Chapter 15.
Monitoring of the Stored Muon Beam

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15.1. Introduction and Overview

The muon decay electron detectors provide substantial information on the beam inside of the storage ring. At early times the beam is bunched, with a base width of less than 10 ns, and a cyclotron period of ~ 150 ns. Each detector reports counts for a brief period each time the bunch goes around the ring (the 'fast rotation' structure). Eventually the bunch spreads out After the initial injection, particles of lower momentum, which are at smaller radii, get ahead of those at higher momentum at larger radii, the beam debunches and fills the ring uniformly. The de-bunching is of interest, since one gains information on the mean radius by measuring the fast rotation frequency and information on the momentum distribution from the debunching process.

This method was used in the third CERN experiment [1] and the computer code developed by Hattersley [2] forms the core of the E821 fast rotation analysis. This method only provides information on the momentum distribution during the first 20 μs, since the fast rotation signal disappears once the beam has debunched. Should the muon distribution change significantly after the debunching time, using the distribution obtained from the fast rotation would introduce an error into the electric field correction which must be made to obtain ω_a.

More recently, Orlov and Soffer [3] have proposed measuring the distribution of rotation frequencies, and thus the muon momentum distribution, by Fourier analysis of the electron time spectra. They claim that with sufficient statistics, the momentum distribution at all times can be measured.

In addition to the electron detectors, there will be several other devices which will monitor the muon beam in the storage ring. Scintillating fiber monitors will be used to measure the beam profile during injection studies, and each time the storage ring is brought back on after a shutdown. Pickup electrodes will measure the moments of the distribution from injection until the beam debunches. In one non-standard vacuum chamber section, a set of wire chambers will measure the decay electron trajectories, and permit us to infer the muon distribution in the ring.

Each of these techniques is discussed in more detail below.

15.2. Scintillating Fiber Monitors

Scintillating fiber monitors to measure the beam profile in the storage ring are being designed and constructed at KEK. [4] A schematic of the overall system is given in Fig. 15.2.1, and sketches of the x and y planes are given in Fig. 15.2.2.

These monitors will be used to monitor the beam immediately after injection into the storage ring, especially during the commissioning of the ring, and when the experiment is brought on in the successive years for running. These monitors will be retractable, and will be removed from the storage region during normal data collection.

There will be two sets of $x - y$ monitors in the ring, one set at 180^0 and one at 270^0 (relative to the inflector exit) around the ring. See Chapter 22, §22.3 for a discussion of their use during commissioning.

The fibers in each plane are equally spaced in a "harp" configuration as can be seen in Fig. 15.2.2. Since the light output of each of the fibers will depend on the number of muons in the bunch passing through that fiber, the pulse height will be proportional to the beam intensity at the fiber location. For example, if 2×10^4 muons are stored in the 90 mm diameter storage aperture, a single fiber of 0.5 mm thickness will see about 40 muons in one revolution, which corresponds to 40 photoelectrons. Each fiber will be read out by a separate photomultiplier, since cross talk problems were encountered with multi-anode phototubes. [4]

Because cross calibration of the relative light yields of the different fibers is essential to obtain an accurate beam profile, it is necessary to have a procedure which can be carried out inside the storage ring. By rotating the "harps" by 90^0, it will be possible to place each fiber just downstream of the next, thus presenting the same portion of the beam to all fibers in a plane. This calibration position is shown in Fig. 15.2.2 for each plane.

These monitors will be quite useful during the intial injection studies, and also when the muon kicker is commissioned, since they will provide a pulse to pulse picture of the beam profile in the storage ring.

Two types of planes are being prepared, planes with seven 0.3 mm thick fibers, and planes with six 0.5 mm planes. Since the fibers are thin, ≤ 0.5 mm, and there are only a few fibers in each plane, the multiple scattering is small and one can monitor the beam for some number of turns before the beam is seriously degraded by these monitors. The beam lifetime from multiple scattering in the fibers is shown in Table 15.2.1, which should

Figure 15.2.1: Schematic View of the Fiber Monitor System

(.a)

(.b)

Figure 15.2.2: **(a)** The x profile monitor showing both the measurement position (upper) and the calibration (lower) positions. **(b)** The y profile monitor showing both the measurement position (left) and the calibration (right) positions.

Table 15.2.1: Beam lifetime τ_{sf} from multiple scattering in the scintillating fiber monitors.

Fiber Thickness (#)	# of planes	τ_{sf} due to scattering
0.3 mm $\times$ (7)	one	1.3ms
	two	0.65 ms
0.5 mm $\times$ (6)	one	0.56 ms
	two	0.29 ms

be compared with the muon decay lifetime of $\gamma\tau = 64.4$ μs and the data collection time of $10\gamma\tau = 644$ μs.

15.3. Monitoring of the Muon Beam with Pickup Electrodes.

15.3.1. Introduction to the Pickup Electrode Principle

The Pickup Electrodes will provide a real time picture of the moments of the muon distribution during data collection, in a non-destructive way. As discussed in Chapter 5, §5.2.1, in order to determine the average magnetic field seen by the circulating muons, it is necessary to know the moments of the muon distribution. [5] A scheme which was proposed to measure the muon moments [5] involves pickup electrodes which form a cylinder with its axis along the muon beam's path. The cylinder consists of 8 independent plates 1 m long, each of which is connected to a separate charge preamplifier. The signals are Fourier analyzed around the cylinder's azimuth in order to determine the moments of the muon distribution up to $N = 4$ for the cosine term and $N = 3$ for the sine term.

Figure 15.3.1: Cross section of the pickup electrode geometry

15.3.2. Hardware

The pickup electrodes will occupy a total of 120 *cm* of the $(g-2)$ ring azimuth. The design of the plates is intimately connected to the beam tube trolley design, and a final geometry will be possible only after the trolley design is finished. The cross section of the pickup electrodes is octagonal. There are eight plates, each ~ 100 *cm* in length, four of the which resemble the quadrupole electrode geometry with the four others occupying the corners. In two of the corners, trolley rails will be added as needed (see Chapter 14). A cross section of the pickup electrodes is shown in Fig. 15.3.1. The supporting cage will be very much like the one used for the quadrupoles. [6] To lower the capacitance of the plates, the trolley rails will be non-metallic.

Figure 15.3.2: The low noise charge pre-amplifier design scheduled to be used in the readout of the pickup electrodes.

15.3.3. Electronics

The charge preamplifier was designed by the BNL instrumentation division, (see Fig. 15.3.2), and the signal to noise ratio for the prototype electronics was found to be adequate on a small prototype. The capacitance of the prototype plates was measured to be about 90 pF total. A full size model has been constructed and will be tested.

The prototype electronics showed promise of being able to resolve the muon distribution in real time, rather than having to average over a number of fills. The charge preamplifiers will be located in the vacuum chamber very close to the plates from which they pick up the charge, which is possible since they had sufficiently low magnetic susceptibility to be approved for use in the main field by the Suceptibility Sheriff. However, the packaging must be changed to meet the outgassing requirements of the ring vacuum system.

The signal from the preamplifier will be transferred to the voltage amplifier with semi-rigid, low loss high bandwidth solid copper coaxial tubes in order to minimize electronic pickup. These second-stage voltage amplifiers will be optimized for the best signal to noise ratio, and will be placed on the outside of the vacuum chamber for more flexibility.

A charge-integrating ADC (Analog to Digital Converter) and a personal computer will read, average, Fourier analyze and display the signals.

15.4. Muon Decay Electron Traceback

The electron traceback system consists of a set of wire chambers which measure the trajectory and momentum of decay electrons in one section of the storage ring. The goal is to propagate the electron back along its trajectory up to the point where its radial component of motion is zero. Leaving aside experimental errors for the moment, the distance of this point to the actual site of the muon decay is typically about 1 mm in the radial and vertical directions. As discussed in Chapter 4, §4.3, a knowledge of the muon phase space distribution is essential for performing the pitch and magnetic field corrections. Furthermore, it provides additional information to supplement the fast rotation data in determining the electric field correction.

Several guidelines have been observed in designing the chambers:

1. The chambers themselves should be made of low Z material to minimize multiple scattering. And, since the chambers themselves cannot be placed in the vacuum, whatever exit window the electrons must pass through before striking the chambers must likewise be made of some low Z material.
2. The arrangement of the chambers should be optimized for decay electrons with energies between 1.5 GeV and 2.5 GeV, where the traceback is most accurate. The highest energy electrons must be traced back the farthest (in fact the traceback distance becomes asymptotically infinite as the electron energy approaches its endpoint) which maximizes the destructive effect of errors caused by multiple scattering.
3. There is an inverse relation between the acceptance of the system and the separation of the chambers, but a direct relation between the accuracy of the momentum measurement and the separation.
4. The outer edge of the exit window should be placed at as large a radius as possible. Electrons which exit from the vacuum tank into the traceback chamber system at the largest radius have traveled the shortest distance and can be traced back with the greatest accuracy.

5. The acceptance of the traceback system will depend on the energy of the electrons, their initial radius and, to a small extent, their initial vertical position.

The accuracy of the traceback technique in obtaining the muon phase space distribution has been investigated with G2GEANT, our GEANT-based simulation program and TRACENON, which uses a slightly modified version of CERN's GEANE package to perform the traceback. The simulations have considered the effect of multiple scattering in the vacuum exit window and the chambers themselves, the effect of the intrinsic position resolution of the chambers, and the effect of the electron selection. The electron selection is determined by the positions and sizes of the exit window and the chambers. The exit window must be thin (0.5 mm) mylar or kapton foil. Each of the chambers must have a position resolution of 100 μm and have a thickness equivalent to no more than 200 μm of mylar. To reconstruct the (approximately) helical path at least three radial (horizontal) and two axial (vertical) coordinates are needed. To provide redundancy, four measurements of each coordinate are planned.

A diagram of the traceback system layout is shown in Fig. 15.4.1. The four chambers are separated by about 25 cm each. The last lies in front of a calorimeter station. The first lies directly behind a chamber exit window which is anchored by an aluminum flange. The window is 14 cm high and its edge extends outward to a radius of 701 cm and inward to 692 cm. Pushing the edge out further in the radial direction would require that the flange obstruct the tracks of high energy electrons headed for the next calorimeter station downstream. The vacuum wall is 2 cm thick as drawn, although this thickness has not been optimized. The thinner the vacuum wall, the more useful will be the data from the calorimeter station which lies just beyond the traceback system. A thinner vacuum wall will also mean a more difficult engineering job.

Although the exact results depend on the parameters of our chamber design and the choice of electrons in the sample, the typical reconstruction error in the vertical and radial directions is 3 mm. To obtain the muon phase space distributions we must also make acceptance corrections with the help of our simulation. Generally, the corrections are not large and their success should be limited by our knowledge of the fringe field and of the precise location of the various obstacles and detector components. Finally, we will need to model the betatron motion of the muons in order to deduce from this local measurement where the muons would be found elsewhere in the storage ring.

We have decided to make the traceback measurement with drift chambers, a well established technology which can provide the required position resolution. (The principal competition, the silicon microstrip detector, has excellent position resolution, but is too thick. The typical thickness is 300 μm, which is equivalent to 1000 μm of mylar. The traceback system requires four radial and four axial measurement planes: smaller (10 $cm \times$

Figure 15.4.1: Traceback system layout

15 cm) in front of the exit window and larger (25 $cm \times 15$ cm) in front of the calorimeter.

These dimensions make no particular demands on the construction of the drift chambers; in fact, the traceback system is not large. Although there are many different varieties of drift chambers, ones of the straw tube geometry may be appropriate. This type of chamber has typically a position resolution of 100 μ for a tube diameter of 10 mm. Straw tubes can be made from aluminized mylar with a wall thickness as small as 25 μm. Thus a stack of straw tubes can have the required resolution and thickness. The four horizontal and vertical planes require a system of about 400 tubes. Straw tubes have the advantage of a simple electric field configuration and independence of the tubes in a stack. The problem of the "right-left" ambiguity, in which the measured drift time can correspond to two positions, can be resolved by using a stack of tubes, the minimum number of layers in the stack is, of course, two. The non-uniform electric field, which results from the cylindrical geometry of the tube, implies that the relation between the drift time and the position of the track is very non-linear. As with other drift chamber geometries, the calibration of straw tubes requires careful consideration. In the traceback application there may be no method of calibration in place, since neither the momentum nor the vertex of the track is known a priori. Thus the calibration of the system must be done in a test beam, a less satisfactory situation.

Another major consideration in the design of the traceback chambers is the magnetic field. The uniform field extends only to a radius of 695 cm, but the traceback chambers must extend to a radius of 681 cm. Thus approximately half of the volume of the drift chambers lies in a highly non-uniform magnetic field. On the median plane outside the uniform field region, the field gradient is of the order of 500 G/cm at the position of the traceback chambers. Seven centimeters off the median plane the radial component of the field is as large as 500 G, and the direction is of the order of 20^o away from the vertical. These are unfavorable conditions for the operation of a drift chamber due to variation in the magnitude and direction of the drift velocity of the electron ionization.

For the measurement of the horizontal coordinate, the sense wires in the drift chamber run vertically, and the drift field lies in a horizontal plane. The drifting electrons then experience a Lorentz force which effects the direction of their drift. This phenomenon is well known in drift chambers and is described by the Lorentz angle, the angle between the velocity vector of the drift electrons and the electric field. Chamber gases with high (low) drift velocity tend to have large (small) Lorentz angles. Since the Lorentz angle and the electron mobility depend, in general, on both the electric and magnetic field, in a non-uniform magnetic and electric field, the relation between the track position and direction and magnitude of the drift velocity is complicated. The same problem arises in the measurement of the vertical coordinate where the sense wires run horizontally, and the drift field is in a vertical plane. Off the median plane the radial component of the magnetic field again causes a Lorentz force. Thus the relation between drift time and the track coordinate will depend on the location of the track. As the design of the traceback chambers continues, the drift time in the non-uniform magnetic field will be simulated.

The effect of the non-uniform magnetic field can be minimized by the choice of chamber gas. A "slow" chamber gas, e.g. CO2-isoC4H10, has a small Lorentz angle regardless of the fields. With a "slow" gas the maximum drift time in a 1 cm straw tube will then be of the order of 500 ns. The estimated number of calorimeter hits per spill for muon injection is 240. The probability of more than one hit within a 500 ns gate, 20 microseconds after injection, is 30%. This is not a large probability; however, for successful track reconstruction, there must be few extraneous hits in the chambers. As mentioned above, extraneous hits are expected from decay electrons which shower in the wall of the vacuum tank, an effect which will be investigated in some detail with our simulation program. The calibration of the traceback system must be done in a non- uniform magnetic field, and, ideally, the field profile should be identical to the field profile of the ring. Thus the fringe field of the ring must be mapped and a test beam magnet must be constructed to reproduce this field. Position resolutions of 100 μm and better have been reported for drift tube systems with **B** fields as non-uniform as our own, but obtaining such accuracy with this chamber system will undoubtedly be challenging.

One should note that if the chambers can be made to work at early times (perhaps by reducing the intensity) we can study the debunching directly with the traceback system.

15.5. Measurement of the Muon Momentum Distribution

Precise knowledge of the muon momentum distribution is needed to calculate the largest correction in the $(g-2)$ experiment, the error caused by the electric field in the presence of a spread of muon momenta. This correction is 0.69 *ppm* and we need to know this correction to about 10% to eliminate its influence on the systematic error on ω_a (see Chapter 4 §4.3.6). The CERN experiment used the fast rotation of the unbunched beam to study the momentum distribution which has been documented by Hattersley. [2] More recently it has been proposed that a Fourier analysis of the time spectra from the electron detectors can provide better information on the momentum distribution.

Fourier analysis is a powerful tool to extract the frequency content from a signal, and the techniques are well understood numerically. [7-9] However, one should be aware that Fourier analysis, while widely used in signal processing, does have its limitations. [10] Fourier analysis works best when a few discrete frequencies are present. When the situation is more complicated, it is a general practice to make a model of the signal and to try to fit the signal directly in the time domain. This technique avoids the bandwidth problems associated with the Fourier spectrum, as well as problems associated with going from a continuous to a discrete transform (see Ref. 8). This is, of course, the tack taken in the fast rotation analysis, where one assumes a model for the radial distribution and then calculates the observed time spectrum of decays during the debunching. While it is true that numerical Fourier techniques are both very powerful and well understood, [8,9] one must take care to understand the limitations of the technique in each specific application. [10]

15.5.1. The Momentum Distribution I: Fast Rotation Analysis

15.5.1.1. The Fast Rotation: General Principles

The bunch of particles injected into the storage ring has a radial distribution over the whole of the aperture but a very narrow time width ($\sim 10\ ns$) which is much less than the rotation time ($149\ ns$) of the particles in the ring. Since all particles travel at essentially the speed of light, those at inner equilibrium radii will get steadily ahead of those at outer equilibrium radii. Thus the time distribution of particles on subsequent turns exhibits an ever increasing width which is directly related to the initial distribution, and which depends mainly on the initial radial distribution. The effect of betatron oscillations can be neglected. If we call the betatron deviations r_B and z_B, the linear r_B/R term is averaged after the first 10 turns, and the quadratic terms $\left(r_B^2 + z_B^2\right)/R^2$, are very small.

The assumption that the radial distribution is constant in time is not true for genuine data; there is an exponential fall-off due to muon decay, which gives a reduction in rate of about 15% over the analysis time, and there is also the ω_a modulation. The analysis program, REVFR, [2] is set up to deal with a constant average rate in the analysis, so the number of counts is multiplied by $e^{t/64.4\mu s}$ to correct for muons lost by decays. In addition, the data are smoothed to bring the general level down to that at the end of the analysis period, which results in a small overestimate in the statistical errors.

With these assumptions and with a know input distribution, it is easy to calculate the time distributions of muons seen in each subsequent revolution of the muon bunch in the storage ring. The distribution is divided into radial bins and from the known rotation time the relative positions (that is the bunch lengths) are calculated on subsequent turns. After a number of turns, the inner edge will have caught up with the outer edge of the previous turn; the peaks will overlap, the picture will be more confused and the time-histogram will tend to smear out. However, the calculation is still quite possible – the only difference being that two (and eventually 3,4, etc.) radial bins will contribute to some of the time bins; the contribution of any radial bin to any time bin is still directly calculable. The time limit comes in principle when a radial bin starts to overlap itself; however the information content in the time histogram is very small long before this occurs, and in practice it seems that it is only worth analyzing data to about twice the overlap time.

The reverse calculation, *i.e.* the determination of the input distribution from the time-histogram, is not so straightforward. Firstly, one must make some assumption as to the input time shape; this fortunately appears not to be critical. Secondly, the final results are needed in the process of calculation; however, by an iterative procedure it is possible to get successively better approximations and in practice it turns out that only one repetition is necessary to reach the accuracy warranted by the data.

The output of REVFR is a parameterization of the input radial distribution normalized to the maximum bin. In order to compare the output distribution with that originally put into the Monte Carlo data generation program, the mean radius is found, together with its standard deviation and the rms width of the distribution. The input and output histograms are also compared directly: an agreement factor is given which consists of the sum of the squares of the deviations weighted according to statistics, and the change in ω_a due to the distribution is given, together with the error in comparison with the input distribution.

15.5.1.2. The Mathematical Details

For the fast rotation analysis, we suppose that the storage aperture is divided into I radial (momentum) bins with the suffix i. Let the contents of the radial bins be f_i; it is this distribution f_i which we want to determine.

We observe the histogram in time and we denote the time bins with the suffix j. Let the counts we observe in bin j be N_j, which form the input data.

The expected contribution from radial bin i to the counts in time bin j is

$$f_i \beta_{ij} \tag{15.5.1}$$

where β_{ij} is a geometrical factor which can and must be calculated separately for each combination of i and j. [11]

We must choose the population of the radial bins, f_i to give maximum agreement between the expected counts, C_j, and the observed counts, N_j, in all time bins. For each time bin j. The expected number of counts is

$$C_j = \sum_{i=1}^{I} f_i \beta_{ij}.$$

We wish to minimize χ^2 where

$$\chi^2 = \sum_j \frac{(C_j - N_j)^2}{Z_j} = \sum_j \frac{\left(\sum_i f_i \beta_{ij} - N_j\right)^2}{Z_j}.$$

Z_j is a weighting factor which should equal the expected counts in bin j. Since the Z_j are initially unknown, we replace them by the N_j in the first pass, taking precautions when they are zero. With the f_i so obtained we carry out a second pass with

$$Z_j = \sum_i f_i \beta_{ij}$$

Experience shows that a third pass does not appreciably alter the results, provided the process converges at all.

To minimize χ^2 by adjusting the $f_n (n = 1, 2, ...I)$ we require

$$\frac{\partial \chi^2}{\partial f_n} = 0$$

for each n. In differentiating χ^2 we assume that the denominators, Z_j are constant, which is not strictly true. However, since the numerators change much faster with f_n than the denominators, it is a good approximation.

We get

$$\frac{\partial \chi^2}{\partial f_n} = \sum_j \left[\frac{\left[\sum_i f_i \beta_{ij} - N_j\right] \beta_{nj}}{Z_j} \right] = 0$$

Hence

$$\sum_j \left[\frac{\sum_i f_i \beta_{ij}}{Z_j} \right] = \sum_j \frac{N_j \beta_{nj}}{Z_j}$$

which, upon reversing the order of summation gives

$$\sum_i f_i \sum_j \left[\frac{\beta_{ij} \beta_{nj}}{Z_j} \right] = \sum_j \frac{N_j \beta_{nj}}{Z_j} \tag{15.5.2}$$

Eq. (15.5.2) represents a set of equations, one for each n, *i.e.* we have I equations for I unknown values of f_i. We solve the equations iteratively to obtain the f_i. The procedure can be represented schematically as:

1. first pass, with $Z_j = N_j$
2. second pass, using the f_i of the first pass and $Z_j = \sum_i f_i \beta_{ij}$.

The I values of f_i can then be used in the so-called electric field corrections, which corrects ω_a for the fact that not all muons have energies which correspond to the magic gamma.

15.5.2. The Momentum Distribution II: Fourier Analysis

From the basic formula,

$$\vec{\omega}_a = \frac{e}{mc}\left[a\vec{B} - \left(a - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right], \tag{15.5.3}$$

where $\beta = v/c$, we can derive the expression for the electric field correction:

$$\frac{\Delta\omega_a}{\omega_a} = -2\langle\frac{\gamma - \gamma_m}{\gamma_m}\frac{p - p_0}{p_0}\rangle\frac{1}{2\pi}\int_0^{2\pi}\psi\left(\theta\right)n\left(\theta\right)d\theta, \tag{15.5.4}$$

where $\gamma_m = \left(\frac{a+1}{a}\right)^{1/2}$ is the magic energy; p_0 is the muon momentum corresponding to the central orbit with the radius R_0, $E(R_0) = 0$; $n(\theta)$ is the actual (not average) field index as a function of azimuth θ,

$$n\left(\theta\right) = \mid\left(\frac{R}{\beta B}\right)\left(\frac{\partial E_R\left(\theta\right)}{\partial R}\right)\mid; \tag{15.5.5}$$

$\psi(\theta)$ is a familiar function in accelerator physics, defining the equilibrium orbit vs. momentum:

$$\frac{R\left(\theta\right) - R_0}{R_0} = \psi\left(\theta\right)\frac{p - p_0}{p_o}; \tag{15.5.6}$$

$$\overline{\psi\left(\theta\right)} \equiv \frac{1}{2\pi}\int_0^{2\pi}\psi\left(\theta\right)d\theta = \alpha \tag{15.5.7a}$$

$$\frac{\Delta L}{L} \equiv \overline{\frac{R\left(\theta\right) - R_0}{R_0}} = \alpha\frac{p - p_0}{p_0} \tag{15.5.7b}$$

and α is the compaction factor, which relates the momentum spread Δp to the circumference spread ΔL, and is defined by [12]

$$\frac{\Delta L}{L} = \alpha\frac{\Delta p}{p_0}$$

which appears in Eq. (15.5.7b). Bars designate kinematic averages over some intervals of the ring, and brackets designate statistical averages.

The integrand in Eq. (15.5.4) is non-zero only inside a focusing quad. Let n_Q, ℓ_Q, $\psi_Q(\theta)$ be the internal field index, length, and ψ-function of a quad. Then,

$$\frac{1}{2\pi}\int_0^{2\pi}\psi\left(\theta\right)n\left(\theta\right)d\theta = n_Q\frac{\ell_Q}{\ell_Q + \ell_F}\overline{\psi_Q} = \frac{\overline{n}}{\ell_Q}\int_0^{\ell_Q}\psi d\ell = \overline{n}\overline{\psi_Q}, \tag{15.5.8}$$

ℓ_F is the interval between quads. Note that $\overline{\psi_Q} > \overline{\psi(\theta)} = \alpha$ because the ψ-functions inside a focusing lens are always bigger than between lenses. We can now rewrite Eq. (15.5.4) in the form

$$\frac{\Delta\omega_a}{\omega_a} = -2\overline{n}\overline{\psi_Q}\langle\frac{\gamma - \gamma_m}{\gamma_m}\frac{p - p_0}{p_0}\rangle, \quad \overline{n} = \frac{1}{2\pi}\int_0^{2\pi}n\left(\theta\right)d\theta. \tag{15.5.9}$$

In the muon $(g-2)$ experiment, the value $1/\gamma^2$ is much less than the accuracy needed to correct the error $\Delta\omega_a/\omega_a$. For this reason, in Eq. (15.5.9) let $\beta \cong 1$, $pc \cong \gamma$, and for muon revolution frequencies,

$$f = 1/T = v/L \cong c/L, \quad L = 2\pi\overline{R}, \tag{15.5.10a}$$

$$\frac{\Delta f}{f} = \frac{\Delta v}{v} - \frac{\Delta L}{L} = \left(\frac{1}{\gamma^2} - \alpha\right)\frac{\Delta p}{p} \cong -\alpha\frac{\Delta p}{p} = -\alpha\frac{\Delta\gamma}{\gamma}. \tag{15.5.10b}$$

Thus, in Eq. (15.5.9) we can use γ instead of p, and vice versa; also, we can let $\gamma_0 = \gamma_m$ in the denominator. From Eq. (15.5.10) we get another expression for $\Delta\omega_a/\omega_a$:

$$\begin{aligned}\frac{\Delta\omega_a}{\omega_a} &= -2\overline{n}\frac{\overline{\psi}_Q}{\alpha^2}\langle\frac{(f-f_m)(f-f_0)}{f_m^2}\rangle \\ &= -2\overline{n}\frac{\overline{\psi}_Q}{\alpha^2}\left[\langle\left(\frac{\Delta f}{f_m}\right)^2\rangle + \langle\frac{\Delta f}{f_m}\rangle\frac{f_m - f_0}{f_m}\right] \qquad (15.5.11a)\\ &= -2\overline{n}\frac{\overline{\psi}_Q}{\alpha^2}\left[\langle\left(\frac{f}{fm}\right)^2\rangle - \langle\frac{f}{f_m}\rangle\left(1 + \frac{f_0}{f_m}\right) + \frac{f_0}{f_m}\right], \qquad (15.5.11b)\end{aligned}$$

where $\Delta f = f - f_m$ with f_m, the frequency associated with the "magic momentum" as the central frequency. Again, we can use L instead of f, and vice versa.

Approximately (but only for evaluations),

$$\overline{\psi}_Q/\alpha^2 \sim (1 - \overline{n})\,. \tag{15.5.12}$$

Here f_m is the frequency associated with the "magic momentum", f_0 is the frequency associated with the actual central momentum. The difference between f_m and f_0, which is not desired, can occur if the central magnetic field is not correctly set.

We plan to use (and compare results of) two methods of measuring $\langle\frac{f}{f_m}\rangle$ and $\langle\left(\frac{f}{f_m}\right)^2\rangle$.

1. The CERN approach [1,2], which was described in the previous section, is based on the observation of the muon debunching in the storage ring. This observation is possible only during a rather small part of the muon lifetime, namely, after the scraping and before the actual spreading of the bunch over the ring, some 50 turns compared with the 400 turns per muon lifetime of 64.4 μs.
2. The new method [3] described below which employs a weighted Fourier analysis of the decay electrons. This method can be applied to the whole period after scraping ($\sim 10\tau$); the longer the duration of the analyzed signal, the better, because the phenomenon on which this method is based is a resonance in a specific sense.

The general theory of this method is given in Ref. 3; here we present a qualitative explanation.

Figure 15.5.1: Schematic of a ring with only one counter.

For clarity of the physical picture, let us consider an initially short bunch of muons injected at time zero onto azimuth $z_0 = 0$, where a single counter C is placed (Fig. 15.5.1). We can assume that after injection, the muons are decaying either outside the counter (which has effective azimuthal size, $\pm\delta z$) and, therefore, are not counted at all, or at point $z_0 = 0 \pm \delta z$, and only in that case are counted. In reality, of course, the counter gathers electrons from some area upstream of the ring. There are two points to note. First, the decay electrons are moving so closely to the muon trajectories ($v = c$ and $\varphi \ll 1$, see Fig. 15.5.1) that their signals come almost simultaneously with those from muons decaying just at $z = z_0$. Second, the average difference in time can be computed and the appropriate correction incorporated into the results; third, and most important, this time difference represents merely a general shift of the time axis and is not essential *if* we measure a distribution of electron signals in time always relative to the first electron signal at C. (For a more detailed definition of time zero for the Fourier analysis of decay electrons, see Ref. 3.)

Thus, every time t_n that an electron hits the counter, we can take the case as a signal from a muon passing the counter (only once in its lifetime).

Figure 15.5.2: Signals from different muons having the same special frequency

Obviously,

$$t_n = Tn_{\text{rev}} = n_{\text{rev}}/f, \tag{15.5.13}$$

where n_{rev} is the number of the muon's revolutions before its decay, and f is its individual frequency. The Fourier analysis in this case includes calculation of the product of the muon signal, $\delta N_\mu(t_n)$ which is the number of muons giving a signal after n revolutions, and a harmonic function $\cos \omega t_n$ with some arbitrary ω. One then calculates the sum, $F(\omega)$:

$$F(\omega) = \sum_{n=0}^{n_{\max}} \delta N_\mu (t_n) \cos \omega t_n. \tag{15.5.14}$$

In our case, $t_{\max} = n_{\max} T \sim 10\gamma\tau$, where $\gamma\tau = 64.4\ \mu s$ is the muon lifetime.

Muon signal $\delta N_\mu(t_n)$ contains, in general, a sum of signals from different muons with different momenta p, i.e., with different revolution frequencies f. Among them there can exist the special frequency (relative to the given ω):

$$f = \omega/2\pi q, \quad T = q\frac{2\pi}{\omega} \tag{15.5.15}$$

where q is an integer.

Figure 15.5.3: Signals from different muons having the same non-special frequency

Signals from different muons having the same special frequency are summarized in Eq. (15.5.14) with the same phases of $\cos \omega t_n$, see Fig. 15.5.2.

$$[F(\omega)]_{f=\frac{\omega}{2\pi q}} = [\delta N_\mu(0)]_{f=\frac{\omega}{2\pi q}} \sum_{n=0}^{n_{\max}} e^{-nT/\tau} \approx \frac{[\delta N_\mu(0)]_{f=\omega/2\pi q}}{\left(1-e^{-T/\tau}\right)} \tag{15.5.16}$$

where the assumption has been made that $n_{max} \to \infty$.

Signals from different muons having the same non-special frequency are summarized with different phases of $\cos \omega t_n$ (see Fig. 15.5.3, where $T < T_{\mathrm{res}}$). In the general case, when $T = 2\pi q/\omega + \delta T$,

$$[F(\omega)]_f = [\delta N_\mu(0)]_f \cdot \sum_{n=0} e^{-nT/\tau} \cos \omega n T \tag{15.5.17a}$$

$$\cong \frac{[\delta N_\mu(0)]_f}{\left(1-e^{-T/\tau}\right)} \left[\frac{1+e^{-T/\tau}\omega^2 \delta T^2/2\left(1-e^{-T/\tau}\right)}{1+e^{-T/\tau}\omega^2\delta T^2/\left(1-e^{-T/\tau}\right)^2}\right] \tag{15.5.17b}$$

the approximation in Eq. (15.5.17b) being valid for $\omega\delta T << 1$. Far from the special frequency, $\mid \delta T/T \mid \gg 1/\omega\tau$, or equivalently $\omega\delta T >> T/\tau$, Eq. (15.5.17) gives

$$\frac{[\delta N_\mu(0)]_f}{\left(1-e^{-T/\tau}\right)} \cdot \frac{T}{2\tau} \to 0 \ \text{ when } \tau \to \infty. \tag{15.5.18}$$

since $\delta N_\mu(0) \sim T/\tau$ also.

We should compare Eq. (15.5.18) and Eq. (15.5.16). Note that $T/2\tau \sim 10^{-3}$. Thus, for $\mid \frac{\delta f}{f} \mid \gg \frac{1}{\omega\tau}$, the non-special frequencies will be averaged and $F(\omega)$ will be proportional to the relative number of muons with revolution frequencies $f = \frac{\omega}{2\pi q}$. That is the main idea of this method.

However, the accuracy of the reconstruction of muon momentum (or frequency) distribution is limited by the finite muon lifetime. Two spectrum lines, f_1 and $f_2 = f_1 + \delta f$ can be separated only if

$$\mid \frac{f_2 - f_1}{f} \mid \gg \frac{1}{\omega\tau} = \frac{1}{2\pi q f \tau}, \tag{15.5.19}$$

with $f\tau \approx 400$. If $q = 1$, for example, the errors of reconstruction exceed 30%. The use of the highest q's (i.e., highest ω's) for the Fourier analysis needs much bigger statistics.

The solution of this problem is to analyze a weighted signal $\delta N_\mu(t)$. If, instead of $\delta N_\mu(t)$, we use

$$e^{+t/\tau} \delta N_\mu(t),$$

then the limitation Eq. (15.5.19) will be eliminated. As a price for this we will need bigger statistics because the relative fluctuations of $\delta N_\mu(t)$ (not shown in Fig. 15.5.2 and Fig. 15.5.3) grow exponentially in time and our factor $e^{t/\tau}$ increases the level of those fluctuations.

Also, we need to use an additional weight function in order to eliminate from our spectrum the side bands created by spin precession which modulates the electron signal intensity at the counter. We can clearly see these FM side bands in Fig. 15.5.4a (the result of simulation). They distort the tails of the distribution we have sought.

Simply stated, any components such as the exponential decay or the $(g-2)$ modulation which make the background over the time region to be analyzed not flat, need to be taken out before the Fourier analysis is carried out to obtain the distribution of rotation frequencies. This is the purpose of the weighting function given below.

The final weighted signal that we will analyze will be

$$S(t) = \frac{e^{t/\tau} \delta N_\mu(t)}{1 + A \sin(\omega_a t + \phi)} = W(t)\, \delta N_{\mu}.(t) \tag{15.5.20}$$

where A and ϕ will be taken from the very earliest results of $(g-2)$ measurements. Fig. 15.5.4b shows the spectra for different q's, and Fig. 15.5.5 a result of a simulation for $q = 1$, $A = 1/3$, $\phi = 0$. The full number of muons N was equal to 10^8 with 24 counters over the ring. The initial bunch had the azimuthal length 6m and the (arbitrary) muon frequency distribution used in the simulation was deliberately chosen to be complicated and asymmetric. Zero time corresponds to the condition,

$$\langle \sin \theta_{0k} \rangle = 0, \tag{15.5.21}$$

15.6.4a

15.6.4b

Figure 15.5.4: **a** The frequency spectrum showing the side bands from the muon spin precession. **b** The frequency spectrum for different q's.

Figure 15.5.5: The result of a simulation for $q = 1$, $A = 1/3$, $\phi = 0$.

where θ_{0k}, $k = 1 \;,\; \ldots \;, N$, are initial azimuths of different muons. The accuracy of the reconstruction of the initial revolution frequency distribution is about 0.3%.

15.6. References

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