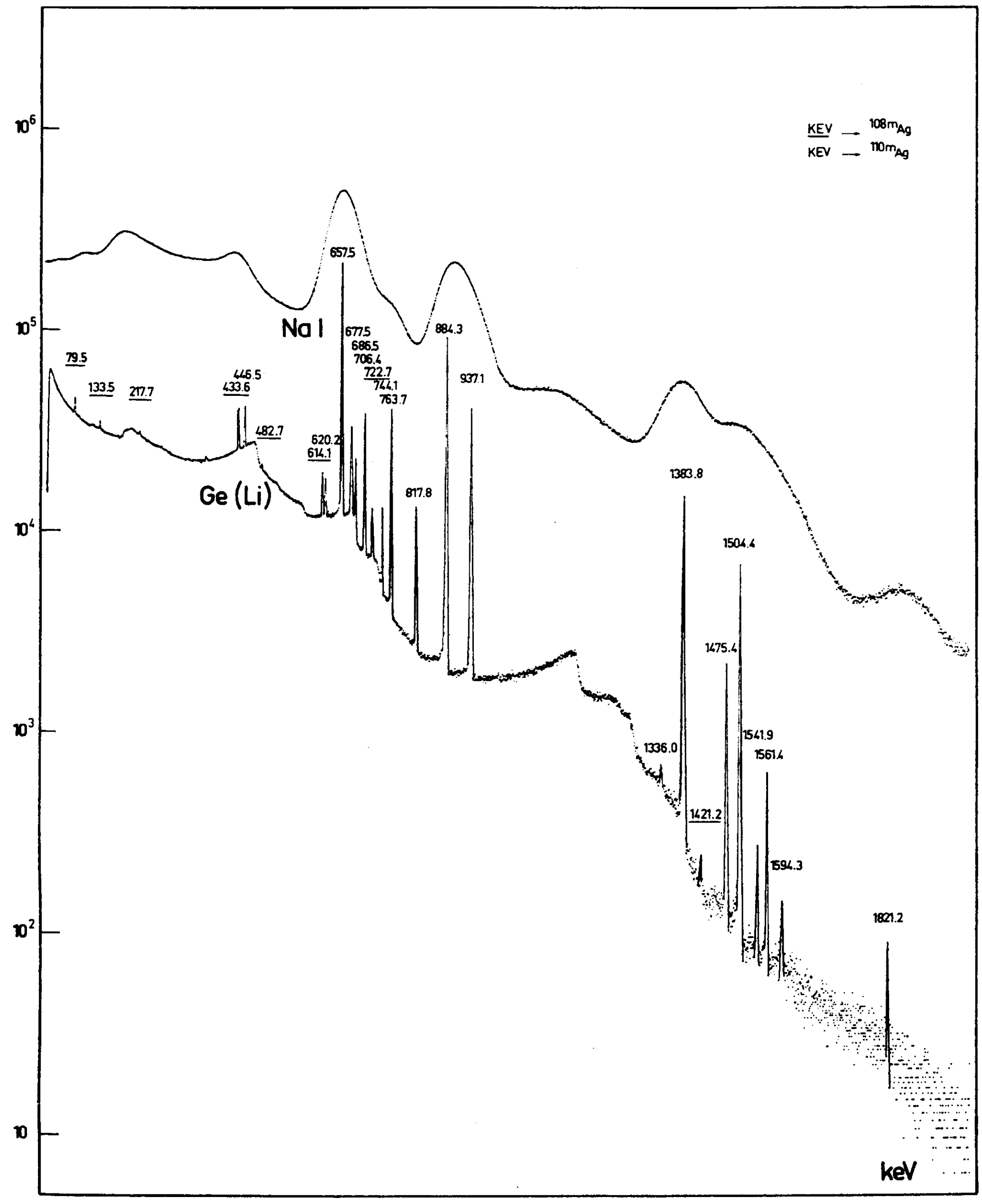
V.3. Resolution and Signal-to-Noise Ratio

Why?

a) Recognize structure in spectra

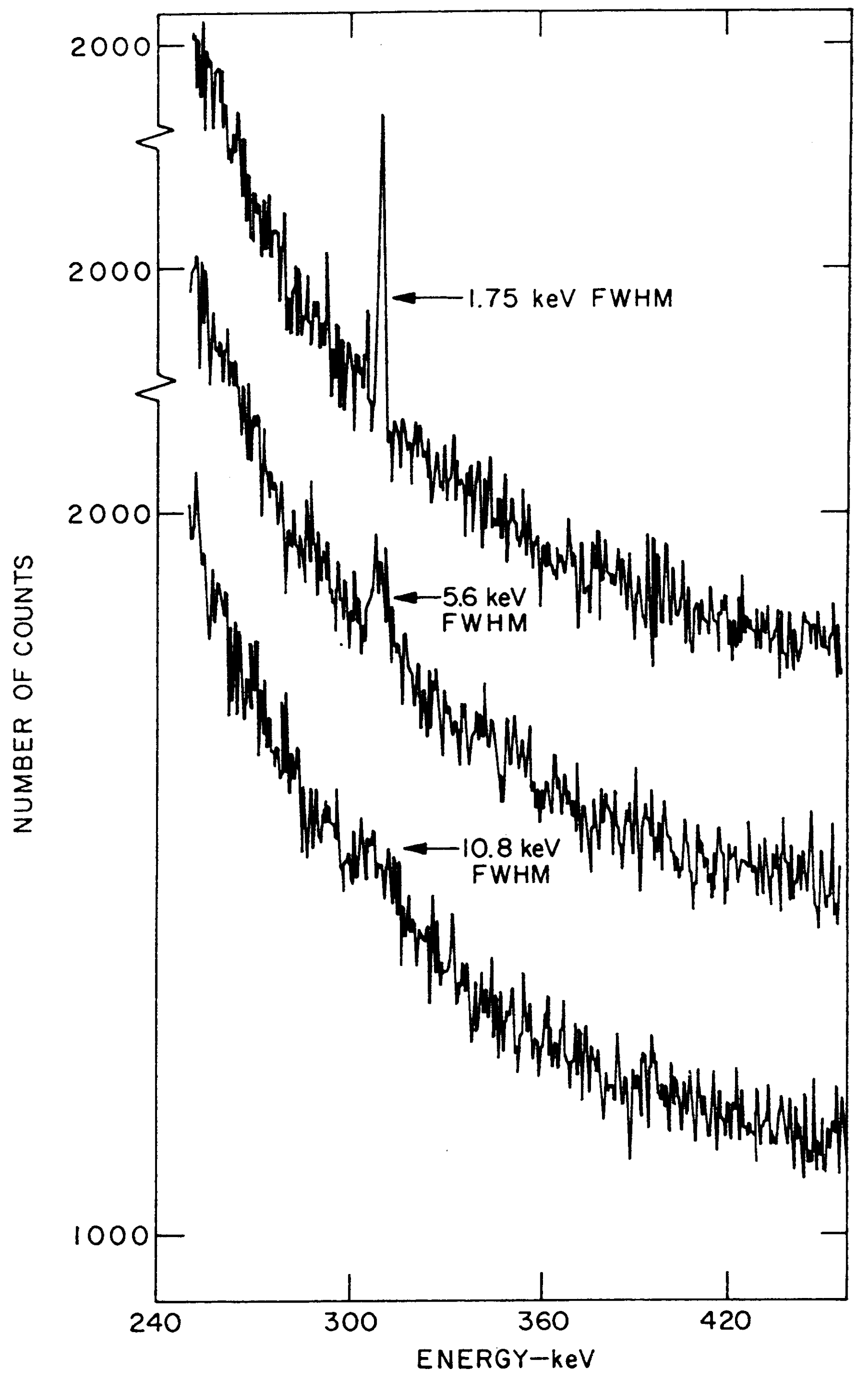
Comparison between NaI(Tl) and Ge detectors



(J.Cl. Philippot, IEEE Trans. Nucl. Sci. **NS-17/3** (1970) 446)

b) Improve sensitivity

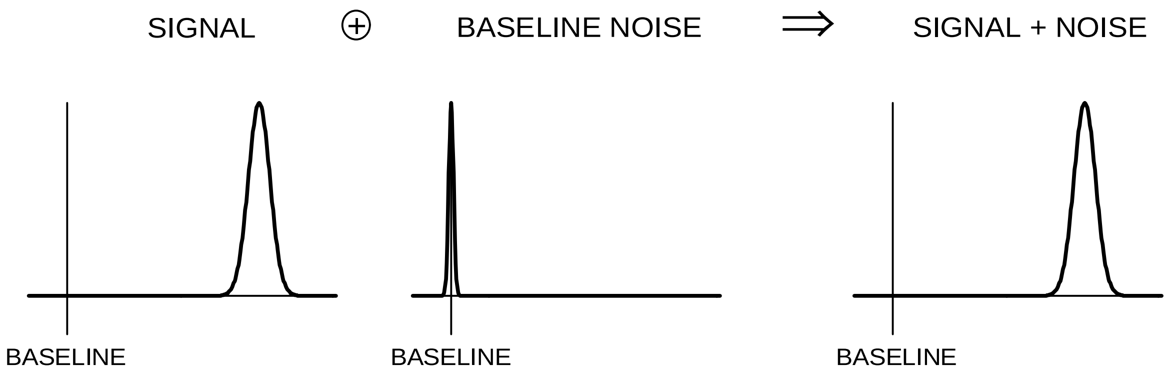
Signal to background ratio improves with better resolution (signal counts in fewer bins compete with fewer background counts)



G.A. Armantrout, *et al.*, IEEE Trans. Nucl. Sci. **NS-19/1** (1972) 107

What determines Resolution?

1. Signal Variance >> Baseline Variance



⟹ Electronic (baseline) noise not important

Examples:

- High-gain proportional chambers
- Scintillation Counters with High-Gain PMTs

e.g. 1 MeV γ-rays absorbed by NaI(Tl) crystal

Number of photoelectrons

$$N_{pe} \approx 8 \cdot 10^4 \text{ [MeV}^{-1}] \times E_\gamma \times QE \approx 2.4 \cdot 10^4$$

Variance typically

$$\sigma_{pe} = N_{pe}^{1/2} \approx 160 \text{ and } \sigma_{pe} / N_{pe} \approx 5 - 8\%$$

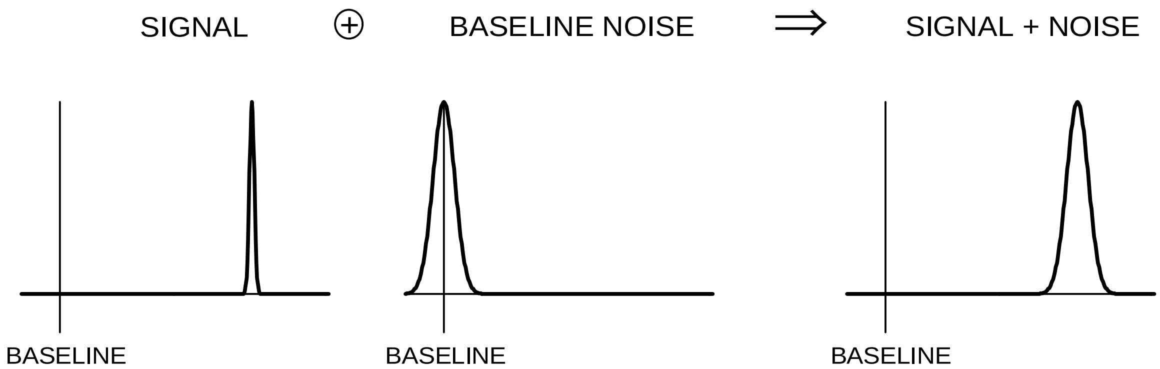
Signal at PMT anode (assume Gain= 10^4)

$$Q_{sig} = G_{PMT} N_{pe} \approx 2.4 \cdot 10^8 \text{ el and}$$

$$\sigma_{sig} = G_{PMT} \sigma_{pe} \approx 1.2 \cdot 10^7 \text{ el}$$

whereas electronic noise easily $< 10^4$ el

2. Signal Variance << Baseline Variance



$\Rightarrow$ Electronic (baseline) noise critical for resolution

Examples

- Gaseous ionization chambers (no internal gain)
- Semiconductor detectors

e.g. in Si

Number of electron-hole pairs

$$N_{ep} = E_{dep} / (3.6 \text{ eV})$$

Variance

$$\sigma_{ep} = \sqrt{F \cdot N_{ep}}$$

(where F= Fano factor ≈ 0.1)

For 50 keV photons

$$\sigma_{ep} \approx 40 \text{ el} \Rightarrow \sigma_{ep} / N_{ep} = 7.5 \cdot 10^{-4}$$

obtainable noise levels are 10 to 1000 el.

Baseline fluctuations can have many origins ...

pickup of external interference

artifacts due to imperfect electronics

… etc.,

but the (practical) fundamental limit is electronic noise.

Basic Noise Mechanisms

Consider n carriers of charge e moving with a velocity v through a sample of length l. The induced current i at the ends of the sample is

$$i = \frac{n\,e\,v}{l} \ .$$

The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left(\frac{ne}{l} \langle dv \rangle \right)^2 + \left(\frac{ev}{l} \langle dn \rangle \right)^2$$

where the two terms are added in quadrature since they are statistically uncorrelated.

Two mechanisms contribute to the total noise:

- velocity fluctuations, *e.g.* thermal noise
- number fluctuations, *e.g.* shot noise
  excess or '1/ f ' noise

Thermal noise and shot noise are both “white” noise sources, i.e.

power per unit bandwidth is constant:

(≡ spectral density)

$$\frac{dV_{noise}^2}{df} = const. \equiv v_n^2$$

or

$$\frac{dP_{noise}}{df} = const.$$

whereas for “1/ f ” noise

$$\frac{dP_{noise}}{df} = \frac{1}{f^{\alpha}}$$

(typically $\alpha = 0.5 - 2$)

1. Thermal Noise in Resistors

The most common example of noise due to velocity fluctuations is the thermal noise of resistors.

Spectral noise power density vs. frequency f

$$\frac{dP_{noise}}{df} = 4kT$$

k = Boltzmann constant
T = absolute temperature

since

$$P = \frac{V^2}{R} = I^2 R$$

R = DC resistance

the spectral noise voltage density

$$\frac{dV_{noise}^2}{df} \equiv e_n^2 = 4kTR$$

and the spectral noise current density

$$\frac{dI_{noise}^2}{df} \equiv i_n^2 = \frac{4kT}{R}$$

The total noise depends on the bandwidth of the system.
For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain $A_v(f)$ is

$$v_{on}^2 = \int_0^\infty e_n^2 \, A_v^2(f) \, df$$

Note: Since spectral noise components are non-correlated, one must integrate over the noise power.

2. Shot noise

A common example of noise due to number fluctuations is "shot noise", which occurs whenever carriers are injected into a sample volume independently of one another.

Example: current flow in a semiconductor diode (emission over a barrier)

Spectral noise current density:

$$i_n^2 = 2q_e I$$

q_e = electron charge
I = DC current

A more intuitive interpretation of this expression will be given later.

Note: Shot noise does not occur in "ohmic" conductors. Since the number of available charges is not limited, the fields caused by local fluctuations in the charge density draw in additional carriers to equalize the total number.

Noise Bandwidth vs. Signal Bandwidth

Consider an amplifier with the frequency response $A(f)$. This can be rewritten

$$A(f) \equiv A_0 G(f)$$

where A_0 is the maximum gain and $G(f)$ describes the frequency response.
For example, in the simple amplifier described above

$$A_V = g_m \left(\frac{1}{R_L} + i\omega C_o \right)^{-1} = g_m R_L \frac{1}{1 + i\omega R_L C_o}$$

and using the above convention

$$A_0 \equiv g_m R_L \quad \text{and} \quad G(f) \equiv \frac{1}{1 + i\,(2\pi f R_L C_o)}$$

If a "white" noise source with spectral density v_{ni} is present at the input, the total noise voltage at the output is

$$V_{no} = \sqrt{\int_0^\infty v_{ni}^2 \left| A_0 G(f) \right|^2 df} = v_{ni} A_0 \sqrt{\int_0^\infty G^2(f)\, df} \equiv v_{ni} A_0 \sqrt{\Delta f_n}$$

Δf_n is the "noise bandwidth".

Note that, in general, the noise bandwidth and the signal bandwidth are not the same. If the upper cutoff frequency is determined by a single RC time constant, as in the "simple amplifier", the signal bandwidth

$$\Delta f_s = f_u = \frac{1}{2\pi RC}$$

and the noise bandwidth

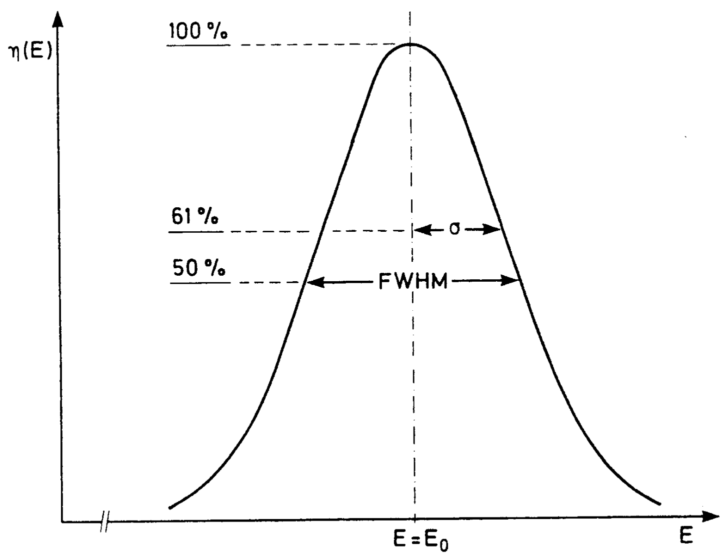
$$\Delta f_n = \frac{1}{4RC} = \frac{\pi}{2} f_u$$

Individual noise contributions add in quadrature
(additive in noise power)

$$V_{n,tot} = \sqrt{\sum_i V_{ni}^2}$$

Both thermal and shot noise are purely random.

⇒ amplitude distribution is gaussian



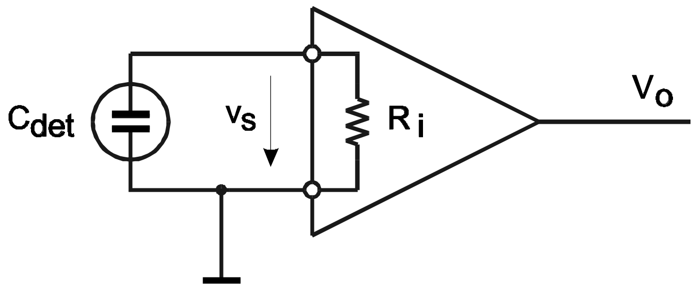
⇒ noise modulates baseline

⇒ baseline fluctuations superimposed on signal

⇒ output signal has gaussian distribution

Signal-to-Noise Ratio vs. Detector Capacitance

a) Voltage-Sensing Amplifier



$$R_i\, C_{det} >> \; t_{coll}$$

Signal (at amplifier input)

$$v_s = \frac{Q_s}{C_{\mathrm{det}}}$$

C_{det}= total capacitance at input

Noise (referred to amplifier input)

$$v_n = v_{ni}$$

for a given preamplifier and shaper

$\Rightarrow$

$$\frac{S}{N} = \frac{v_s}{v_n} = \frac{1}{C_{\mathrm{det}}} \frac{Q_s}{v_{ni}}$$

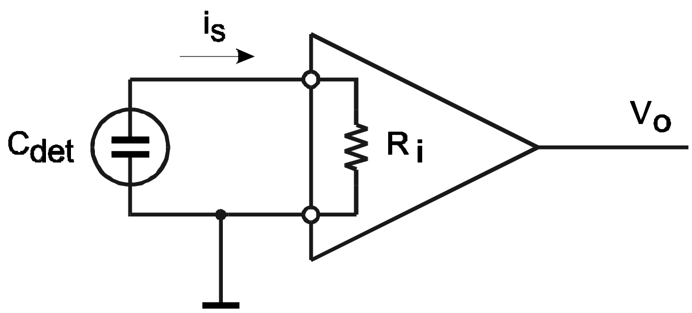
$\uparrow$

!

b) Current-Sensing Amplifier

Assume that the current induced on the detector electrodes is a delta pulse with charge Q_s.

The measured current pulse depends on the input time constant $\tau = C_{det} R_i$ that discharges the detector capacitance.



The signal current flowing into the amplifier is

$$i_s = i_0 e^{-t/\tau}$$

Since $Q_s = \int_0^\infty i_s dt \;\Rightarrow\; i_0 = Q_s / \tau$ and $i_s = \frac{Q_s}{\tau} e^{-t/\tau}$

Assume that only the peak signal at $t = 0$ will be sensed, so $i_s = Q_s / \tau$.

If the noise level of the amplifier - expressed as a current and referred to the amplifier input - is i_{ni} , the signal-to-noise ratio is

$$\frac{S}{N} = \frac{Q_s / \tau}{i_{ni}} = \frac{Q_s}{C_{\mathrm{det}}} \frac{1}{R_i i_{ni}}$$

$\Rightarrow$ *For a given amplifier system* (parameters R_i , i_{ni}), the signal-to-noise ratio of the current signal is inversely proportional to the total input capacitance.

Charge-Sensitive Preamplifier
Noise vs. Detector Capacitance

In a voltage-sensitive preamplifier the noise voltage at the output is essentially independent of detector capacitance,

i.e. the *equivalent input noise voltage* $v_{ni} = v_{no} / A_v$.

The signal-to-noise ratio depends on detector capacitance, since the input signal decreases with increasing input capacitance.

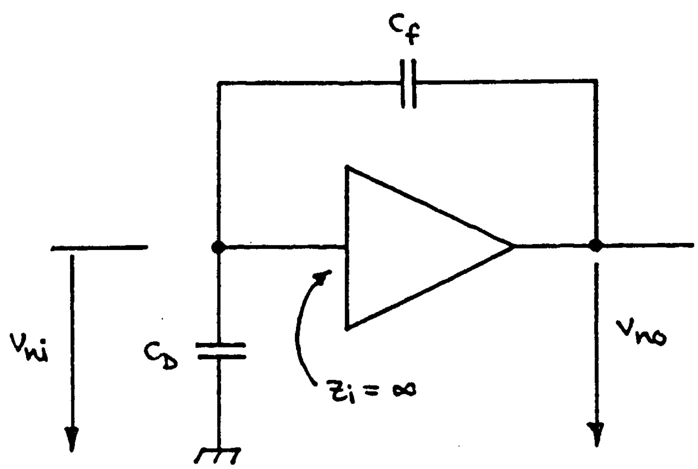
In a charge-sensitive preamplifier, the signal at the amplifier output is independent of detector capacitance (if $C_i >> C_{det}$).

What is the noise behavior?

> Noise appearing at the output of the preamplifier is fed back to the input, decreasing the output noise from the open-loop value $v_{no} = v_{ni} A_{v0}$. The magnitude of the feedback depends on the shunt impedance at the input, i.e. the detector capacitance.
>
> Note, that although specified as an equivalent input noise, the dominant noise sources are typically internal to the amplifier. Only in a fed-back configuration is some of this noise actually present at the input. In other words, the primary noise signal is not a physical charge (or voltage) at the amplifier input, to which the loop responds in the same manner as to a detector signal.
>
> $\Rightarrow$ **S/N at the amplifier output depends on feedback.**

Noise in charge-sensitive preamplifiers

Start with an output noise voltage v_{no}, which is fed back to the input through the capacitive voltage divider $C_f - C_d$.



$$v_{no} = v_{ni}\frac{X_{C_f} + X_{C_D}}{X_{C_D}} = v_{ni}\frac{\frac{1}{\omega\, C_f} + \frac{1}{\omega\, C_D}}{\frac{1}{\omega\, C_D}}$$

$$v_{no} = v_{ni}\left(1 + \frac{C_D}{C_f}\right)$$

Equivalent input noise charge

$$Q_{ni} = \frac{v_{no}}{A_Q} = v_{no} C_f$$

$$Q_{ni} = v_{ni}\left(C_D + C_f\right)$$

Signal-to-noise ratio

$$\frac{Q_s}{Q_{ni}} = \frac{Q_s}{v_{ni}(C_D + C_f)} = \frac{1}{C} \frac{Q_s}{v_{ni}}$$

Same result as for voltage-sensitive amplifier,

but here noise grows with increasing C.

As shown previously, pulse rise time at the amplifier output also increases with total capacitive input load C, because of reduced feedback.

In contrast, the rise time of a voltage sensitive amplifier is not affected by the input capacitance, although the noise increases with C just as for the charge-sensitive amplifier.

Conclusion

In general

- optimum S/N is independent of whether the voltage, current, or charge signal is sensed.

- S/N cannot be *improved* by feedback.

Practical considerations, i.e. type of detector, amplifier technology, can favor one configuration over the other.