Chapter 4 Magnitude Determinations

Both local Richter magnitude (XMAG) and coda duration magnitude (FMAG) may be computed by HYPOELLIPSE. The computation of these magnitudes is described below.

4.1 Coda duration magnitude, FMAG

FMAG is calculated according to an empirical equation (Lee and others, 1972; Lahr and others, 1975; Bakun and Lindh, 1977) which can be adjusted to agree with the local Richter magnitude scale. The equation for FMAG is

$$FMAG = C_1 + C_2 \log_{10}(F_c) + C_3 D + C_4 Z + C_5 \left(\log_{10}(F_c)\right)^2$$

where:

C_1, C_2, C_3, C_4, and C_5 are found empirically and correspond to the RESET TEST variables 31, 32, 33, 40, and 43, respectively (see 2.2.4).

F = F-P time (s), by USGS convention defined to be the time between the P p-phase arrival and the time when the coda drops to 1 cm peak-to-peak amplitude on a Geotech Model 6585 film viewer.

D = Epicentral distance in km.

Z = Hypocentral depth in km.

c = Station FMAG correction from TIME DEPENDENT STATION parameter record (see 2.2.5.3)

For California,	$C_1 = -0.87$	for Alaska,	$C_1 = -1.15$
	$C_2 = 2.0$		$C_2 = 2.0$
	$C_3 = 0.0035$		$C_3 = 0.0$
	$C_4 = 0.0$		$C_4 = 0.007$
	$C_5 = 0.0$		$C_5 = 0.0$

The station correction (c) is not added to the coda magnitude calculated, but is used as a multiplier of the observed coda. Therefore a station with no FMAG correction should have FMAG correction equal to 1.0. If the STATION record has no entry for FMAG correction, then the correction will be set equal to 1.0. The fifth term in the formula,

$$C_5 \left(\log_{10}(F_c)\right)^2$$

can be used to compensate for the nonlinear relationship of log(F) with magnitude (Bakun and Lindh, 1977).

If the MAGNITUDE OPTION code is negative then the F minus P coda length from the ARRIVAL TIME record will be converted into the F minus S coda length by subtraction of the S-P time. This formulation may be useful for earthquakes with wide depth variations, but is now only experimental.

Coda magnitude will not be computed for a station if the portion of the coda following the S arrival is less than 20 per cent of the total coda duration. This prevents an FMAG from being computed for stations without a significant coda following the S phase.

4.2 Local Richter magnitude, XMAG

4.2.1 Equation used to compute Richter magnitude

The formula for computing Richter magnitude is:

$$XMAG = \underbrace{\log\left(\frac{A}{2}\right)}_{\substack{\text{log of maximum zero}\\ \text{to peak amplitude in mm}\\ \text{as recorded on a}\\ \text{standard Wood-Anderson}\\ \text{seismograph.}}} + \underbrace{\left[-B_1 + B_2 \log X^2\right]}_{\substack{\text{Approximation}\\ \text{to Richter's}\\ \log A_o \text{ from}\\ \text{Eaton (1970).}\\ \text{See Figure 4.1.}}} + \underbrace{G}_{\substack{\text{Station}\\ \text{Correction}}}$$

where:

A = Maximum peak-to-peak amplitude in mm.

For $1 \text{ km} \leq D < 200 \text{ km}$
$B_1 = 0.15$
$B_2 = 0.80$

For $200 \text{ km} \leq D \leq 600 \text{ km}$
$B_1 = 3.38$
$B_2 = 1.50$

$$X = \sqrt{D^2 + Z^2}$$

where D is the epicentral distance and Z the focal depth in km.

G = Station XMAG correction, as specified on the TIME DEPENDENT STATION parameter record (see 2.2.5.3).

XMAG is not computed if X is not in the range 0.1 to 600 km.

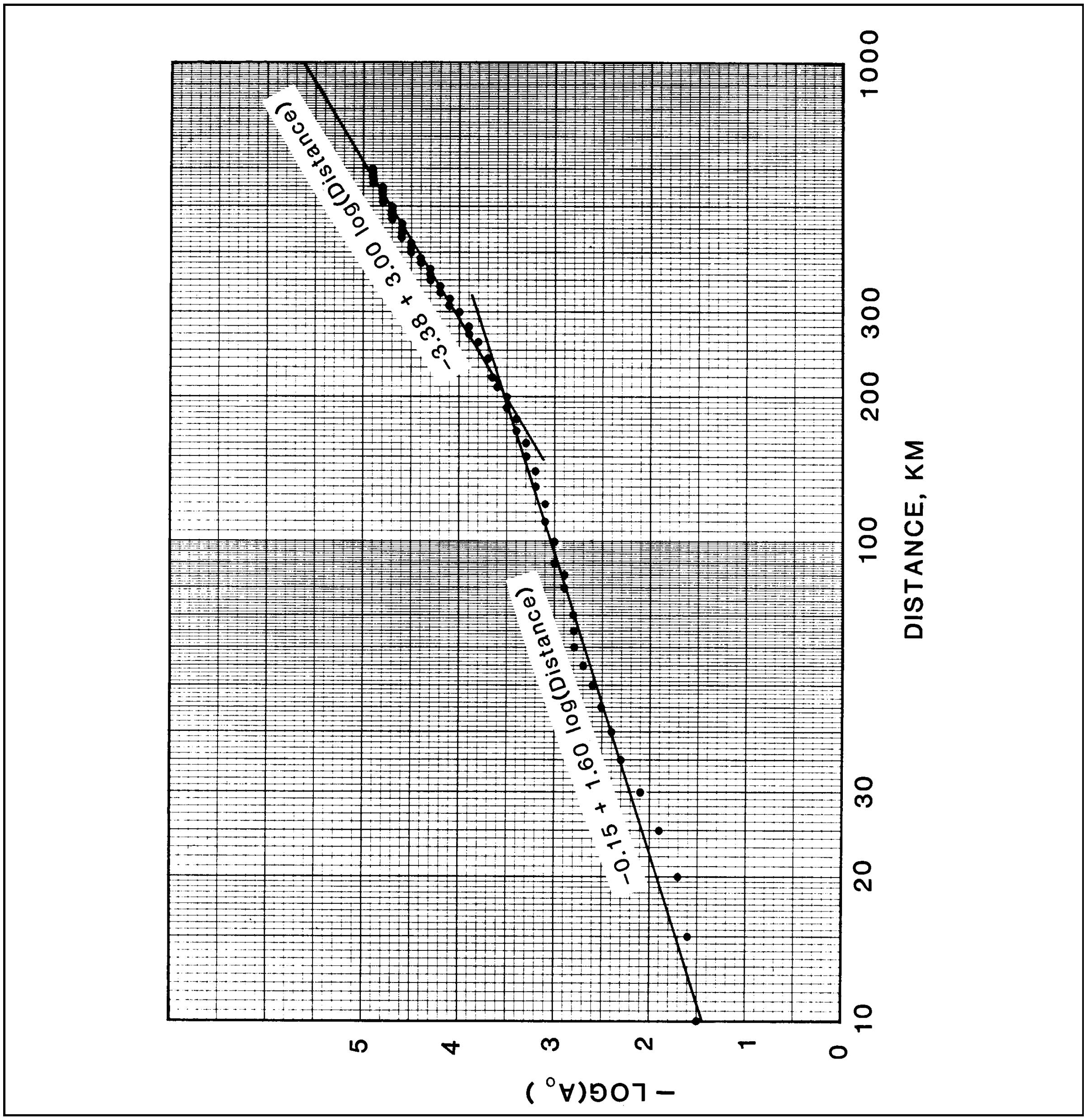

Figure 4-1. Comparison of $\log_{10}A_o$ from Richter (1958) (dots) with the approximation (straight lines) used in this program.

4.2.2 USGS style system calibration and XMAG formulation

A system of station calibration has been developed by the USGS that is based on the assumption that the only difference between the response of stations using the same type of instrumentation is the gain level. On this basis, a series of calibration curves has been developed for use in computing XMAG that need only be adjusted for an individual station's overall gain. The development of these calibration parameters is summarized below, and follows the method described by Eaton (1970).

At any frequency f:

Ground Motion 2h mm -> Geophone E_G µV -> Telemetry -> Viewer screen 2A mm

Seismometer Response $R_s(F) = \frac{E_G}{2h}$ microVolts/mm

System Response (Telemetry + Viewer) $V(f) = \frac{2A}{E_G}$ mm/microVolt

Total Harmonic Magnification $M_T(f) = R_s\ V = \frac{A}{h}$ mm(viewer)/mm(ground)

The system response at some frequency f may be calculated from the response at a particular frequency f_p multiplied by the ratio of the response at f to the response at f_p

$$V(f) = V(f_p)\,\frac{V(f)}{V(f_p)}$$

If the viewer amplitude A is measured for input signals E_G of constant amplitude and varying frequency f then the ratio

$\frac{V(f)}{V(f_p)} = \frac{A(f)}{A(f_p)}$ may be calculated as a function of frequency f.

Thus $V(f) = V(f_p)\,\frac{A(f)}{A(f_p)}$

$V(f_p)$ is calculated for $f_p = 5$ hz for a 10 µV rms input signal. 10 µV rms equals 28.28 µV p-p. Let the amplitude, A(5), for this signal be C10.

$$V(f) = \frac{C10}{28.28}\ \frac{A(f)}{A(5)}$$

The formula for seismometer response is:

$$R_s(f) = \frac{2\pi f^3 G_{LE}}{\sqrt{(F_o^2 - f^2)^2 + 4B^2F_o^2f^2}}$$

where: F_o = natural frequency
B = damping constant
G_{LE} = motor constant µV/mm/s

TGN(f) is defined to be $R_s(f)$ divided by 28.28. Thus the total harmonic magnification is:

$$M_T(f) = R_S V = \frac{2\Pi f^3 G_{LE}}{\sqrt{(F_o^2 - f^2)^2 + 4B^2F_o^2f^2}} \quad \frac{C10}{28.28} \quad \frac{A(f)}{A(5)}$$

Using the definition of TGN(f) this becomes:

$$M_T(f) = C10 \quad \frac{A(f)}{A(5)} \quad TGN(f)$$

			System		Seismometer	
$M_T(f) =$	"Gain"	*	frequency	*	response divided	mm(viewer)/mm(ground)
			response		by 28.28	

To calculate magnitudes equivalent to the local Richter magnitude it is necessary to calculate the amplitude B(f) that would have been read on the seismogram from a Wood-Anderson seismograph. The magnification of the Wood-Anderson is

$$M_{WA}(f) = \frac{2800f^2}{\sqrt{(F_o^2 - f^2)^2 + 4B^2F_o^2f^2}} \quad mm(WA\ record)/mm(ground)$$

with $F_o = 1.25$ and $B = 0.8$. (Urhammer and Collins (1990) found the static magnification to be 2080 rather than 2800. For this reason, RESET TEST 52 is provided to adjust this parameter.)
The amplitude that would be measured on a Wood-Anderson record is:

$$B(f) = \frac{A(f)}{M_T(f)} M_{WA}(f)$$

$$B(f) = \frac{A(f)}{M_T(f)} M_{WA}(f)$$

$$B(f) = A(f) \quad * \quad \frac{M_{WA}(f)}{TGN(f)} \frac{A(5)}{A(f)} \quad / \quad C10$$

Amplitude p-p in mm measured on viewer. f is also measured on the viewer.	Value is determined by inter- polation in the table corresponding to the system in use.	Station calibration from station record.

It is convenient to carry out the interpolation in terms of the table:

$$RSPA(n) = \log\frac{TGN(f_n)A(f_n)}{M_{WA}(f_n)A(5)} \quad vs \quad \log(f_n)$$

$$where \ \log(f_n) = -2.0 + 0.1n.$$

Using the table of RSPA(n) the following formula is used to compute XMAG:

$$XMAG = \log\left(\frac{A}{2C10}\right) - R_{kf} \quad + \quad \left[-B_1 + B_2\log^2 X\right] \quad + \quad G$$

Log of maximum zero to peak amplitude in mm as recorded on a standard Wood-Anderson seismograph.	Approximation to Richter's $\log A_o$	Station Correction

where:

R_{kf} = frequency response of system number k for frequency f interpolated from table of RSPA(n).

A = Maximum peak-to-peak amplitude in mm.

C10 = USGS peak-to-peak calibration amplitude for 10 µV rms, 5 hz preamplifier input.

B_1 and B_2 were defined above.

X = SQRT($D^2 + Z^2$),
where D is the epicentral distance and Z the focal depth in km.
G = Station XMAG correction.

4.2.2.1 Calibration curves for additional systems

For eight systems previously used at the USGS these values are stored for n = 15,34 corresponding to 0.316 to 25.119 Hz or to periods of 3.162 to 0.040 s. Up to nine additional calibration tables may be used, as described in section 2.2.10. Using this option, five such tables (corresponding to system response codes 9-13) have been defined for use with the USGS A1VCO (Rogers and others, 1980), and are described below. The A1VCO gain is automatically reduced by a factor of 10 or a factor of 500 when large signals are being recorded. In order to take these gain states into account in computing magnitude, column 62 of the ARRIVAL TIME record (see 2.2.6.2) contains either a 0, 1, or 2, corresponding to normal gain, gain reduced by 10, and gain reduced by 500.

<u>System response code 9</u> has been assigned to an A1VCO recorded on a Develocorder and viewed with 20x magnification.

$$RSPA(n) = LOG_{10}\left(\frac{M_T(F_n)}{M_{WA}(F_n)}\right), \text{ where}$$

Fn is the frequency defined by $\log(F_n) = -2.0 + 0.1n$,
$M_T(F_n)$ is the magnification of an ideal A1VCO at frequency F_n and gain zero, &
$M_{WA}(F_n)$ is the magnification of a Wood-Anderson instrument at frequency F_n.

To account for variations in geophone motor constant, gain setting and other factors between different stations, the C10 value is defined to be:

$$C10 = 2^N \frac{MAMP}{PAMP}, \text{ where}$$

MAMP is the measured peak-to-peak amplitude in mm of the 5 Hz calibration signal,
PAMP is 49.12 mm, the theoretical peak-to-peak 5 Hz calibration signal level for an ideal A1VCO with Develocorder attenuator setting of 2, and N is the A1VCO field gain minus 2.

<u>System response code 10</u> has been assigned to an A1VCO recorded on FM tape and processed on the USGS CUSP digital playback system.

$$C10 = 2^N \frac{MAMP}{PAMP}, \text{ where}$$

MAMP is the peak-to-peak amplitude of the 5 Hz calibration signal in counts, measured on a high gain Siemens playback.
PAMP is 2620 counts, the theoretical peak-to-peak 5 Hz calibration signal level for an ideal A1VCO played back on the Siemens, and
N is the A1VCO field gain.

System response code 11 has been assigned to an A1VCO recorded by the PC system.

$$C10 = 2^{N}\frac{MAMP}{PAMP}, \text{ where}$$

MAMP is the peak-to-peak amplitude of the 5 Hz calibration signal in counts, as recorded by the PC,
PAMP is 2620 counts, the theoretical peak-to-peak 5 Hz calibration signal level for an ideal A1VCO recorded by the PC, and
N is the A1VCO field gain.

System response code 12 has been assigned to an A1VCO-Siemens high gain playback.

$$C10 = 2^{N}\frac{MAMP}{PAMP}, \text{ where}$$

MAMP is the amplitude of the 5 Hz calibration signal in mm, measured on a high gain Siemens playback.
PAMP is 128 mm, the theoretical 5 Hz calibration signal level for an ideal A1VCO played back on the Siemens, and
N is the A1VCO field gain.

Siemens "ink squirt" playbacks are made at the USGS with one of two gain settings, high or low (reduced by a factor of 4 from high). In order to take the Siemens playback gain into account in computing magnitude, column 61 of the ARRIVAL TIME record (see 2.2.6.2) contains either 0 or 1, corresponding to high and low gain, respectively.

System response code 13 has been assigned to an A1VCO recorded on the University of Alaska Masscomp computer system.

$$C10 = 2^{N}\frac{MAMP}{PAMP}, \text{ where}$$

MAMP is the amplitude of the 5 Hz calibration signal after conversion to millivolts,
PAMP is 4000 millivolts, the theoretical 5 Hz calibration signal level for an ideal A1VCO recorded on the Masscomp system, and
N is the A1VCO field gain.

4.2.2.2 Magnitude determination when more than one source is possible

ARRIVAL TIME records include a code in column 108 (see 2.2.6.2) that indicates the source of the amplitude measurement. For example, a given station could be recorded simultaneously on Develocorder and on a computer system. Interpretation of the amplitude will clearly depend upon the data source being used to make the measurements. The TIME-DEPENDENT STATION records (see 2.2.5.3) allow up to five sources to be used simultaneously for a given station. The magnitude subroutine requires that amplitudes read on ceratin sources have the appropriate response calibration information available in the station list in order to compute a magnitude.

The source codes used for Alaska data processing are given below. Also shown is the corresponding System Response Code, when a specific code is required for magnitude calculation.

Source Code	Response Function	
V	PRIME	- USGS 20x film viewer
1	PRIME	- USGS 20x one-film digitizer
4	PRIME	- USGS 20x four-film digitizer
*	PRIME	- Assumed to be USGS 20x film
H		- USGS Helicorder
2	10	- USGS FM tape, digitized and processed on CUSP
E	PRIME	- USGS FM tape, digitized and processed on Eclipse computer
S	12	- USGS FM tape played back into Siemens "ink-squirt"
L		- USGS portable RCA/COSMAC ELOG portable digital recorder
P,O,U,I,G,K	11	- AEIC PC digital recorder (PCSEIS) P = Cordova, Petersburg, Yakutat, and Willie1 @ Fairbanks O = Willie2 @ Fairbanks U = FOCUS I = CALIBRATE G = EVENT (Willie3) @ Fairbanks K = CODA (Willie4) @ Fairbanks
Z		- AEIC Spurr Broad-Band Digital PC recorder (PCSEIS)
5		- USGS five-day tape recorder
M		- SMA1 and SSA strong motion recorder
A		- UAGI 20x film
%		- Assumed to be UAGI 20x film
R		- UAGI Helicorder
D	13	- UAGI Masscomp digital data from DAQ or DAN
J	13	- Same as D, but measured automatically by JadePost
X	13	- Same as D, but measured automatically by Xpick
F		- UAGI film read by USGS while visiting UAGI
W		- Alaska Tsunami Warning Center (ATWC) 20x film
T		- ATWC teletype, corrected for satellite delay
B		- Published bulletin (EDR, ISC etc)
C		- Canadian data from magnetic tape or digital format
N		- NEIS, corrected for satellite delay
Q		- PDAS100 portible digital recorder
Y		- Field data loger such as Quanterra, K2, or UNIX box

If the amplitude source code on the arrival time record is blank then the code entered on the BLANK SOURCE option record, which may be any character including a blank (see 2.2.3.21), will be used instead. If the amplitude source code is a blank, "V", "1", "4", or "*" then the primary system response function and XMAG calibration constant specified in columns 10-20 of the TIME-DEPENDENT STATION record will be used in computing XMAG. If the amplitude source code is not blank, and either does not appear in the above table or does not have a response function associated with it, then XMAG values will not be computed for that amplitude. Otherwise, a C10 value for the system response function listed in the table above is sought from the TIME-DEPENDENT STATION records. If the station archive does not have the required C10 value, then XMAG will not be computed for this station.

4.2.3 University of Alaska computation of XMAG

The University of Alaska, Geophysical Institute, periodically calibrates each of its seismic stations at a series of periods. These data are maintained in a table that consists of up to 1000 logical records for the SUN version and 100 for the PC version. The data on each record is ordered in the following manner: station name, source code, begin yrmody, end yrmody, npairs, period(1), sysmag(1), period(2), sysmag(2), ..., period(npairs), sysmag(npairs). Pairs must be in order of increasing period. The format is fixed for the station name and source code. The station name begin in column 1 and is separated by one blank from the source code, which is followed by another blank. The remainder of the record is in free format.

Item	Format
Station name	A4 (columns 1-3 or 1-4)
Source code	A1 (column 5 or 6
Begin yrmody of calibration (eg., 881129)	Free format integer
End yrmody of calibration	Free format integer
Npairs - number of pairs to follow	Free format integer
Period	Free format real (s)
Sysmag - system magnification	Free format real (counts/mm ground displacement)

The name of this file must be entered on a UOFACAL record included with the input data (See 2.2.3.18). Stations with a primary system response code equal to 18 (in columns 10-11 of the TIME-DEPENDENT STATION record, see section 2.2.5) will have an amplitude magnitude (XMAG) computed only if calibration data for the corresponding time period is found in the table.

4.3 Summary of parameters that control magnitude calculations

The overall preferred magnitude is entered on the SUMMARY record in columns 35-36 (see 2.4.1). This magnitude may be either the average XMAG, average FMAG, or the average of XMAG and FMAG, as determined by the MAGNITUDE OPTION code (see 2.2.3.3).

4.3.1 FMAG

The computed FMAG for each station is entered on the ARCHIVE ARRIVAL TIME record (columns 103-104; see 2.2.15) and the average FMAG for the event is entered on the SUMMARY record (columns 70-71; see 2.4.1).

The parameters for the FMAG equation are set by RESET TEST variables 31, 32, 33, 40, and 43.

The TIME-DEPENDENT STATION parameter record (2.2.5.3) includes:

> FMAG weight, which determines how each station's FMAG is weighted in computing the average FMAG for the earthquake.

FMAG correction, a multiplicative correction factor.

The ARRIVAL TIME record (2.2.6.2) includes the measured coda duration (F-P) in seconds and the coda duration source code. The source code is just for documentation and is not used by HYPOELLIPSE.

4.3.2 XMAG

The XMAG for each station is entered on the ARCHIVE ARRIVAL TIME record (columns 101-102; see 2.2.15) while the average XMAG for the earthquake is entered on the SUMMARY record (columns 68-69; see 2.4.1).

4.3.2.1 USGS XMAG

The TIME DEPENDENT STATION parameter record (see 2.2.5.3) may include up to five system response codes and corresponding calibration constants. This allows magnitude calculations for a station to be based on amplitudes measured on any of five recording systems.

Calibration curves for nominal gain settings may be entered into the program (see 2.2.10)

The ARRIVAL TIME record (see 2.2.6.2) includes maximum peak-to-peak amplitude, period of maximum amplitude, A1VCO gain state, Siemens playback gain state, and amplitude source code.

4.3.2.2 University of Alaska XMAG

The TIME DEPENDENT STATION parameter record must have system response code 18 specified (columns 10-11; see 2.2.5.3).

A table of station calibrations must be specified with a UOFACAL OPTION record (see 4.2.3).

The ARRIVAL TIME record (see 2.2.6.2) includes maximum peak-to-peak amplitude, period of maximum amplitude, A1VCO gain state, and amplitude source code.