Relativistic Stellar Aberration Requirements for the Space Interferometry Mission

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Abstract. We summarize here results of our recent study of the relativistic stellar aberration requirements for the Space Interferometry Mission (SIM). Based on a simplified model for differential astrometric observations with a free-flying single-baseline interferometer, we analyzed the astrometric errors introduced by imperfect metrology, inaccurate baseline length estimations, and those due to relativistic orbital motion of the spacecraft. We have shown that motion of the spacecraft in the direction perpendicular to a tile provides the most stringent requirement on the accuracy of knowledge of the barycentric velocity of the spacecraft. We consider a number of non-gravitational forces acting on the spacecraft and estimate their impact on the navigational accuracy. We show that, if not properly accounted, the solar radiation pressure and the anisotropic thermal radiation of the spacecraft may have a significant impact on the accuracy of future astrometric observations with SIM.

Introduction

A future model for the high accuracy astrometric observations with SIM should necessarily account for a large number of physical phenomena affecting the propagation of light. These phenomena are due to the interstellar media, the solar system dynamics, as well as due to the motion of the free-flying interferometer itself. The accuracy of astrometric observations expected with SIM requires a number of dynamical parameters to be precisely known. One of such a parameters is the barycentric velocity of the spacecraft which is responsible for the annual relativistic stellar aberration. This effect is a very important component of the dynamical model for any high accuracy astrometric observations made with the instrument in solar orbit. Thus, the annual aberration due to the barycentric motion of the spacecraft is the second largest term in the SIM astrometric model and will amounts to ~ 20.5 arcsec. For observations with accuracy of about 4 microarcseconds (μas), this effect will play an important role not only for the wide angle astrometry, but also will produce a significant astrometric distortion even for the narrow angle case. As a result, one will have to know the barycentric velocity of the spacecraft throughout the entire mission and account for the relativistic stellar aberration inside every single tile.

1. Astrometric errors introduced by the orbital motion

To derive the expression necessary to analyze the astrometric errors in future observations with SIM, we will use a simple model for the optical path difference for a single-baseline interferometer. Our derivations are different from the ones obtained earlier by the fact that for each tile we account for effects of barycentric motion of the spacecraft, for the errors in the components of the baseline vector, and for the error in estimating the instrumental offset (or, calibration) term. The expression for the astrometric errors that include a number of contributions due to various physical phenomena affecting the astrometric accuracy was obtained as follows (Turyshev & Unwin, 1998):

$$\sigma^2_\alpha = \frac{\sigma^2_{\delta_{c_0}}}{b^2\cos^2\frac{\alpha}{2}} + \frac{4\,\sigma^2_b}{b^2}\tan^2\frac{\alpha}{2} + 4\Big[\frac{\sigma^2_v}{c^2}\sin^2\psi + \sigma^2_\psi\,\frac{v^2}{c^2}\cos^2\psi\Big]\sin^2\frac{\alpha}{2} +$$

$$+\ \frac{4\,\sigma_{\delta_{c_0}}\sigma_b}{b^2}\rho(c_0,b)\,\frac{\sin\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}} - \frac{4\,\sigma_{\delta_{c_0}}}{b}\Big[\sigma_\psi\,\rho(c_0,\psi)\,\cos\psi\Big]\frac{v}{c}\tan\frac{\alpha}{2}, \qquad (1)$$

where α, σ_α is the angular separation between the two stars of interest and the corresponding error; b, σ_b is the interferometer's baseline length and the corresponding measurement error; c_0, $\sigma_{\delta_{c_0}}$ is the calibration term across the tile and the corresponding error; v, σ_v is magnitude of barycentric velocity of the spacecraft and the corresponding error; ψ, σ_ψ is the sky-angle of the barycentric velocity of the spacecraft and the corresponding error; $\rho(c_0,b)$ is the correlation coefficient between the constant term and the baseline length, $\rho(c_0,\psi)$ is the correlation coefficient between the constant term and the velocity's sky-angle.

It is evident from Eq.(1) that the worst case observation scenario is realized for the motion of spacecraft in the direction perpendicular to a tile (e.g $\psi = \pm\frac{\pi}{2}$). This case of motion will provide the most stringent requirement on the accuracy of knowledge of the velocity's magnitude:

$$\sigma_v = \sigma_\alpha\,\frac{c}{2\sin\frac{\alpha}{2}}. \qquad (2)$$

Motion parallel to the tile, (e.g. $\psi = 0$), produces another constraint, namely on the accuracy of the sky-angles of the spacecraft's barycentric velocity:

$$\sigma_\psi = \sqrt{\frac{\sigma^2_v}{v^2} + \Big[\frac{\sigma_{\delta_{c_0}}}{b}\,\frac{\rho(c_0,\psi)}{\sin\alpha}\,\frac{c}{v}\Big]^2} - \frac{\sigma_{\delta_{c_0}}}{b}\,\frac{\rho(c_0,\psi)}{\sin\alpha}\,\frac{c}{v}. \qquad (3)$$

These two extremal cases of orbital motion are bribing the two sets of requirements that we will discuss in more details.

The wide angle astrometric observations with SIM are expected to be with a mission accuracy of $\sigma_\alpha = 4\ \mu$as. Assuming that contribution of any component of the total error budget, Eq.(1), should not exceed 10% of the total variance a single accuracy of σ^2_α, one may derive a set of necessary requirements for the spacecraft's velocity knowledge. Thus, neglecting the correlation, one obtains:

$$\sigma_v = \ 7.0\ \ \mathrm{mm/s}, \qquad \sigma_\psi = \ 48.7\ \mathrm{mas}. \qquad (4)$$

However, accounting for a possible correlation between the calibration term c_0 and the two velocity sky-angles σ_ψ may be an important factor that may result in tightening the aberration requirements. It is important to point out that for the worst case of high correlation of ψ and c_0 the obtained results will have to be further reduced by a factor of $\sqrt{2}-1=0.4$, thus tightening the requirements for the two sky-angles as $\sigma_\psi \sim 20$ mas. This fact suggests that a possible correlation between the velocity components and the constant term may put an additional demand on the quality of the navigational data. Thus, for the worst case scenario, the data for all three components of the velocity vector will have to delivered with a considerably smaller errors, say $\sigma_v^{\tt corr}=2.9$ mm/s, $\sigma_\psi^{\tt corr}=$ 20.2 mas. These two numbers are the pessimistic estimates for the required accuracy of the spacecraft's velocity determination.

Analysis of not only the stellar aberration, but also a number of the other important issues is complicated by the fact that a realistic model for the spacecraft and the instrument are not yet available. The obtained results Eq.(4) were based on the assumption that the errors on the right-hand side of the Eq.(1) are forming an ellipsoid with half-axes partially given by Eqs.(2)-(3). In reality, one will have to minimize each constituent of the total error budget in a such a way that in any given time the sum of the terms on the right-hand side of the Eq.(1) will not be larger then the expected variance σ_α^2. Currently, the error budget allocates $\sigma_{\alpha_v}=36$ pm for the stellar aberration, which corresponds: $\sigma_v=3.9$ mm/s, and $\sigma_\psi=27.0$ mas. Despite the fact that we have a reasonable gap between our estimates, there some other factors that are necessary to consider. Thus a possible correlation between the constant term and the two velocity sky-angles may completely eliminate this gap. This minimizes the tolerable errors, reducing those down to the values above and suggests that the currently adopted error allocation for the stellar aberration of $\sigma_v=4$ mm/s is well justified.

2. Dynamical environment of the SIM Earth-trailing orbit

How difficult will be to meet these navigational requirements? To answer this question, we will study different phenomena that will affect the motion of the spacecraft and, therefore, the astrometric measurements. The figure against which we compare the effects of non-gravitational accelerations is that the expected error in the velocity estimations must be equal or smaller than $\sigma_v=4$ mm/s at integration times of about two weeks (this is a characteristic time that is necessary to meet the requirement for σ_v). We therefore require that the fluctuations in non-gravitational accelerations over a time scale of $\tau=1.21\times10^6$ s be less or equal to

$$\sigma_a \le \frac{\sigma_v}{\tau}=3\times10^{-7}\ \ \mathrm{cm/s}^2. \tag{5}$$

Let us consider the influence of five largest sources of non-gravitational accelerations: (i) solar radiation and (ii) solar wind pressure, (iii) emitted radio power, (iv) anisotropic thermal radiation of the spacecraft and (v) attitude motions. Although one could in principle set up complicated engineering models to predict at least some of the effects, their residual uncertainty may be unacceptable for the experiment, in spite of the significant effort required. A better approach seems to accept our ignorance about non-gravitational accelerations and assess to what extent these can be assumed constant over the time scale necessary to meet the relativistic stellar aberration requirements. In fact, a constant acceleration produces a linear frequency drift that can be accounted for in the data analysis be a single unknown parameter.

2.1. Direct solar radiation pressure

At the distance of $D = 0.983$ AU, from the Sun, the solar radiation pressure is determined by the cross-section of the spacecraft of total area $A = 4.5 \times 10^5$ cm^2 and produces an acceleration along the line of sight

$$a_{\mathrm{s.p.}} = (\alpha + 2\epsilon)\frac{f_\odot A\cos\theta}{cM\,D^2} = (\alpha + 2\epsilon)\,6.01 \times 10^{-6}\left(\frac{A}{45\ \mathrm{m}^2}\right)\ \mathrm{cm/s}^2, \quad (6)$$

where $M = 3,500$ kg is the total mass of the spacecraft and $f_\odot = 1367$ W m^{-2} is the solar constant; α and ϵ are, respectively, the absorption and reflection coefficients of the spacecraft, and θ is the angle between the largest surface (solar shade) and the direction to the Sun. For the worst case we have $(\alpha+2\epsilon)_{\mathrm{max}} \approx 1.7$, which makes the maximal value for $a_{\mathrm{s.p.}} = 1.02 \times 10^{-5}$ cm/s^2. Also, one may expect that the variations of the thermo-optical coefficients of the spacecraft's surface may be significant over the mission duration. This is why this particular contribution of the non-gravitational forces may pose a serious problem.

2.2. Fluctuations in the solar wind

The acceleration caused by the solar wind has the same expression (6), with $f_\odot$ replaced by $m_p v^3 n$, where $n \approx 5$ cm^{-3} is the proton density at 1 AU and $v \approx 400$ km/s is the speed of the wind. Thus, $a_{\mathrm{s.w.}} \approx 4.03 \times 10^{-12}$ cm/s^2. Because the density can change by as much as 100%, this acceleration is totally unpredictable, but is about 10^{-6} times smaller than the direct solar radiation pressure, and therefore completely negligible.

2.3. Emitted radio power

The recoil due to the emitted radio power P_0 produces an acceleration $a_0 = P_0/(Mc)$ of the spacecraft away from the Earth. For a total radiated power of 23 W (simultaneously at S and X bands) the resulting acceleration is $a_0 = 2.2 \times 10^{-9}$ cm/s^2, a level of two orders of magnitude smaller than the limit (5).

2.4. Anisotropic thermal radiation of the spacecraft

Any difference in temperature between parts of the spacecraft produces a force and a torque. The torque is counteracted by the attitude control system, through a set of reaction wheels. Thermal anisatropies are due to a complicated internal heat exchange driven by the absorbed solar radiation and the internally generated power. The latter is dominated by the thermal emission from the spacecraft solar panels. SIM will exploit one large solar cell panel mounted on the boom. Most of the power will be radiated from the panel in a symmetrical pattern, but the residual anisotropy could still produce a significant effect. Let us assume an extreme case in which the radiation pattern is so anisotropic that, say, $\delta P = 10$ kW are emitted along a constant but unknown direction with respect to the spacecraft. The resulting change in acceleration will amount to $a_\delta \approx \delta P/(Mc) = 9.45 \times 10^{-7}$ cm/s^2, which is almost three times larger than the threshold (5) and, therefore, may present a significant obstacle. The exact evaluation of this force would require an accurate thermal model of the spacecraft.

2.5. Attitude motions

As the center of phase of the antenna does not coincide with the center of mass of the spacecraft, attitude control motions give rise to a Doppler shift in the received ground signal. The 1.5 m paraboloid will be constantly pointed toward the Earth by means of reaction wheels or thrusters, which provide three axis stabilization of the spacecraft. A spacecraft rotation with angular velocity Ω (almost orthogonal to the Earth-pointing vector $\vec{e}$) produces a two-way relative frequency shift

$$\frac{\Delta\nu}{\nu} = \frac{2}{c}(\vec{\Omega} \times \vec{r}) \cdot \vec{e} \approx \frac{2\Omega d}{c}, \tag{7}$$

$\vec{r}$ being the position of the phase center in the spacecraft frame and $d = 2$ m its distance from Earth–center-of-mass line. What matter for the frequency stability are the variations of the angular velocity. For a given accuracy σ_v, the requirement on the fluctuation $\sigma_\Omega(\tau)$ is $\sigma_\Omega(\tau) < \sigma_v/(2d) = 1$ mrad/s.

Although thrusters will not be used during the astrometric mode, the effect of thruster leakage has to be assessed. This effect is highly unpredictable. However, a previous study of this effect on the high-accuracy navigation, based on data from various space missions, suggested that this effect will be well below the limit expected for the velocity aberration knowledge.

Conclusions

Thus the contributions of a number of most prominent perturbing forces, besides the solar radiation pressure and the anisotropic thermal radiation of the spacecraft, to the measurement error is negligible. There are few proposals that may minimize the impact of these perturbing factors and, therefore, improve the overall navigation accuracy: 1). on-board processing of data, as oppose to increasing the use of the Doppler tracking time; 2). the use of a precisely positioned solar shade for maintaining constant solar pressure loading; and 3). velocity determination using integrated and time averaged accelerometers. This issue is currently being addressed and results soon will be available.

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References

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