Dispersion and Resolution Requirement

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1 Redshift Accuracy

The BAO redshift uncertainty requirement is

$$\sigma_z \leq 0.001(1+z) \tag{1}$$

where σ_z is the 1-σ uncertainty in the redshift determination. This can be expressed as a requirement on the determination of the observed wavelength of a line

$$\frac{\sigma_z}{1+z} = \frac{\sigma_\lambda}{\lambda} \leq 0.001 \tag{2}$$

where σ_λ is the 1-σ uncertainty with which one can determine the wavelength of a peak. The resolving power of a spectrometer is typically defined as

$$R_\lambda \equiv \frac{\lambda}{\Delta\lambda}. \tag{3}$$

where $\Delta\lambda$ is the width of a resolution element. However, this quantity is not very useful for a slitless spectrometer when the sources are resolved. In the case of JDEM, we wish to locate the angular centroid of an Hα line relative to either a 0-order image or to a counter-dispersed copy of the spectrum. In these terms, the redshift accuracy requirement may be written as

$$\frac{\sigma_\theta}{\lambda\, d\theta/d\lambda} \leq 10^{-3}, \tag{4}$$

where $d\theta/d\lambda$ is the angular dispersion in the spectrum, and σ_θ is the angular uncertainty in the line centroid, discussed below. Note that this is a joint requirement on angular centroid uncertainty and dispersion.

In the case where we compare the line centroid to the image position (without regard for the roll angles of the component spectra) we have

$$\sigma_\theta = \sqrt{\sigma_{\theta_\pm}^2 + \sigma_{\theta_0}^2} \tag{5}$$

where $\sigma_{\theta_\pm}$ is the 1-σ uncertainty in the line centroid determination from a co-added spectrum, and σ_{θ_0} is the uncertainty in the image centroid, which is assumed to be uncorrelated with the line error. In the case of counter-dispersed spectra we have

$$\sigma_\theta = \frac{1}{2}\sqrt{\sigma_{\theta_+}^2 + \sigma_{\theta_-}^2} \tag{6}$$

where σ_{θ_+} and σ_{θ_-} are the line centroid uncertainties in the two counter-dispersed spectra, and the factor of 1/2 accounts for the fact that $\sigma^2_{\theta_+} + \sigma^2_{\theta_-}$ measures the variance of 2λ. In general, all of these uncertainties will have statistical and systematic components, which are assumed to be uncorrelated.

1.1 Statistical Errors

For a given signal to noise ratio, S/N, the statistical precision with which one can centroid a peak is roughly

$$\sigma_{\theta,\mathrm{stat}} = \frac{\Delta\theta_{\mathrm{FWHM}}}{S/N} \tag{7}$$

where $\Delta\theta_{\mathrm{FWHM}}$ is the FWHM of the convolved line in arcsec, which has contributions from the finite pixel size, p, the finite size of a typical source galaxy, expressed in terms of the half-light radius r_h, and the dynamic optical spot size σ_{spot}. Assuming a top-hat pixel profile and an exponential galaxy profile, these contributions lead to a convolved line width of

$$\Delta\theta_{\mathrm{FWHM}} = 2.35\sqrt{0.2\,p^2 + 0.71\,r_h^2 + \sigma^2_{\mathrm{spot}}}, \tag{8}$$

where the factor of 2.35 converts a 1-σ Gaussian dispersion to a FWHM. Then, assuming

$$\begin{aligned} p &= 0.25'' \quad \text{(image)} && (9)\\ &= 0.50'' \quad \text{(spectrum)} && (10)\\ r_h &= 0.35'' && (11)\\ \sigma_{\mathrm{spot}} &= 0.15'' \quad (2.0\,\mu\mathrm{m}), && (12) \end{aligned}$$

we obtain

$$\sigma_{\theta_0,\mathrm{stat}} = \frac{0.821''}{S/N|_0} \qquad \text{(image)} \tag{13}$$

$$\sigma_{\theta_\pm,\mathrm{stat}} = \frac{0.938''}{S/N|_\pm} \qquad \text{(spectrum)} \tag{14}$$

1.2 Systematic Errors

There are many sources of systematic uncertainty in the determination of the angular position of the image and line centroid (which will not necessarily be Gaussian distributed). We tabulate allocations to each source in Table 1.

2 Results

For the case where we centroid relative to the 0-order position, we obtain the following for the relative centroid error

$$\sigma^2_\theta = \sigma^2_{\theta_\pm,\mathrm{stat}} + \sigma^2_{\theta_\pm,\mathrm{sys}} + \sigma^2_{\theta_0,\mathrm{stat}} + \sigma^2_{\theta_0,\mathrm{sys}} \tag{15}$$

$$= \left(\frac{0.938''}{S/N|_\pm}\right)^2 + (0.077'')^2 + \left(\frac{0.821''}{S/N|_0}\right)^2 + (0.056'')^2. \tag{16}$$

Table 1: Allocated contributions to $\sigma_{\theta,\mathrm{sys}}$ (**Preliminary**)

Image - error source	value ($''$)
Optical distortion	0.045
Mean plate scale	0.025
Flat field and dark current	0.015
PSF asymmetry	0.015
Pixel to pixel cross talk	0.005
Total (root-square-sum)	0.056
Spectrum - error source	value ($''$)
Filter bandpass edge fit	0.040
Optical distortion	0.045
Mean plate scale	0.025
Prism dispersion	0.025
Filter bandpass edge knowledge	0.025
Flat field and dark current	0.015
PSF asymmetry	0.015
Pixel to pixel cross talk	0.005
Total (root-square-sum)	0.077

Then, demanding $S/N|_{\pm} = 6.5$ and assuming $S/N|_0 = 20$, we get $\sigma_\theta = 0.177''$ and

$$\lambda\,(d\theta/d\lambda) \geq 177'', \tag{17}$$

which is dominated by the statistical uncertainty in the line centroid.

For the case where we use the counter-dispersed spectrum, we obtain

$$\sigma_\theta^2 = \frac{1}{4}\left[\sigma^2_{\theta_+,\mathrm{stat}} + \sigma^2_{\theta_+,\mathrm{sys}} + \sigma^2_{\theta_-,\mathrm{stat}} + \sigma^2_{\theta_-,\mathrm{sys}}\right] \tag{18}$$

$$= \frac{1}{2}\left[\left(\frac{0.938''}{S/N|_+}\right)^2 + (0.077'')^2\right], \tag{19}$$

where we have assumed $S/N|_- = S/N|_+$. Further, assuming $S/N|_+ = S/N|_{\pm}/\sqrt{2} = 4.6$, we get $\sigma_\theta = 0.154''$ and

$$\lambda\,(d\theta/d\lambda) \geq 154''. \tag{20}$$

3 Requirement

The dispersion values given above represent the minimum dispersion needed to meet the top-level redshift accuracy requirement for a given set of allocated (not estimated) errors. These minimum values depend on the angular centroid accuracy which, in turn, depends on the achieved signal to noise ratio, and on many optical performance parameters. While preliminary allocations have been made, we note that dispersion values higher than the minimum allow one to set more forgiving - and less expensive - specifications on many parameters without sacrificing redshift accuracy. However, too high a dispersion results in loss of signal as one begins to resolve the NII line from the Hα line and the Hα line itself. Given these considerations, we require a dispersion in the range

$$200'' \leq \lambda\,(d\theta/d\lambda) \leq 240''. \tag{21}$$

The requirement applies everywhere over the full focal array. It specifies the product $\lambda\,(d\theta/d\lambda)$, where λ is the observed wavelength in Angstroms, and $d\theta/d\lambda$ is the dispersion in arcsec per Angstrom. It derives from the redshift accuracy requirement

$$\frac{\sigma_z}{1+z} = \frac{\sigma_\lambda}{\lambda} \leq 0.001, \tag{22}$$

which can be recast in terms of the angular centroid uncertainty and the dispersion

$$\frac{\sigma_\theta}{\lambda\,(d\theta/d\lambda)} \leq 0.001. \tag{23}$$

Here σ_θ is the 1-σ uncertainty in angular centroid of the line, which has statistical and systematic contributions

$$\sigma_\theta^2 = \frac{1}{4}\left[\sigma^2_{\theta_+,\mathrm{stat}} + \sigma^2_{\theta_+,\mathrm{sys}} + \sigma^2_{\theta_-,\mathrm{stat}} + \sigma^2_{\theta_-,\mathrm{sys}}\right] \tag{24}$$

$$\approx \frac{1}{2}\left[\left(\frac{0.94''}{S/N}\right)^2 + (0.08'')^2\right]. \tag{25}$$

In this expression, S/N is the signal to noise ratio in the positively dispersed spectrum (2 of the 4 rolls).