coefficient

δ_e elevator deflection with respect to stabilizer chord line, degrees (positive with T.E. down)

i_t angle of incidence of stabilizer (stabilizer setting) with respect to horizontal reference line of model, degrees (positive with T.E. down)

$\left(dC_m/dC_L\right)_x$ slope of curve of C_m against C_L at any C_L and stabilizer setting for center of gravity at x

x original center-of-gravity location about which data are given, chords behind leading edge of mean aerodynamic chord

x_n new center-of-gravity location about which $C_m = 0$, chords behind leading edge of mean aerodynamic chord

x_o location of neutral point, chords behind leading edge of mean ~~a~~erodynamic chord

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$\left(dC_m/dC_L\right)_{x_n}$ slope of curve of C_m against C_L for center of gravity at x_n

$\left(dC_m/dC_L\right)_1$ slope of stabilizer curve 1 at C_L (measured from horizontal)

$\left(dC_m/dC_L\right)_2$ slope of stabilizer curve 2 at C_L (measured from horizontal)

C_{m_1} untrimmed pitching-moment coefficient at C_L for stabilizer setting 1 (measured from $C_m = 0$)

C_{m_2} untrimmed pitching-moment coefficient at C_L for stabilizer setting 2 (measured from $C_m = 0$)

C_{m_a} pitching-moment coefficient at original center-of-gravity level for a given set of conditions (C_{m_1}, C_{m_2}, etc.)

C_{m_b} C_{m_a} transferred vertically (with respect to horizontal reference line of model) to a lower center of gravity

$\left(dC_m/dC_L\right)_a$ slope of curve at C_{m_a}

$\left(dC_m/dC_L\right)_b$ slope of curve at C_{m_b}

C_C chord-force coefficient ($C_D \cos \alpha - C_L \sin \alpha$)

C_D drag coefficient

y vertical center-of-gravity movement, chords downward from original center of gravity

α angle of attack of horizontal reference line of model, degrees

α_{l_o} angle of attack for zero lift, degrees

dC_D/dC_L rate of change of drag coefficient with lift coefficient

Δx horizontal change in neutral point for a vertical shift in center of gravity of y chords, chords

C_{m_P}, C_{L_P} coordinates of point of intersection of tangents to a series of stabilizer curves at a given C_L

$$K = \frac{C_L}{C_L - C_{L_P}}$$

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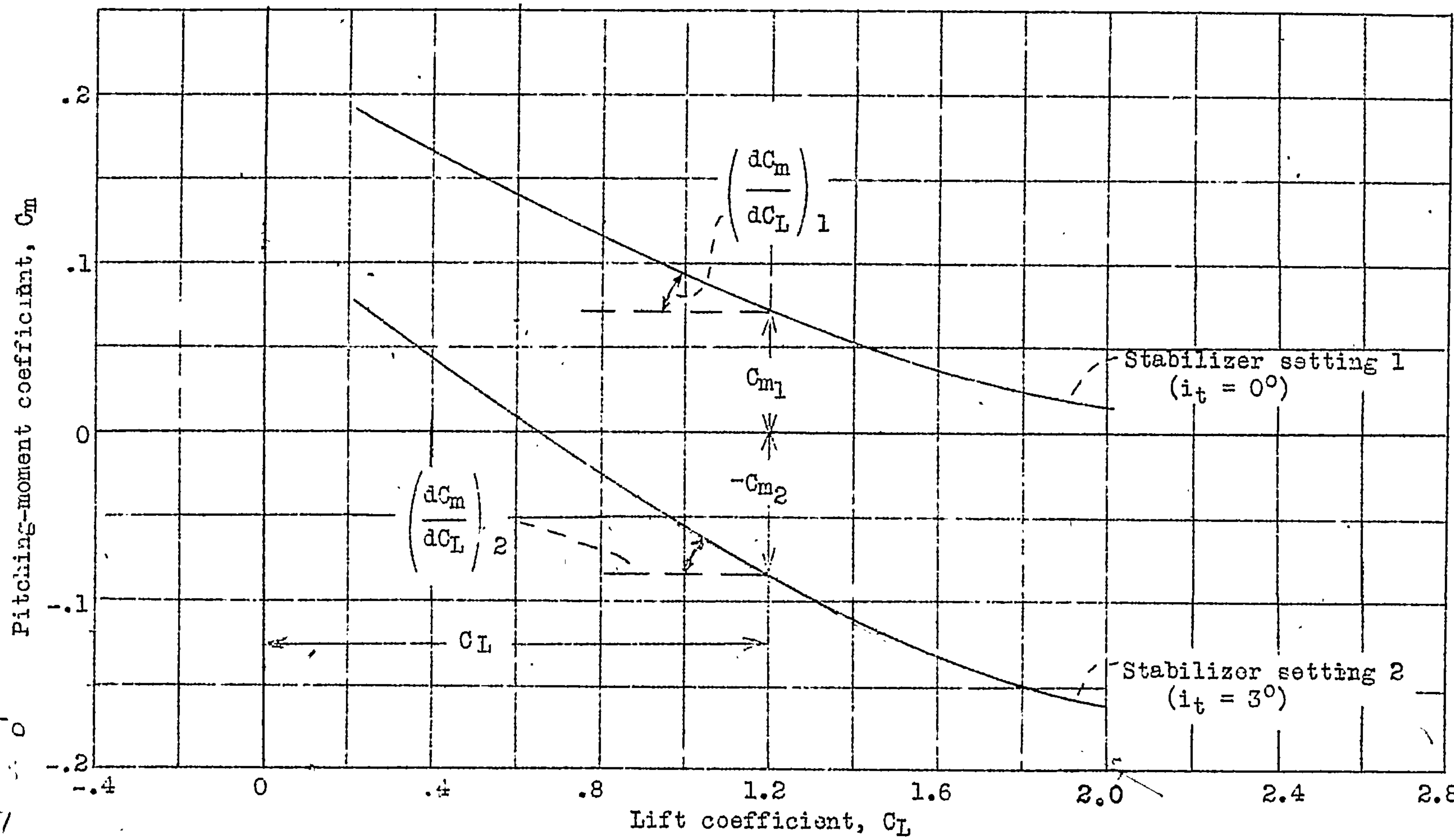

Figure 1.- Typical variation of C_m with C_L obtained in wind tunnel. Center of gravity at 20 percent of mean aerodynamic chord, on thrust line. Power on; flaps down; $\delta_e = 0°$.

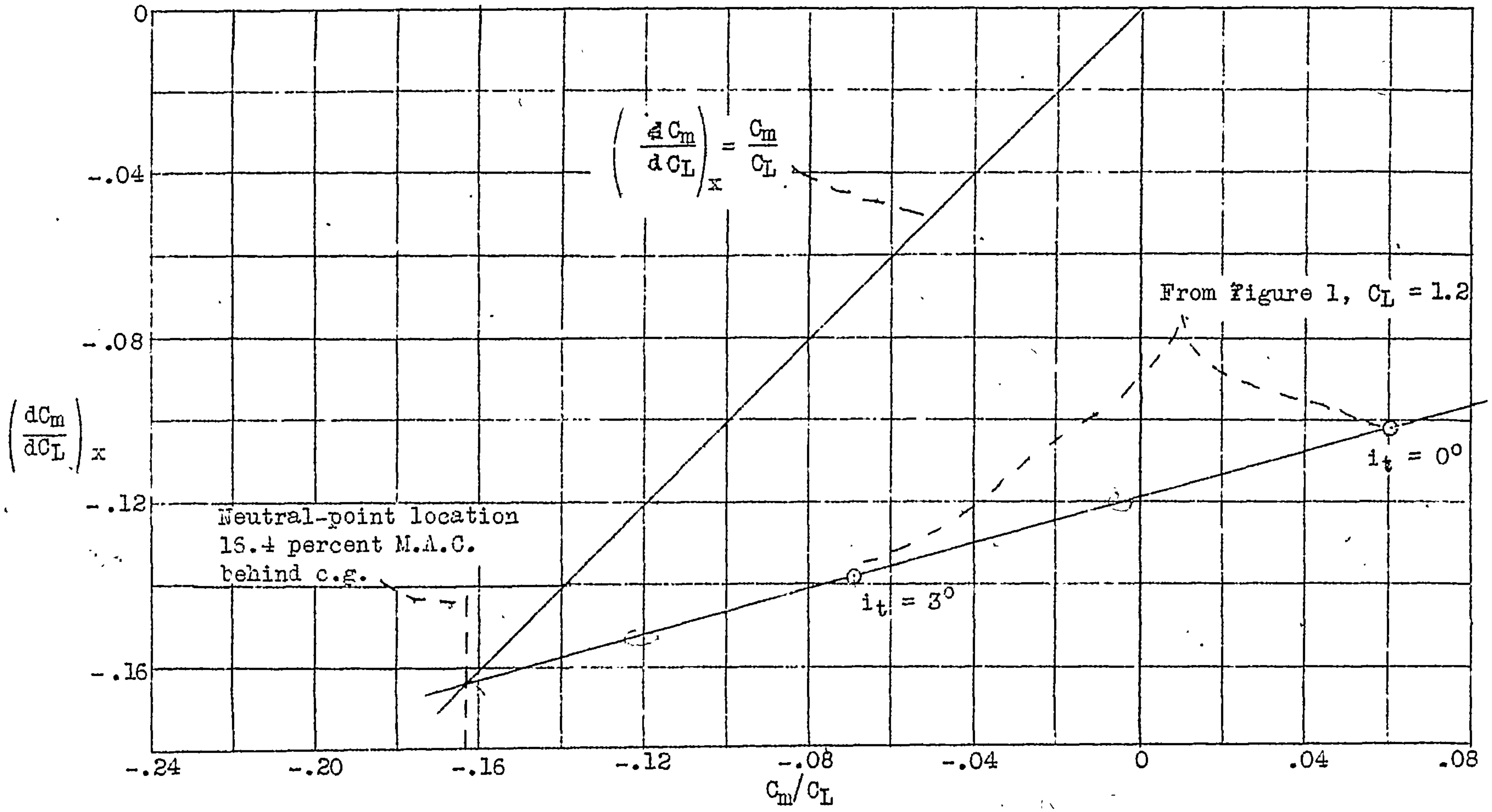

Figure 2.- Graphical construction for neutral-point determination.

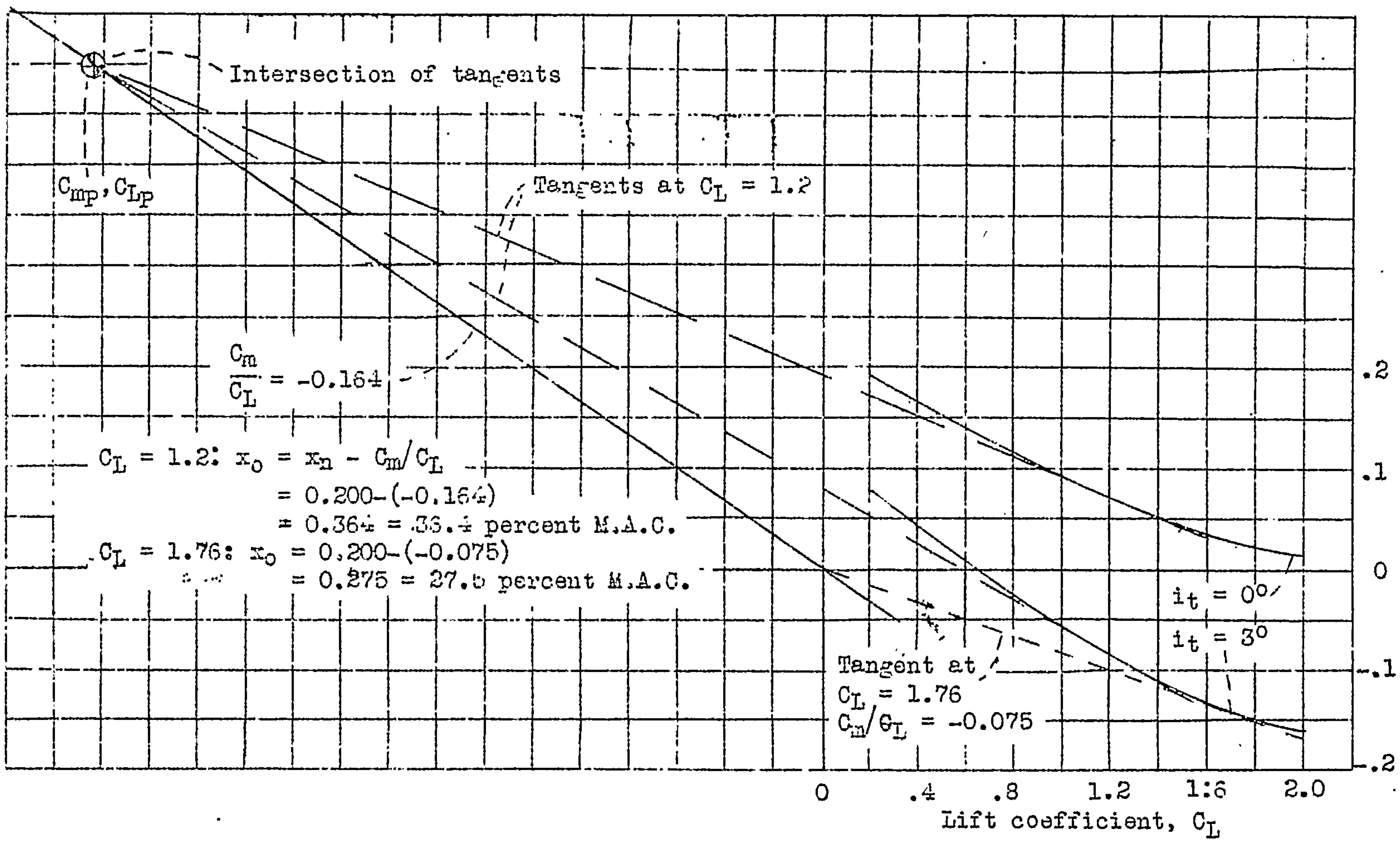

Figure 3.- Graphical determination of horizontal location of neutral point by intersection method.

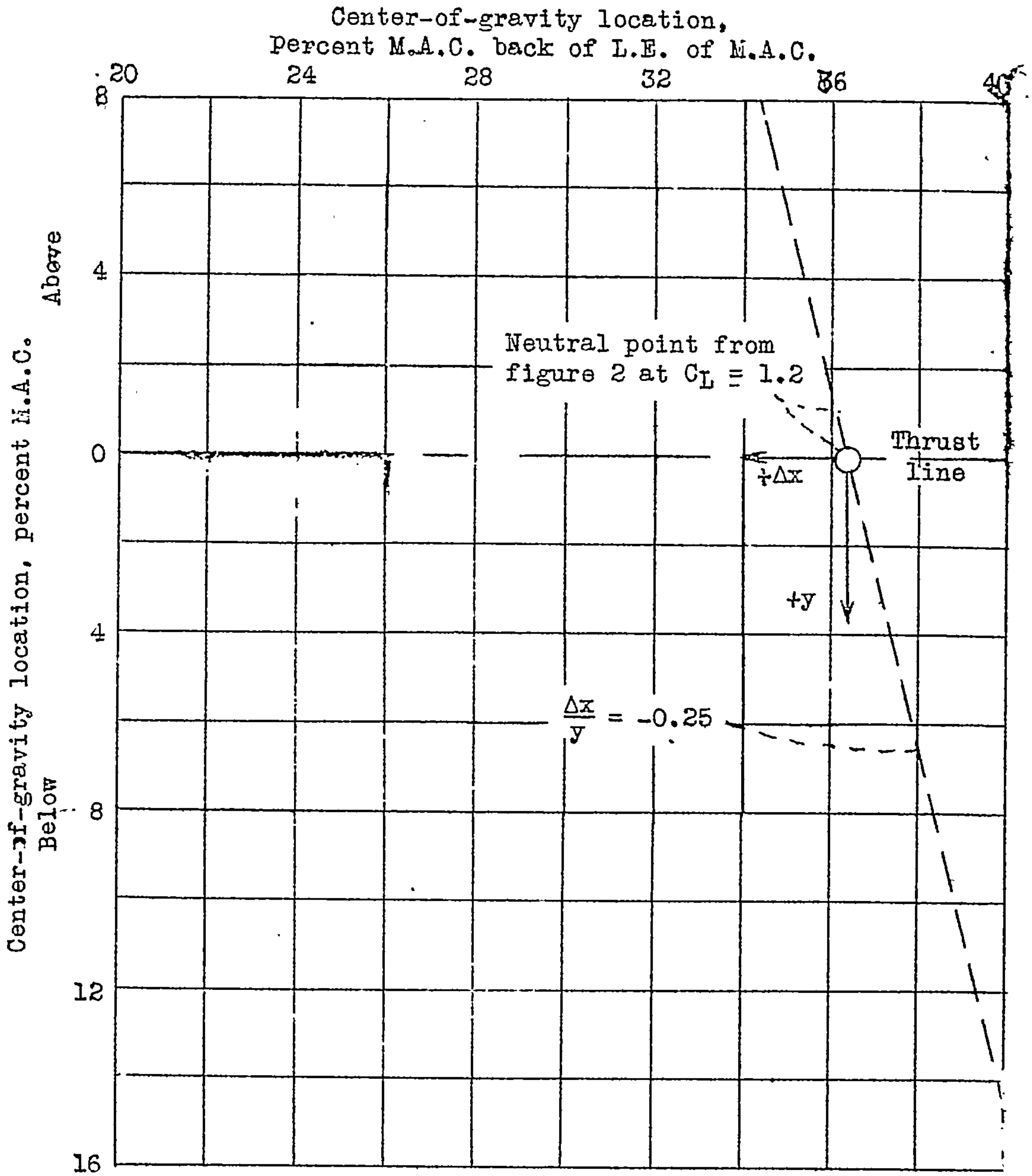

Figure 4.- Locus of center-of-gravity locations for neutral stability, that is, locus of neutral points.