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Radiative energy loss and $p_\perp$-broadening of high energy partons in nuclei

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Abstract

The medium-induced $p_\perp$-broadening and induced gluon radiation spectrum of a high energy quark or gluon traversing a large nucleus is studied. Multiple scattering of the high energy parton in the nucleus is treated in the Glauber approximation. We show that $-dE/dz$, the radiative energy loss of the parton per unit length, grows as L, the length of the nuclear matter, as does the characteristic transverse momentum squared of the parton $p^2_{\perp W}$. We find $-dE/dz = \frac{1}{8}\alpha_s N_c p^2_{\perp W}$ holds independent of the details of the parton-nucleon scatterings so long as L is large. Numerical estimates suggest that $p_\perp$-broadening and energy loss may be significantly enhanced in hot matter as compared to cold matter, thus making the study of such quantities a possible signal for quark-gluon plasma formation.

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1 Introduction

In a recent paper [1], hereinafter referred to as BDMPS II or simply as II, we investigated the problem of induced gluon radiation and radiative energy loss of a high energy quark or gluon traversing hot QCD matter of finite volume. This induced radiation may be an important tool for studying the physics of relativistic heavy ion collisions. In particular, contrasting hot and cold matter results may provide a signal for quark-gluon plasma formation. The induced gluon radiation may be measurable by studying the medium dependence of the characteristics of high-$p_\perp$ jet and hadron production.

The present paper is devoted to the study of radiative energy loss and jet $p_\perp$-broadening in nuclear matter. We shall show that these two phenomena are related in a model-independent way. Scatterings encountered by a fast parton in a nuclear medium may be described within a diffusion picture. The jet $p_\perp$-broadening is determined by a classical diffusion equation. Energy loss is described in a picture where the fast parton propagation is treated classically whereas the induced gluon radiation is a quantum mechanical phenomenon. Interference effects play a crucial role in the production of gluons which determine the induced radiation spectrum.

The Gyulassy-Wang model [2] used in II to describe multiple scatterings in hot matter is here replaced by the Glauber model [3] of multiple scattering in which parton-nucleus scattering is given in terms of independent scatterings off the nucleons making up the nucleus.

The result which we find for radiative energy loss is similar to that for hot matter, namely that the total energy loss is proportional to the square of the length of the traversed nuclear matter, L. The coefficient of L^2 may be estimated from other, nonperturbative but measurable, quantities such as the small-x gluon distribution of the nucleon or, as discussed by Luo, Qiu and Sterman in [4], the transverse momentum which a jet, or dijet, receives as it passes through a nucleus. In this latter case, we find an interesting general relation between the radiative energy loss per length and the $p_\perp$-broadening (defined as the characteristic width $p_{\perp W}^2$ of the transverse momentum distribution) of a high energy parton passing through a nucleus, $-dE/dz = \frac{1}{8}\alpha_s N_c p_{\perp W}^2$. Numerical estimates suggest possibly large differences for $p_\perp$-broadening and energy loss for partons in hot matter as compared to nuclear matter.

The outline of the paper is as follows. In section 2 the basic equations are established for jet $p_\perp$-broadening and for the induced gluon radiation spectrum. These equations are solved in section 3 to logarithmic accuracy. Parameters appearing in the solutions are related to (other) phenomenological quantities, such as the gluon distribution of the nucleon, in section 4. Numerical estimates are given in section 5. In Appendix A, our basic equation for jet broadening is derived in detail. In Appendix B we relate the elementary parton nucleon scatterings, as they are used for $p_\perp$-broadening and radiative energy loss, to the gluon distribution of the nucleon.

2 The equations

In this section we give the basic equations for jet transverse momentum broadening and induced gluon radiation for a high energy jet traversing cold nuclear matter. The equation for jet broadening is straightforward to write down and represents diffusion, in transverse momentum, due to independent multiple scatterings of the jet as it encounters the various nucleons of the nucleus. The equation for induced energy loss has exactly the same form as found in II. In each case the fundamental assumption is that a parton is multiply scattered off uncorrelated scattering centres in the material. That is, we suppose that the Glauber picture of multiple scattering holds for a high energy parton passing through a nucleus. At extremely high energies the Glauber approximation breaks down due to coherent effects in inelastic particle production [5]. We limit the energies which we consider in order to avoid this complication. It is an important goal for the future to better understand how severe these limitations are in practice.

2.1 The basic elements of the equations

We formulate the multiple scattering in close analogy with the procedure used in [6] within the Gyulassy-Wang model. Thus, we first need to find the cold matter quantity corresponding to the single scattering probability distribution.

In jet $p_\perp$-broadening the high energy parton receives a large transverse momentum due to the many scatterings in passing through a large nucleus. This means that in the individual parton-nucleon collisions all gluon couplings to the high energy parton occur with a small impact parameter separation between the parton in the amplitude and in the complex conjugate amplitude. This, perhaps surprising, result can be seen by referring below to (B.8) . If one inserts a complete set of scattering states between the two F^a_{+i}'s in the right-hand side of that equation, the cross section is given explicitly in terms of a production amplitude times a complex conjugate amplitude. The amplitude F^a_{+i} is evaluated at $\vec{y} = 0$ while in the complex conjugated amplitude $\vec{y}^{\,2} = B^2/\mu^2$ with μ a scale to be defined shortly. With such small impact parameter separations, $B^2 \propto L^{-1}$, the one-gluon approximation becomes accurate. (The argument of the parton-gluon coupling is just the impact parameter separation.)

In general, we allow breakup of the struck nucleon in the elementary parton-nucleon collision. The basic high energy quark-nucleon scattering amplitude is shown in Fig.1 and the corresponding cross section is denoted by $d\sigma/d^2\vec{q}$.

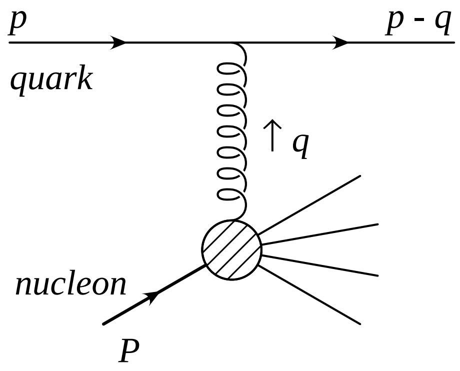

Figure 1: *Basic quark-nucleon scattering amplitude.*

A crucial assumption of the Glauber model we are using to describe multiple scattering is the fact that successive scatterings are independent. That is, the collision process at a given centre does not depend on the collisions which occur before or after. If the particles produced in the elementary collision in Fig.1 have a formation time smaller than the average time between two scatterings (the mean free path λ) then the independent scattering picture is clear. If that formation time is longer one expects corrections to independent scattering [5]. In the following sections and Appendix B we shall relate an elementary scattering to the gluon distribution of the nucleon.

An estimate of the size of the collective effects which undermine the Glauber model is the strength of shadowing corrections for the gluon distribution of a nucleus. Phenomenologically, these corrections are always less than a factor of 2 even for quite small values of x [7,8]. Given this uncertainty, we expect the independent scattering picture to be a reasonable framework for studying jet $p_\perp$-broadening and the induced gluon spectrum for high energy partons.

Because $d\sigma/d^2\vec{q}$ obeys diffractive kinematics, the four-momentum transfer squared, q^2, is equal to $\vec{q}^{\,2}$ the two-dimensional momentum transfer squared. It is then convenient to define

$$\sigma = \int \frac{d\sigma}{d^2\vec{q}}\, d^2\vec{q} \tag{2.1}$$

as the total elementary cross section. The reader may worry about the meaning of σ since the integration over small transverse momenta is not under perturbative control. We may imagine that the integral on the right-hand side of (2.1) has an infrared cutoff. The parton-nucleon cross section cannot be reliably calculated in perturbative QCD. However, we shall show that the cutoff dependence disappears (in the logarithmic approximation) when one calculates quantities insensitive to small momentum transfer scattering such as jet $p_\perp$-broadening and induced gluon radiation. This cutoff independence is due to a cancellation of long-distance effects between the real inelastic production shown in Fig.1 and forward elastic scattering, a virtual contribution. This cancellation is required by the probability conserving formalism which we use to describe multiple scattering. Any divergence in real production must be compensated by a corresponding virtual contribution (the no-scattering term on the right-hand side of (2.7)). This virtual contribution in our formalism shows up in the (cutoff dependent) mean free path for the quark given by

$$\lambda = [\rho\sigma]^{-1}\,, \tag{2.2}$$

with ρ the nuclear matter density.

In order to deal with dimensionless variables, and functions, it is convenient to introduce a (somewhat arbitrary) scale μ^2, with μ representing a *typical* momentum transfer to the quark in a quark-nucleon collision. Then $\vec{Q} = \vec{q}/\mu$ and $\vec{U} = \vec{p}/\mu$ are the dimensionless transverse momentum transfer and transverse momentum, respectively. The probability distribution for the scattered quark is given by

$$V(Q^2) = \frac{1}{\pi\sigma}\ \frac{d\sigma}{dQ^2}\,, \tag{2.3}$$

where we note that $\int d^2\vec{Q}V(Q^2) = 1$. The "potential" V, defined by (2.3), will play exactly the same role in cold matter calculations as did a similar quantity, also called V, for hot matter [6].

As was done earlier for hot matter it is convenient to define the multiple scattering of a high energy parton in terms of the probability distribution for a single scattering, $V(Q^2)$, and the probability distribution for distances Δ between successive scatterings [2,6],

$$P(\Delta)d\Delta = e^{-\Delta/\lambda}\frac{d\Delta}{\lambda}\,. \tag{2.4}$$

2.2 Equation for parton $p_\perp$-broadening

In calculating the transverse momentum given to a high energy parton by multiple scattering with the nucleons of a nucleus it is not necessary, in the leading approximation, to include the transverse momentum given to the parton by induced gluon emission since the emission is formally of higher order in α_s. Thus the transverse momentum given to the jet in passing through a nucleus comes directly from the scattering of the projectile parton off the nucleons in the nucleus.

Suppose a high-energy parton is produced in the nucleus in a hard collision at $z=0$ with an initial transverse momentum distribution $f_0(U^2)$ with

$$\int d^2\vec{U} f_0(U^2) = 1\,. \tag{2.5}$$

(If the parton initially has no transverse momentum, then $f_0(U^2) = \frac{1}{\pi}\delta(U^2)$.) In terms of the scaled variable

$$t = \frac{z}{\lambda_R}\,, \quad \lambda_R = \lambda\frac{C_F}{C_R} \tag{2.6}$$

for a parton of colour representation R, the transverse momentum distribution at $z = t\lambda_R$ is

$$f(U^2,t) = f_0(U^2)e^{-t} + \int_0^t dt' e^{-(t-t')}\int d^2\vec{Q} f\left((\vec{U}-\vec{Q})^2, t'\right) V(Q^2)\,. \tag{2.7}$$

The distribution $f(U^2,t)$ satisfies the normalization condition (2.5) for all t. We derive (2.7) using a multiple scattering formalism in Appendix A. The first term on the right-hand side of (2.7) corresponds to no interaction with the nuclear medium where the e^{-t} represents the probability that no interactions have occurred in a distance $z = \lambda_R t$. The second term on the right-hand side of (2.7) corresponds to a final scattering at t' before the parton reaches t with transverse momentum $\vec{U}$. One can recast (2.7) as a differential equation by taking a "time" derivative on both sides of that equation. One finds

$$\frac{\partial f(U^2,t)}{\partial t} = -f(U^2,t) + \int d^2\vec{Q} f((\vec{U}-\vec{Q})^2,t)V(Q^2)\,. \tag{2.8}$$

A different, but equivalent, form of (2.8) for f, which makes its interpretation as a kinetic master equation more transparent, is

$$\frac{\partial f(U^2,t)}{\partial t} = -\int f(U^2,t)V((\vec{U}-\vec{U}')^2)d^2\vec{U}' + \int f(U'^2,t)V((\vec{U}'-\vec{U})^2)d^2\vec{U}'\,. \tag{2.9}$$

When partons pass through the distance dt the first term on the right-hand side (loss term) accounts for partons which are scattered *out* of the direction $\vec{U}$, $\vec{U} \to \vec{U}'$, weighted by the (normalized) scattering cross section V. The second term (gain term) counts those partons which are scattered *into* the direction $\vec{U}$ from all other directions $\vec{U}', \vec{U}' \to \vec{U}$. One can diagonalize (2.8) by defining

$$\tilde{f}(B^2,t) = \int d^2\vec{U} e^{-i\vec{B}\cdot\vec{U}} f(U^2,t)\,, \tag{2.10a}$$

and

$$\tilde{V}(B^2) = \int d^2\vec{Q} e^{-i\vec{B}\cdot\vec{Q}} V(Q^2)\,, \tag{2.10b}$$

in which case

$$\frac{\partial \tilde{f}(B^2,t)}{\partial t} = -\frac{1}{4} B^2 \tilde{v}(B^2) \tilde{f}(B^2,t)\,, \tag{2.11}$$

where

$$\tilde{v}(B^2) = \frac{4}{B^2}(1 - \tilde{V}(B^2))\,. \tag{2.12}$$

Note that $\tilde{V}(0) = 1$ because of our normalization of $V(Q^2)$ as a probability distribution.

Instead of searching for an exact solution one often approximates (2.8), in case of soft "potentials", by expanding $f((\vec{U} - \vec{Q})^2, t)$ inside the integral through second order in $\vec{Q}$. This leads to the diffusion equation [9]

$$\frac{\partial f(U^2,t)}{\partial t} = \frac{1}{4}\, \hat{q}_U\, \nabla^2_U\, f(U^2,t)\,, \tag{2.13}$$

where the "transport coefficient" $\hat{q}_U$ is defined by

$$\hat{q}_U = \int d^2\vec{Q}\, Q^2 V(Q^2)\,. \tag{2.14}$$

The transport coefficient with respect to the longitudinal variable z (see (2.6)) and the transverse momentum $\vec{q}$ is then given by

$$\hat{q} = \frac{\mu^2}{\lambda_R} \hat{q}_U\,. \tag{2.15}$$

In cases where the integral in (2.14) exists one finds that

$$\hat{q}_U = \tilde{v}(0)\,. \tag{2.16}$$

For example, if V were gaussian, $V(Q^2) = \frac{1}{\pi} e^{-Q^2}$, (2.7) and (2.14) would be completely equivalent to standard treatments of parton transverse momentum broadening in nuclei [10] with μ^2 the average transverse momentum in an elementary collision.

However, in QCD (2.14) diverges logarithmically since $V(Q^2) \propto 1/Q^4$ at large Q^2. Correspondingly, $\tilde{v}(B^2)$ has no finite limit for $B^2 \to 0$. Nevertheless, $\tilde{v}(B^2)$ may be related to the *first moment* of $V(Q^2)$ in a logarithmic approximation. To see this we use (2.10b) and (2.12) to write

$$\tilde{v}(B^2) = \frac{4}{B^2} \int d^2\vec{Q} \left(1 - e^{-i\vec{B}\cdot\vec{Q}}\right) V(Q^2) \,. \tag{2.17}$$

Doing the angular integral gives

$$\tilde{v}(B^2) = \frac{4\pi}{B^2} \int_0^\infty dQ^2 (1 - J_0(BQ))\, V(Q^2) \,. \tag{2.18}$$

This integral is convergent in the ultraviolet. For $BQ \ll 1$, we have $1 - J_0(BQ) \approx \frac{1}{4} B^2 Q^2$ so that there is a logarithmic region of integration in (2.18) extending up to $Q^2 \approx 1/B^2$. Thus, in the logarithmic approximation, for small B^2

$$\tilde{v}(B^2) \approx \pi \int_0^{1/B^2} dQ^2 V(Q^2) Q^2 \,. \tag{2.19}$$

2.3 Equation for the induced energy spectrum

Because of the close correspondence between the multiple scattering in hot matter, as discussed in II, and the present circumstance we need not rederive the equation for the induced radiation spectrum. We may simply take over equations (4.16), (4.24) and (5.1) of that paper which read

$$\frac{\partial}{\partial \tau} \vec{f}(\vec{U}, \tau) = -(1 - i\tilde{\kappa} U^2) \vec{f}(\vec{U}, \tau) + \int d^2\vec{Q} \vec{f}(\vec{U} - \vec{Q}, \tau) V(Q^2) \,, \tag{2.20}$$

$$\frac{\omega dI}{d\omega dz} = \frac{3\alpha_s C_R}{2\pi^2 \lambda_g} 2\mathrm{Re} \left\{ i \int_0^{\tau_0} d\tau \left(1 - \frac{\tau}{\tau_0}\right) \int \frac{d^2\vec{B}}{2\pi} \, \frac{1 - \tilde{V}(B^2)}{B^2} \vec{B} \cdot \widetilde{\vec{f}}\,(\vec{B}, \tau) \bigg|_{\tilde{\kappa}}^{\tilde{\kappa}=0} \right\} , \tag{2.21}$$

and

$$i \frac{\partial}{\partial \tau} \widetilde{\vec{f}}\,(\vec{B}, \tau) = \left[\tilde{\kappa} \nabla_B^2 - \frac{i}{4} B^2 \tilde{v}(B^2) \right] \widetilde{\vec{f}}\,(\vec{B}, \tau) \,; \quad \widetilde{\vec{f}}\,(\vec{B}, 0) = -\frac{i\pi}{2} \vec{B}\, \tilde{v}(B^2) \,, \tag{2.22}$$

respectively. In (2.20)–(2.22)

$$\tau = z/\tilde{\lambda}, \quad \tau_0 = L/\tilde{\lambda} \,; \qquad \tilde{\kappa} = \frac{2C_F}{N_c} \kappa \,, \quad \tilde{\lambda} = \frac{2C_F}{N_c} \lambda = 2\lambda_g \,. \tag{2.23}$$

Equations (2.21) and (2.22) give the induced gluon radiative energy spectrum for high energy partons having colour representation R. The importance of the parameter $\kappa = \lambda\mu^2/2\omega$ has been discussed in [6].

3 Energy loss and jet broadening

In this section, we give the solutions to (2.20) – (2.22). We then compare the resulting formulas for dE/dz and $p_{\perp W}^2$ and note a simple relationship between these two quantities.

3.1 Jet transverse momentum broadening

It is straightforward to solve (2.11) as

$$\tilde{f}(B^2,t) = \tilde{f}(B^2,0)\exp\left\{-\frac{1}{4}B^2\tilde{v}(B^2)t\right\} . \tag{3.1}$$

In this paper we only consider media whose length L obeys $L/\lambda \gg 1$. Thus we need only consider t large in which case only small values of B^2 give a result which is not exponentially small. If $\tilde{v}(0)$ were finite we could choose μ^2 to be the mean momentum transfer squared per collision so that $\tilde{v}(0) \equiv 1$. In this case the transverse momentum distribution, $f(U^2,t)$, would be a gaussian in U with *mean transverse momentum squared* $\langle p_\perp^2\rangle = \mu^2 t$. However, we expect a logarithmic dependence of $\tilde{v}(B^2)$ for small B^2 because of the approximate scale invariance of QCD. We suppose that $B^2\frac{\partial}{\partial B^2}\ln\tilde{v}(B^2) \ll 1$ for small B^2 in which case it is clear from (3.1) that the values of B^2 for which $\tilde{f}(B^2,t)$ is not small are

$$B^2 \leq \frac{4}{t\tilde{v}(1/t)} . \tag{3.2}$$

Using $\tilde{f}(0,0) = 1$, coming from the normalization of $f(U^2,t)$ as a probability distribution in transverse momenta, we get

$$\tilde{f}(B^2,t) \approx \exp\left\{-\frac{1}{4}B^2 t\tilde{v}(1/t)\right\} . \tag{3.3}$$

Transforming back to $\vec{U}$-space gives

$$f^{(G)}(U^2,t) \approx \frac{1}{\pi\, t\tilde{v}(1/t)}\exp\left\{-\frac{U^2}{t\,\tilde{v}(1/t)}\right\} . \tag{3.4}$$

In neglecting the $B^2=0$ singularity of $\tilde{v}$ in (3.4) we have lost the large-U^2 tail of $f(U^2,t)$ which is

$$f(U^2,t) \;\simeq\; V(U^2)\,t \;\sim\; \frac{t}{U^4} .$$

The gaussian approximation (3.4) is valid up to U^2-values where the exponential is as small as $1/t \ll 1$. This means that to evaluate the *characteristic width* of the distribution one can use (3.4). Defining $p_{\perp W}^2$ as the value at which the distribution falls to $1/e$ of its peak value, we have

$$p_{\perp W}^2 = \mu^2\int d^2\vec{U}\,\vec{U}^2 f^{(G)}(U^2, L/\lambda_R) , \tag{3.5}$$

which results in

$$p_{\perp W}^2 = \frac{\mu^2}{\lambda_R}\tilde{v}(\lambda_R/L)\,L . \tag{3.6}$$

The linear growth of typical transverse momenta squared with L is expected and has been used for some time to discuss $p_\perp$-broadening of high energy partons in nuclei [10,13,4].

The individual parts on the right-hand side of (3.6) which multiply L are not uniquely defined. Nevertheless, we note that the quantity $\frac{\mu^2}{\lambda_R}\tilde{v}(\lambda_R/L)$ is independent of the scale μ^2 and of any infrared cutoff we may have used to define σ. Indeed $\tilde{v}$ scales as $1/\mu^2$ so that $\mu^2\tilde{v}$ is independent of μ^2. More importantly, the factors of σ cancel in $\tilde{v}/\lambda$, and the dangerous small-$\vec{Q}$ region of integration is suppressed for small $\vec{B}$ in taking the difference indicated in (2.17) leaving the cutoff only as a scale for logarithmic dependence. Indeed, from (2.19)

$$\frac{\tilde{v}(B^2)}{\lambda} \approx \rho \int^{1/B^2} dQ^2 \frac{d\sigma}{dQ^2} Q^2 \,, \tag{3.7}$$

which explicitly shows the suppression of long-distance contributions.

It is worth mentioning that the basic evolution equations (2.11) and (2.22), and thus their solutions, are also cutoff independent for large z if we substitute $t = z/\lambda_R$ everywhere in the equations and view $\tilde{f}$ as a function of the physical variable z rather than t. We prefer to continue working with the variable t, however, as the correspondence with the procedure used for hot matter is more transparent in this variable.

3.2 Energy loss

Since (2.21) and (2.22) are equations identical to those encountered in II we may immediately write down the solution to (2.22) as

$$\widetilde{f}\,(\vec{B},\tau) \;=\frac{2\pi i\vec{B}}{B^2}\,\frac{\partial}{\partial\tau}\exp\left\{-\frac{i}{2}m\;\omega_0 B^2\tan\omega_0\tau\right\}\,, \tag{3.8}$$

and we may write (2.21) as

$$\frac{\omega dI}{d\omega dz} = \frac{6\alpha_s C_R}{\pi L}\ln\left|\frac{\sin\omega_0\tau_0}{\omega_0\tau_0}\right|\,, \tag{3.9}$$

where

$$m = -\frac{1}{2\tilde{\kappa}}\,, \quad \omega_0 = \sqrt{i\tilde{\kappa}\tilde{v}}\,. \tag{3.10}$$

The argument of $\tilde{v}$ is B^2 in (3.8) and $\sqrt{\tilde{\kappa}/\ln(1/\tilde{\kappa})}$ in (3.9). Exactly as in II one may give an analytic expression for dE/dz, the energy loss per unit length, as

$$-\frac{dE}{dz} = \frac{\alpha_s C_R}{8}\,\frac{\mu^2}{\lambda_g}\,\tilde{v}(\tilde{\lambda}/L)\;L = \frac{\alpha_s N_c}{8}\,\frac{\mu^2}{\lambda_R}\,\tilde{v}(\tilde{\lambda}/L)L\,, \tag{3.11}$$

and the total energy loss as

$$-\Delta E = \frac{\alpha_s C_R}{8}\,\frac{\mu^2}{\lambda_g}\tilde{v}(\tilde{\lambda}/L)\;L^2\,. \tag{3.12}$$

In the above, $\alpha_s = \alpha_s(k^2)$ is evaluated at a large momentum scale k^2 proportional to L. Indeed, as shown in II, the effective value of the impact parameter B^2 is of order $\tilde{\lambda}/L$.

3.3 Relationship between energy loss and $p_\perp$-broadening

Comparing (3.6) and (3.11) we note that

$$-\frac{dE}{dz} = \frac{\alpha_s N_c}{8}\, p_{\perp W}^2 \tag{3.13}$$

relating the energy loss per unit length with the typical transverse momentum squared that a jet receives in passing through a length L of nuclear matter. It is interesting that the relationship between these two quantities is completely independent of the dynamics of the multiple scattering, at least in the framework of our approach to these problems. Thus (3.13) holds equally well in finite length hot matter as well as cold matter. Perhaps even more surprising the coefficient relating $p_{\perp W}^2$ to dE/dz is independent of the nature of the high energy parton passing through the hot or cold matter. Equation (3.13) makes more precise the bound $-dE/dz \leq \frac{1}{2}p_{\perp W}^2$ suggested sometime ago by Brodsky and Hoyer [11] on the basis of the uncertainty relation.

4 Determining $\frac{\mu^2}{\lambda}\tilde{v}$, estimates

In this section, we shall discuss how $\frac{\mu^2}{\lambda}\tilde{v}$ can be determined phenomenologically and we shall give a crude estimate of this quantity in terms of the nucleon gluon distribution.

4.1 Determining $\frac{\mu^2}{\lambda}\tilde{v}$ phenomenologically

The broadening of the transverse momentum spectrum of high energy partons in nuclei has been widely discussed in the literature. In μ-pair and J/ψ-production in high energy proton-nucleus collisions it is the incoming high energy quark (or gluon) which multiply scatters in the nucleus and in so doing broadens the μ-pair and J/ψ transverse momentum spectra as compared to the corresponding production in nucleon-nucleon collisions. Transverse momentum broadening has also been studied for outgoing jets produced in high energy proton-nucleus and photon-nucleus collisions.

The phenomenological situation is somewhat confused at the moment. It would appear that *outgoing* partons receive much more transverse momentum due to multiple scattering in a nucleus [12] than do *incoming* partons [13]. Theoretically, this difference is not understood [4,14]. It is not the purpose of this section to give a comprehensive study of the phenomenology of multiple scattering of high energy partons in nuclei but rather to show how $\frac{\mu^2}{\lambda}\tilde{v}$ can be determined, so we limit ourselves here to relating our approach to that of Luo, Qiu and Sterman [4] who studied the $p_\perp$-broadening of dijets produced in photon-nucleus collisions at Fermilab.

Luo, Qiu and Sterman consider a single rescattering in the nucleus of dijets produced in high energy photon-nucleus collisions. They find that the transverse momentum squared given to the dijet by summing over all single rescatterings in the nucleus is (see equation (21) in [4]):

$$\left\langle k_T^2(R)\right\rangle = \frac{4}{3}\pi^2\alpha_s(Q^2)\, A^{1/3}\, \lambda_{LQS}^2\, . \tag{4.1}$$

Here Q is the relative transverse momentum of the two jets making up the dijet, A is the atomic number of the nucleus and λ^2_{LQS} is a dimensional parameter which characterizes the momentum transfer squared in a single collision. An expression for λ^2_{LQS} is given in [4] in terms of a new QCD nuclear matrix element, but the actual determination of λ^2_{LQS} is made by comparing (4.1) to experiment. A value $\lambda^2_{LQS} \approx 0.05 - 0.1\mathrm{GeV}^2$ is found.

We may compare (3.6) to (4.1) by noting the following:

(i) In the Luo, Qiu and Sterman calculation, the dijet consists of a high transverse momentum quark and a gluon with the quark and gluon transverse momenta nearly balancing. It is the nuclear contribution to the imbalance, namely, the transverse momentum of the whole dijet, which is expressed in (4.1). Since the quark and gluon have very high transverse momentum they form a (colour triplet) system which is compact in transverse coordinate space. Thus, in a relatively soft rescattering with a nucleon in the nucleus this system should act like a single quark and this indeed is the case in the calculation done in [4]. We may then expect that our calculation of jet broadening of a single parton should apply to the dijet system.

(ii) Luo, Qiu and Sterman only consider a single rescattering while we have summed arbitrary numbers of rescatterings in arriving at (3.6) from (2.7). However, in a multiple scattering, where the scatterers are uncorrelated, the resulting transverse momentum squared is just the sum of the transverse momenta squared of the individual scatterings. In [4] only a single scattering is explicitly done but (4.1) results from summing over all possible single scatterings. Thus (3.6) and (4.1) should be directly comparable. However, here we do sidestep the difficult issue as to exactly which $\langle k_T^2 \rangle$-average has been done experimentally in evaluating λ^2_{LQS} and how close that average is to our $p^2_{\perp W}$.

The last piece of information we need is to relate L to the nuclear radius R. Since the dijet is produced in a hard collision it is produced uniformly in the volume of the nucleus. After being produced the average length of material that the dijet passes through is $L = \frac{3}{4}R$. With this identification of L we may equate (3.6) to (4.1) and obtain

$$\frac{\mu^2}{\lambda}\tilde{v}(\lambda/L) = \frac{16}{9}\pi^2\alpha_s(Q^2)\,\frac{A^{1/3}}{R}\,\lambda^2_{LQS}\,. \tag{4.2}$$

We shall give a numerical estimate of $\frac{\mu^2}{\lambda}\tilde{v}$ from (4.2) a little later on.

4.2 Relating $\frac{\mu^2}{\lambda}\tilde{v}$ to the gluon distribution

One can get a crude relation between $\frac{\mu^2}{\lambda}\tilde{v}$ and the nucleon gluon distribution [15] and that is our purpose in this section. Indeed, Luo, Qiu and Sterman relate their parameter λ^2_{LQS} to the average colour field strength squared that a parton *sees* as it passes through a nucleon in the nucleus. In order to relate $\frac{\mu^2}{\lambda}\tilde{v}$ to the gluon distribution we use (2.19) which gives

$$\tilde{v}(B^2) = \frac{1}{\sigma_R}\int_0^{1/B^2} dQ^2\, Q^2 \frac{d\sigma_R}{dQ^2} = \frac{1}{\mu^2\sigma_R}\int_0^{\mu^2/B^2} dq^2\, q^2 \frac{d\sigma_R}{dq^2}\,, \tag{4.3}$$

where now $d\sigma_R/dQ^2$ refers to the scattering of a parton of colour representation R with a nucleon. But in the one-gluon exchange approximation

$$\int_0^{\mu^2/B^2} dq^2 q^2 \frac{d\sigma_R}{dq^2} = \frac{4\pi^2\alpha_s C_R}{N_c^2-1}\, xG(x,\mu^2/B^2)\,. \tag{4.4}$$

(We postpone until later on the prickly question of what value of x we intend to use in (4.4). We do note however that in the region that (4.4) will be used, a region of small but not too small x, the quantity $xG(x,\mu^2/B^2)$ should have little dependence on x.) The simplest way to see that (4.4) should be true is to consider the case where the target and jet are both quarks. In this case

$$\frac{d\sigma}{dq^2} = \frac{2\pi C_F\alpha_s^2}{N_c\,(q^2)^2} \tag{4.5}$$

to lowest order in α_s while

$$xG_q(x,\mu^2/B^2) = \frac{\alpha_s C_F}{\pi}\ \ln\frac{\mu^2}{B^2}\,. \tag{4.6}$$

Using (4.5) in the left-hand side of (4.4) gives the right-hand side of that equation after using (4.6), thus fixing the $2\pi^2\alpha_s/N_c$ factor on the right-hand side of (4.4), for a quark jet, and a factor $4\pi^2\alpha_s C_R/(N_c^2-1)$ for a jet of colour representation R. (A more complete, and perhaps more satisfactory, derivation of (4.4) is given in Appendix B.) The α_s in (4.4) should be evaluated at the scale μ^2/B^2 as B/μ is the short distance over which the scattering occurs. Using (4.4) in (4.3) along with $\lambda_R = [\rho\sigma_R]^{-1}$ gives

$$\frac{\mu^2}{\lambda_R}\tilde v(B^2) = \frac{4\pi^2\alpha_s C_R}{N_c^2-1}\,\rho\, xG(x,\mu^2/B^2)\,. \tag{4.7}$$

5 Some numerical estimates

The basic results for $p_\perp-$ broadening and energy loss are given in (3.6) and in (3.11), (3.12) respectively. The parameter controlling these quantities is $\frac{\mu^2}{\lambda}\ \tilde v$, for which we have found the expressions (4.2) and (4.7). Our task here is to make rough estimates of this quantity.

(i) In II, for hot matter having $T = 250\,\mathrm{MeV}$, we took $\mu^2/\lambda = 1\,\mathrm{GeV/\,fm^2}$ and, for $L \simeq 10$ fm, $\tilde v \approx 2.5$ giving

$$\frac{\mu^2\tilde v}{\lambda} \approx 0.5\,\mathrm{GeV^2/\,fm}\,.$$

We get

$$p_{\perp W}^2 \simeq 5\,\mathrm{GeV}^2\,\frac{L}{10\ \mathrm{fm}}$$

from (3.6) and, taking $\alpha_s = 1/3$,

$$-\Delta E \simeq 30\,\mathrm{GeV}\,\left(\frac{L}{10\ \mathrm{fm}}\right)^2$$

for a quark jet. These are rather big numbers and one should not take their exact values too seriously. However, they do suggest that hot matter may be rather effective in stimulating radiative energy loss and in broadening the $p_\perp$-distribution of high energy partons.

(ii) For cold matter from (4.7), with $C_R = C_F = 4/3$, using $\rho = 0.15$ fm^{-3} one finds

$$\frac{\mu^2 \tilde{v}}{\lambda} \approx \frac{1}{25} \cdot \alpha_s \cdot [xG(x)]\, \mathrm{GeV}^2/\,\mathrm{fm}\,.$$

Taking $\alpha_s = 1/2$ and $xG = 1$ results in much smaller values

$$p_{\perp W}^2 \simeq 0.2\, \mathrm{GeV}^2\, \frac{L}{10 \text{ fm}}$$

and

$$-\Delta E \approx 2\, \mathrm{GeV} \left(\frac{L}{10 \text{ fm}}\right)^2 .$$

These numbers may be reasonable. In particular, $dp_{\perp W}^2/dz = \frac{\mu^2 \tilde{v}}{\lambda} \approx \frac{1}{50}\, \mathrm{GeV}^2/\,\mathrm{fm}$ is in agreement with the $p_\perp$-broadening of the μ-pair spectrum[13] so $-dE/dz \approx 0.2\, \mathrm{GeV}/\,\mathrm{fm}(L/10 \text{ fm})$ may be a sensible, if small, estimate of radiative energy loss for high energy quarks.

(iii) From (4.2), with $\lambda_{LQS}^2 = 0.05\, \mathrm{GeV}^2$, one finds $\frac{\mu^2 \tilde{v}}{\lambda} \approx 0.8 \alpha_s(Q^2)\, \mathrm{GeV}^2/\,\mathrm{fm}$. Taking $\alpha_s(Q^2) = 1/3$ gives

$$\frac{\mu^2 \tilde{v}}{\lambda} \simeq 0.3\, \mathrm{GeV}^2/\,\mathrm{fm}$$

and leads to

$$p_{\perp W}^2 \simeq 3\, \mathrm{GeV}^2\, \frac{L}{10\, \mathrm{fm}}$$

and

$$-\Delta E \simeq 15\, \mathrm{GeV} \left(\frac{L}{10 \text{ fm}}\right)^2 .$$

These numbers seem too large. (They would correspond to $xG \approx 20 - 25$ if (4.7) is used.) The problem here, as alluded to earlier, is that present experiments find a large $p_\perp$-broadening and energy loss for outgoing partons giving $\lambda_{LQS}^2 \approx 0.05 - 0.1\, \mathrm{GeV}^2$ while much smaller numbers are found for the broadening of the μ-pair spectrum.

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A

Appendix A. Basic equation for jet broadening

Here we give a brief derivation of (2.7). The probability density, $p_n(U^2,t)$, for the incoming parton to have a transverse momentum squared U^2 at time t after n collisions at times $t_1, t_2, \dots t_n$ and involving fixed transverse momentum transfers $\vec{Q}_1, \vec{Q}_2, \dots \vec{Q}_n$ is [6]

$$p_n(U^2,t) = f_0\left((\vec{U} - \sum_{i=1}^{n} \vec{Q}_i)^2\right) \prod_{\ell=1}^{n} \left[V(Q_\ell^2) e^{-(t_\ell - t_{\ell-1})}\right] e^{-(t-t_n)} . \tag{A.1}$$

Thus,

$$p_n(U^2,t) = e^{-t} f_0\left((\vec{U} - \sum_{i=1}^{n} \vec{Q}_i)^2\right) \prod_{\ell=1}^{n} V(Q_\ell^2) , \tag{A.2}$$

where $t_0 = 0$.

The probability density, $P_n(U^2,t)$, to have momentum $\vec{U}$ at time t is obtained by integrating over the $\vec{Q}_i$ and t_i leading to

$$P_n(U^2,t) = \int \prod_{\ell=1}^{n} d^2\vec{Q}_\ell \int_0^t dt_n \int_0^{t_n} dt_{n-1} \cdots \int_0^{t_2} dt_1\, p_n(U^2,t) . \tag{A.3}$$

The momentum distribution f is given by the sum over all possible number of interactions,

$$f(U^2,t) = \sum_{n=0}^{\infty} P_n(U^2,t) = P_0(U^2,t) + \sum_{n=1}^{\infty} P_n(U^2,t) , \tag{A.4}$$

where the probability to have no interactions with the medium between times 0 and t is

$$P_0(U^2,t) = f_0(U^2) e^{-t} . \tag{A.5}$$

It is easy to relate P_n to P_{n-1}. Inserting (A.2) into (A.3) gives

$$\begin{aligned} P_n(U^2,t) &= \int d^2\vec{Q}_n V(Q_n^2) \int_0^t dt_n e^{-(t-t_n)} \prod_{\ell=1}^{n-1} d^2\vec{Q}_\ell V(\vec{Q}_\ell^2) \int_0^{t_n} dt_{n-1} \cdots \int_0^{t_2} dt_1 \\ &\cdot \quad e^{-t_n} f_0\left((\vec{U} - \vec{Q}_n - \sum_{\ell=1}^{n-1} \vec{Q}_\ell)^2\right) . \end{aligned} \tag{A.6}$$

Relabeling $\vec{Q}_n$ as $\vec{Q}$ and t_n as t',

$$P_n(U^2,t) = \int_0^t dt' e^{-(t-t')} \int d^2\vec{Q} V(Q^2) P_{n-1}((\vec{U} - \vec{Q})^2, t') , \tag{A.7}$$

which has an obvious probabilistic interpretation. Finally, using (A.7) in (A.4) leads to (2.7).

B

Appendix B. Relating elementary scatterings to the gluon distribution

In this appendix we derive (4.4). Much of our discussion here follows [4]. We assume a one-gluon exchange approximation at the outset (see Fig.1), the justification of that assumption being the small coupling we find for the coupling of the exchanged gluon to the incident high energy parton. In order to have expressions for the gluon distribution which conform to conventional notation we suppose that the incident parton is moving in the negative z-direction so that $\vec{p}=0$ and, using light-cone variables, $p_- \gg p_+$. (In order to fix (approximately) the value of x in (4.4) we must consider the parton propagating through the medium to be virtual though, in our covariant gauge discussion which follows, we may continue to assume that the exchanged gluon couples in an eikonal manner.) We suppose the nucleon, labeled by P, is at rest. Then

$$\frac{d\sigma}{d^2\vec{q}} \propto g^2 \int dq_+ dq_- d^4y (P|A_+^a(y)A_+^a(0)|P)\delta((p+q)^2)e^{-iqy}\,, \tag{B.1}$$

with A_μ^a the gluon field with colour index a. The process is illustrated in Fig.2 where 0 and y are the coordinates where the gluon attaches to the high energy parton in the production amplitude and complex conjugate amplitude, respectively.

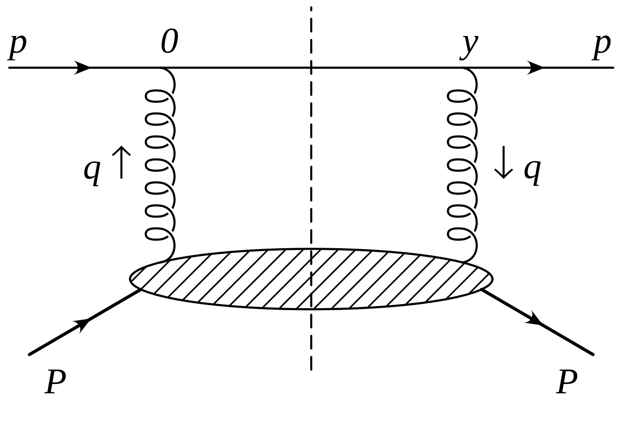

Figure 2: *Diagrammatic representation of the cross section (B.1).*

With $(q+p)^2 \approx q^2+p^2+2p_-q_+$ the q_+-integration is easily done using the $\delta-$ function in (B.1):

$$\frac{d\sigma}{d^2\vec{q}} \propto g^2 \int dq_- d^4y (P|A_+^a(y)A_+^a(0)|P)e^{-iq_+y_- - iq_-y_+ + i\vec{q}\cdot\vec{y}}\,, \tag{B.2}$$

with $q_+ \approx -(p^2+q^2)/2p_- \approx -p^2/2p_-$ in (B.2). (We shall see later that $q^2 \lesssim p^2$.) The q_--integration gives a δ-function in y_+ (because of our large-p_- limit) so we find

$$\frac{d\sigma}{d^2\vec{q}} \propto g^2 \int d^2\vec{y} dy_- (P|A_+^a(y_+=0,y_-,\vec{y})A_+^a(0)|P)e^{-iq_+y_- + i\vec{q}\cdot\vec{y}}\,. \tag{B.3}$$

Now multiply both sides of (B.3) by $\vec{q}^2$. The $\vec{q}^2$ on the right-hand side of (B.3) can be written in terms of derivatives of the A's giving

$$\vec{q}^2 \frac{d\sigma}{d^2\vec{q}} \propto g^2 \int d^2\vec{y}\, dy_- (P|\, \frac{\partial}{\partial y_i} A_+^a(y_+ = 0, y_-, \vec{y}) \frac{\partial}{\partial y_i} A_+^a(0)\,|P)\; e^{-iq_+ y_- + i\vec{q}\cdot\vec{y}} . \tag{B.4}$$

At lowest order in g

$$F_{i+}^a(y) = \frac{\partial}{\partial y_i} A_+^a(y) - \frac{\partial}{\partial y_-} A_i^a(y) . \tag{B.5}$$

The second term on the right-hand side of (B.5) is small when used in (B.4) because the y_--derivative can be integrated by parts to give a q_+ while the $+$ index on A_+ in the first term naturally yields a factor of P_+ when used in (B.4). (We shall shortly identify $x \equiv q_+/P_+ \approx -p^2/2p_- P_+ = -p^2/s \ll 1$ with the Bjorken x-variable of the gluon distribution.) Thus,

$$\vec{q}^2 \frac{d\sigma}{d^2\vec{q}} \propto g^2 \int d^2\vec{y}\, dy_- (P|\, F_{+i}^a(y_+ = 0, y_-, \vec{y}) F_{+i}^a(0)\,|P) e^{-ixP_+ y_- + i\vec{q}\cdot\vec{y}} . \tag{B.6}$$

Integrating (B.6) over $\vec{q}^2$ between 0 and μ^2/B^2 gives

$$\int_0^{\mu^2/B^2} d^2\vec{q}\,\vec{q}^2 \frac{d\sigma}{d^2\vec{q}} \propto g^2 \int_0^\infty d\rho J_1(\rho) \int dy_- e^{-ixP_+ y_-} (P|F_{+i}^a(y_+ = 0, y_-, \vec{y}) \cdot F_{+i}^a(0)|P)\Big|_{\vec{y}^2 = \rho^2 \frac{B^2}{\mu^2}} , \tag{B.7}$$

where we have used $zJ_0(z) = d/dz(zJ_1(z))$. Values of ρ on the order of 1 dominate the integral in (B.7). Therefore in the leading logarithmic approximation we may write

$$\int_0^{\mu^2/B^2} d\vec{q}^{\,2} \vec{q}^{\,2} \frac{d\sigma}{d^2\vec{q}} \propto g^2 \int dy_- e^{-ixP_+ y_-} (P|F_{+i}^a(y_+ = 0, y_-, \vec{y}) F_{+i}^a(0)|P)\Big|_{\vec{y}^2 = B^2/\mu^2} . \tag{B.8}$$

The integral on the right-hand side of (B.8) is proportional to a standard expression for the gluon distribution $xG(x, \mu^2/B^2)$ at a momentum scale μ^2/B^2 [16]. The g^2 in (B.8) (the α_s in (4.4)) should also be evaluated at a scale μ^2/B^2 since this is the limit on the $\vec{q}^2$ integration set by the *hard* scattering. Once we know that the left-hand side of (B.8) is proportional to xG, the constant of proportionality is easily set using (4.5) and (4.6).

Finally, we come to the value of x to be used. To determine the approximate value of x we must decide what value of p^2 should be used for the incident parton. We suppose that the incident parton (jet) is produced in the nucleus in a hard collision with $\vec{p} = 0, p_-$ fixed and p_+ varying with the distance from the production point as $p_+ \simeq 1/z$. (We recall that due to the uncertainty principle one cannot specify p_+ more accurately in a distance z.) Then for our elementary scattering

$$\frac{2p_-}{L} \lesssim |p^2| \lesssim \frac{2p_-}{\lambda_R} , \tag{B.9}$$

where L is the length of the nuclear material and λ_R the mean free path. Since

$$\vec{q}^{\,2} \lesssim \frac{\mu^2}{B^2} \sim \frac{L}{4\lambda_R} \tilde{v}(\lambda_R/L)\mu^2 \tag{B.10}$$

(see(3.2)),

$$\frac{\vec{q}^{\,2}}{|p^2|} \lesssim \frac{\mu^2 L^2 \tilde{v}}{8\lambda_R p_-}, \tag{B.11}$$

a quantity which we assume not to be large. (Note that for $L = L_{cr}$, with L_{cr} defined in [1,6], one has $\mu^2 L^2/\lambda_R p_- \approx 1$.) Thus, we are justified in dropping q^2 compared to p^2 in determining x from $(q+p)^2 = 0$.

From (B.9) we come to the conclusion that x should be chosen in the range

$$\frac{1}{ML} \leq \; x \; \leq \frac{1}{M\lambda_R}\,, \tag{B.12}$$

with M the nucleon mass.

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