Energy Loss and v_2

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Qualitative understanding $p_\perp$ dependence and b dependence of v_2 at RHIC (large $\sqrt{s}$)

- Jet quenching and azimuthal anisotropy of large $p_\perp$ spectra in non-central heavy-ion collisions.

- Reaction operator approach to non-abelian energy loss, Submitted to NPB.

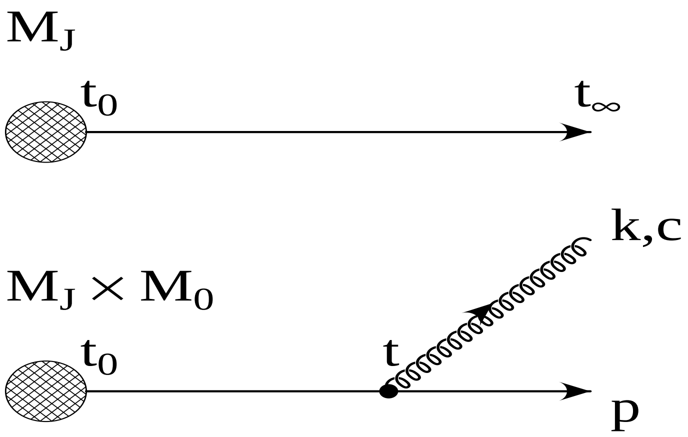
- Non-abelian energy loss at finite opacity, Accepted in PRL.

Hard Radiation and Self Quenching

The finite time t_0 creation of hard probes in high Q^2 processes can be modeled by $J(P)$

$$\gamma^* \to q\,\bar{q}\,, \qquad \gamma^* \to q\,\bar{q}\,g$$

M_J and $M_J \otimes M_0$ amplitudes:



The Gluon Distribution – Eikonal Approximation

$$\mathcal{M}_J = J(P)e^{iPx_0}\,,$$

$$\mathcal{M}_0 = -2ig_s\frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k_\perp^2}e^{it_0\frac{k_\perp^2}{2xE_0}}c\,,$$

$$x\frac{d^3N_g^{(0)}}{dx\,d^2\vec{k}} \approx \frac{C_R\alpha_s}{\pi^2}\frac{1}{k_\perp^2} \equiv R_g^{(0)}(\vec{k})\,.$$

Recall the $\gamma^* \to q\bar{q}g$ kinematics and correct for the quark Splitting Function

$$f_{Sp}(0) \equiv 1 \;\Rightarrow\; f_{Sp}(x) = 1 - x + \frac{x^2}{2}$$

"Self Quenching" effect even with NO scatterings

$$\begin{aligned}
\Delta E_0 &= \frac{2C_R\alpha_s}{\pi} \int_0^1 \frac{dx}{x} \left(1 - x + \frac{x^2}{2}\right) \int_{k^2_{\perp\,\mathrm{min}}}^{k^2_{\perp\,\mathrm{max}}} \frac{dk_\perp^2}{k_\perp^2}\, xE_0 \\
&\simeq \frac{4C_R\alpha_s}{3\pi} E_0 \log\frac{E_0}{\mu} \,.
\end{aligned}$$

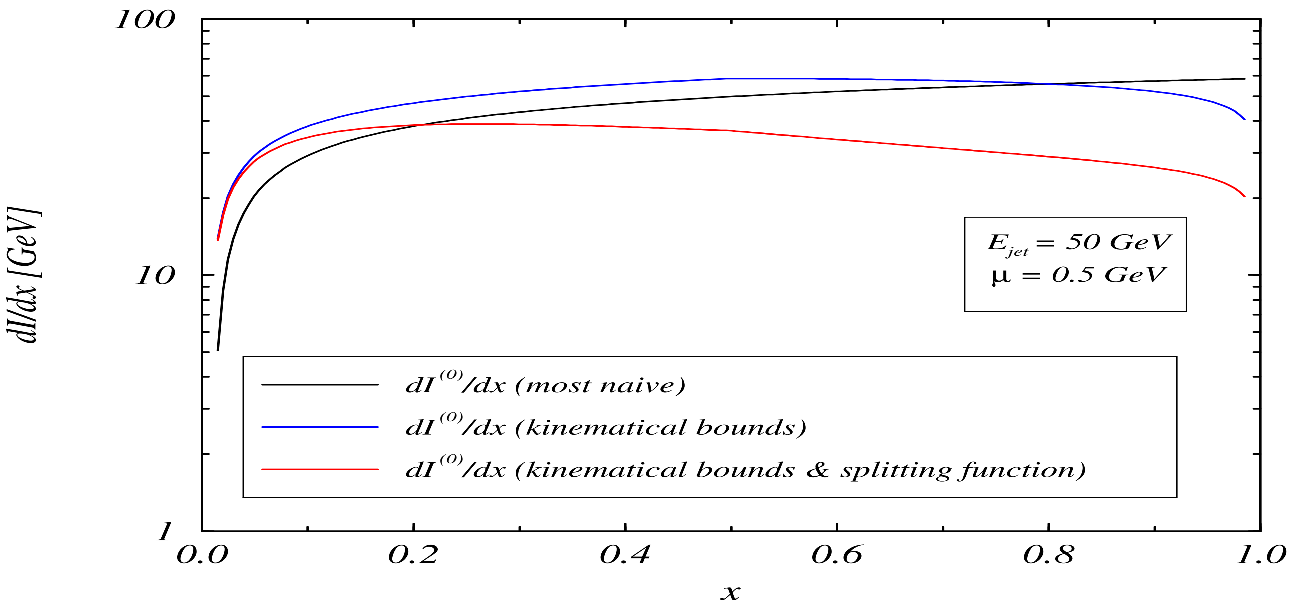
In the Leading Pole of the Leading Log Approximation (LLA). One can do better than that by keeping the kinematical bounds

$$k^2_{\perp\,\mathrm{min}} = \mu^2 \,, \quad k^2_{\perp\,\mathrm{max}} = \min\left[4E_0^2x^2, 4E_0^2x(1-x)\right]$$

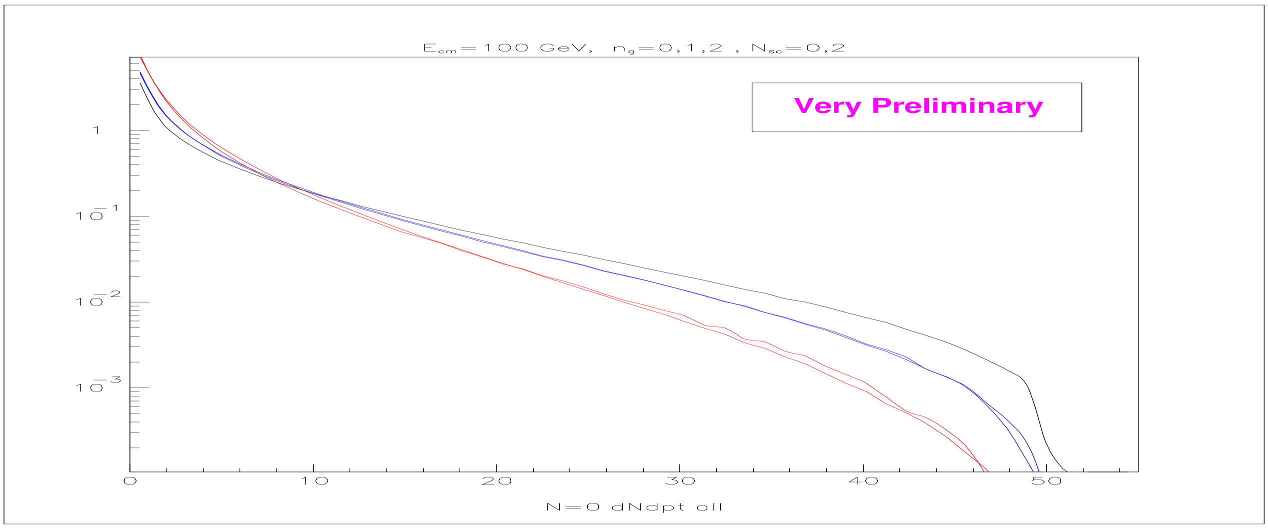
Conclusion: The "Self Quenching" effect at RHIC and LHC is HUGE $\sim 50\% - 80\%$.

Hard Radiation Intensity Distributions and "Self Quenching" Effect

The Hard Radiation intensity distribution. Cases of naive kinematics, three body corrections and splitting function incorporation



High $P_\perp$ tail shifts in hadronic spectra – jet quenching evidence (Lund JETSET model)



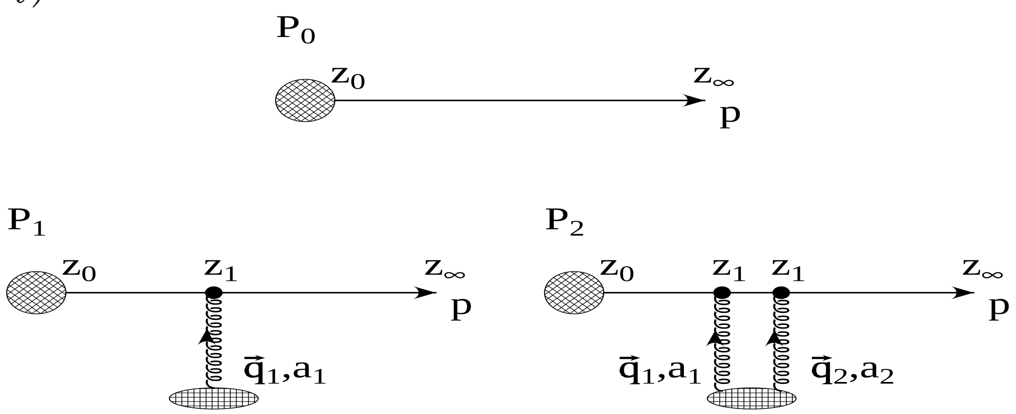
Inclusive Measurements
Elastic Scattering Example

Example: Consider a scalar field and interaction Hamiltonian

$$\phi(\mathbf{x},t) = \sum_{\mathbf{p}} c_p (e^{-ipx} a_p + e^{+ipx} b_p^\dagger)$$

$$H_I(t) = \int d^3\mathbf{x}\, V(\mathbf{x}-\mathbf{x}_1)\phi^\dagger(\mathbf{x},t) D_t \phi(\mathbf{x},t)\ , \quad D_t = i \overleftrightarrow{\partial_t}$$

Graphs contributing up to first order in $\sigma_{el}/A_\perp$ $(z_i = ct_i)$



The Non-perturbed jet production probability

$$d^3N_0 = |P_0|^2 \frac{d^3\mathbf{p}}{2|\mathbf{p}|(2\pi)^3} = |j(p)|^2 \frac{d^3\mathbf{p}}{2|\mathbf{p}|(2\pi)^3}$$

Including both Direct and Double Born interactions - key to probability conservation at higher orders in opacity

Up to first order in opacity $\frac{N\sigma_{el}}{A_\perp} = \frac{L}{\lambda} = \bar{n}$

$$d^3N^{(0+1)} = d^3N_0 + d^3N_1 + [2\,\mathrm{Re}(P_1P_0^*) + 2\,\mathrm{Re}(P_2P_0^*)]\,\frac{d^3\mathbf{p}}{2|\mathbf{p}|(2\pi)^3} + \cdots ,$$

Working in the "Gyulassy-Wang static model"

$$V_n = 2\pi\delta(q^0)\frac{4\pi\alpha_s}{\mathbf{q}_n^2+\mu^2}e^{-i\mathbf{q}_n\cdot\mathbf{x}_n}$$

The remaining contributions Add to Zero

$$d^3N_1 = \frac{\sigma_{el}}{A_\perp}|j(p)|^2\,\frac{d^3\mathbf{p}}{2|\mathbf{p}|(2\pi)^3}$$

$$2\mathrm{Re}\,\langle P_1P_0^*\rangle \approx 2\mathrm{Re}\left(-i|j(p)|^2\frac{v(0)}{A_\perp}\right) \equiv 0$$

$$2\mathrm{Re}\,\langle P_2P_0^*\rangle\frac{d^3\mathbf{p}}{2|\mathbf{p}|(2\pi)^3} \approx -\frac{\sigma_{el}}{A_\perp}|j(p)|^2\frac{d^3\mathbf{p}}{2|\mathbf{p}|(2\pi)^3}$$

Therefore $d^3N_0 \equiv d^3N^{(0+1)}$

Q.E.D.

Gluon Double Differential Distributions to All Orders in Opacity

1. Do first and second order and generalize based on physical intuition
2. Present a rigorous proof through Reaction Operator Formalism

$$x\frac{dN^{(n)}}{dx\, d\mathbf{k}^2} = \frac{C_R\alpha_s}{\pi}\frac{1}{n!}\left(\frac{L}{\lambda_g}\right)^n \prod_{i=1}^{n}\int d\mathbf{q}_i \left(\bar{v}^2(\mathbf{q}_i) - \delta^2(\mathbf{q}_i)\right)$$

$$\left[-2\,\mathbf{C}_{(1,\cdots,n)} \cdot \sum_{j=1}^{n} \mathbf{B}_{(j+1,\cdots,n)(j,\cdots,n)} \right.$$

$$\left. \left(\cos\left(\sum_{k=2}^{j} \omega_{(k,\cdots,n)} \Delta z_k \right) - \cos\left(\sum_{k=1}^{j} \omega_{(k,\cdots,n)} \Delta z_k \right) \right) \right]$$

where

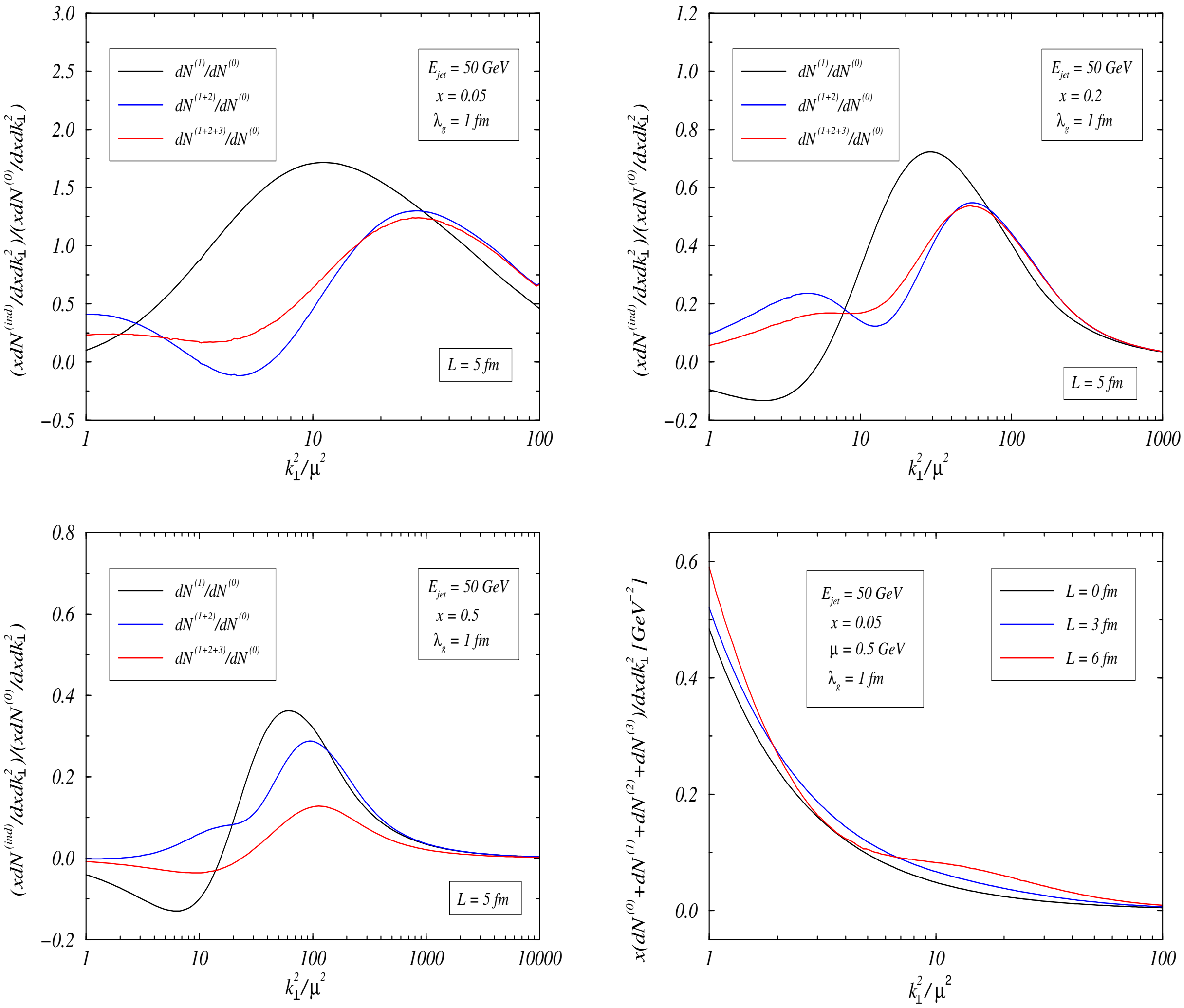
$$\omega_{(j,\cdots,n)} = \frac{(\mathbf{k} - \mathbf{q_j} - \cdots - \mathbf{q_n})^2}{2xE}$$

$$\mathbf{C}_{(j,\cdots,n)} = \frac{\mathbf{k} - \mathbf{q_j} - \cdots - \mathbf{q_n}}{(\mathbf{k} - \mathbf{q_j} - \cdots - \mathbf{q_n})^2}$$

$$\mathbf{B}_{(j+1,\cdots,n)(j,\cdots,n)} = \mathbf{C}_{(j+1,\cdots,n)} - \mathbf{C}_{(j,\cdots,n)}$$

Transverse Momentum Distributions of Induced Radiation

Competition between production, LPM suppression and redistribution of medium-induced radiation



For modest plasmas and hard production the simple diffusion idea

$$\langle \mathbf{k}^2 \rangle = N_{scat} \;\; \mu^2$$

Doesn't seem to quite work

Gluon Intensity Distribution
Analytical Approach

The induced radiation to first order in opacity

$$\frac{dI^{(1)}}{dx} = \frac{C_R \alpha_s}{\pi} \frac{L}{\lambda_g} E \int_{\mathbf{k}^2_{\min}}^{\mathbf{k}^2_{\max}} \frac{d\mathbf{k}^2}{\mathbf{k}^2} \int_0^{\mathbf{q}^2_{1\,\max}} d^2\mathbf{q}_1 \frac{\mu^2_{eff}}{\pi(\mathbf{q}_1^2+\mu^2)^2} \frac{2\,\mathbf{k}\cdot\mathbf{q}_1(\mathbf{k}-\mathbf{q}_1)^2 L^2}{16x^2E^2+(\mathbf{k}-\mathbf{q}_1)^4 L^2}$$

Approximate

$$|\mathbf{k}|_{\min} = 0\ , \quad |\mathbf{k}|_{\max} = \infty\ , \quad |\mathbf{q}_1|_{\max} = \infty$$

$$\frac{dI^{(1)}}{dx} = \frac{C_R \alpha_s}{\pi} \frac{L}{\lambda_g} E\, f(\gamma)\ , \qquad \gamma = \frac{L\mu^2}{4xE}$$

Two important limits govern the physics

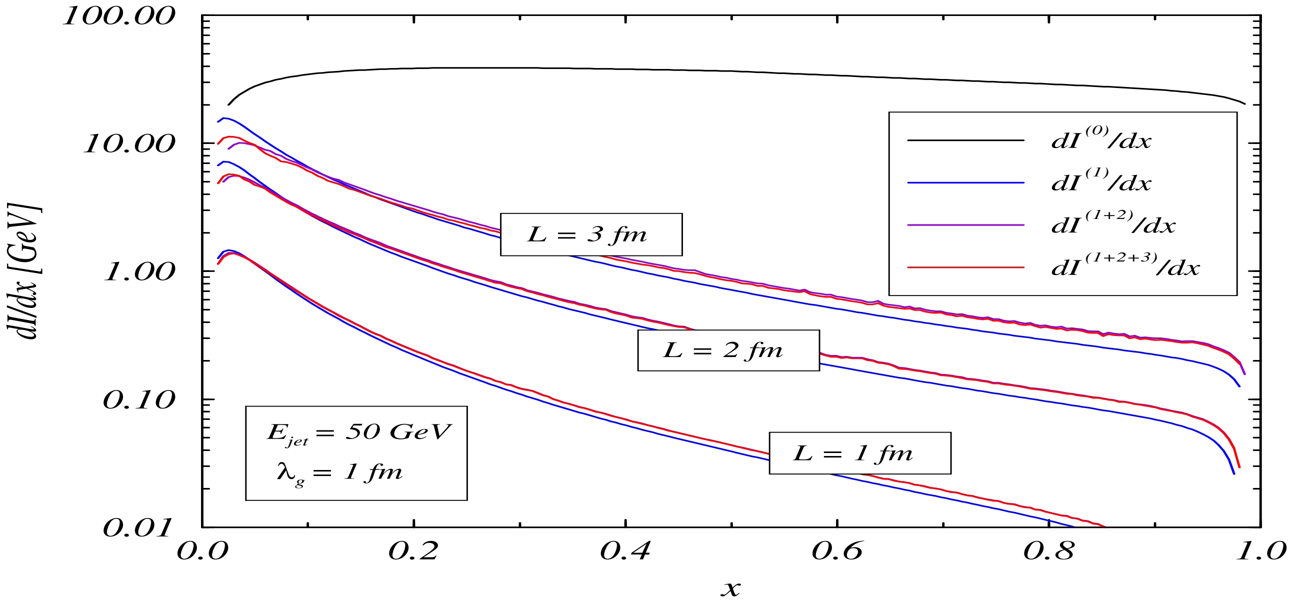
$$f(\gamma) = \frac{\gamma\ (\pi + 2\gamma \log \gamma)}{(1+\gamma^2)} \approx \begin{cases} \pi\gamma & \text{if} \quad \gamma \ll 1 \\ 2\log\gamma & \text{if} \quad \gamma \gg 1 \end{cases}$$

For E = 50 GeV, L=5 fm, μ=0.5 GeV, x=0.2

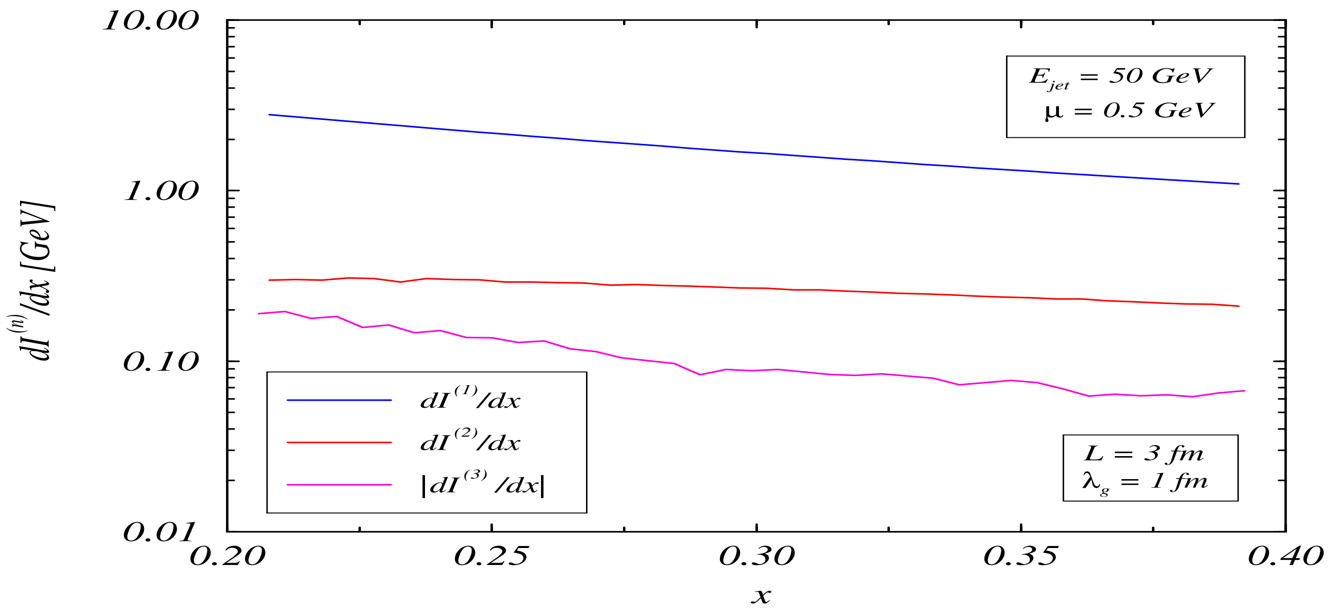
$$\gamma \approx 0.156$$

Gluon Intensity Distribution
Numerical Results

Keep all kinematical bounds. Monte Carlo averaging



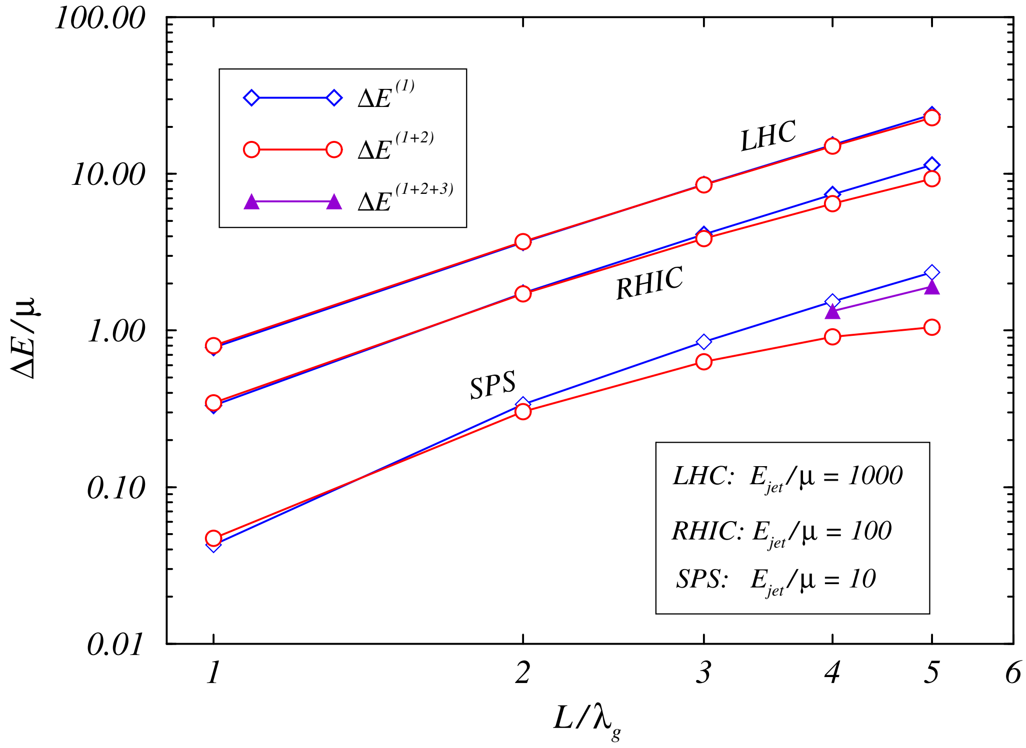
Higher orders in opacity have little if any effect for all practical purposes



Jet Energy Loss $\propto L^2$. First through Third Orders in Opacity

Simple no $\mathbf{k}^2$ bounds approximation

$$\Delta E^{(1)} = \frac{C_R \alpha_s}{4} \frac{L^2}{\lambda_g} \log \frac{E}{\mu}$$



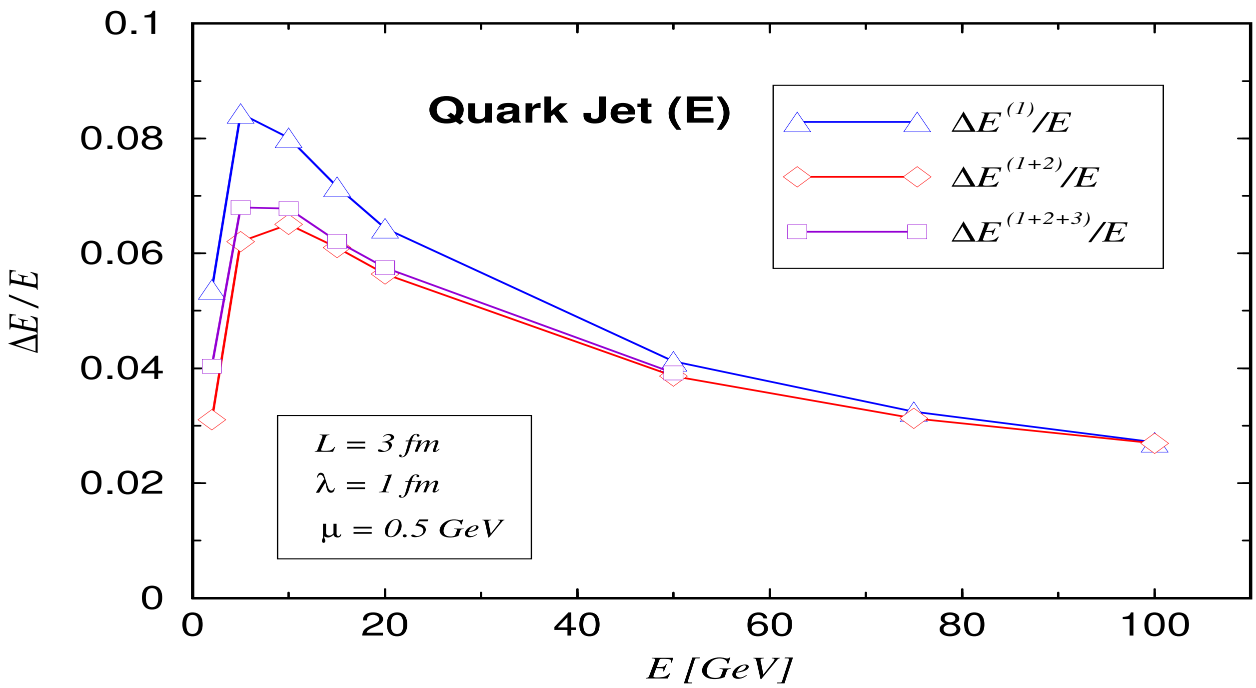
Higher orders in opacity don't matter too much In reality

$$\Delta E^{(1)} = \frac{C_R \alpha_s}{N(E)} \frac{L^2}{\lambda_g} \log \frac{E}{\mu}$$

With $N(E) =$ 7.3, 10.1 and 24.4 for $E =$ 500, 50 and 5 GeV

Parameterizing the Effect of Higher Order Contributions

Higher order contributions play a role at smaller jet energies where phase space cuts are most important



Parameterize the difference between $\Delta E^{(tot)}/E$ and $\Delta E^{(1)}/E$

$$\frac{\Delta E^{(tot)}}{E} = \left[0.75 \left(\frac{2\,GeV}{E} \right)^3 + \left(1 - \frac{2\,GeV}{E} \right)^3 \right] \frac{\Delta E^{(1)}}{E}$$

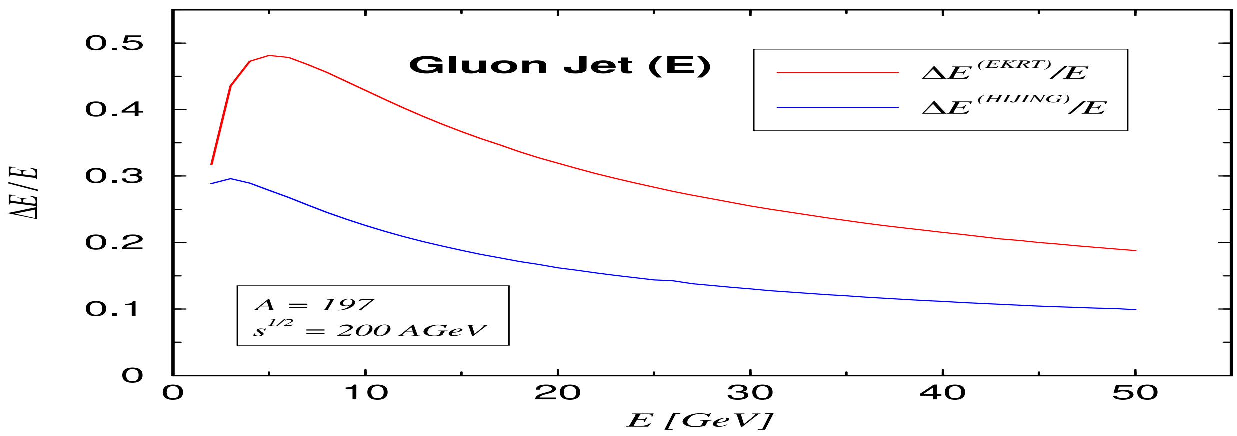
We can achieve precision of $\sim$2%-5%

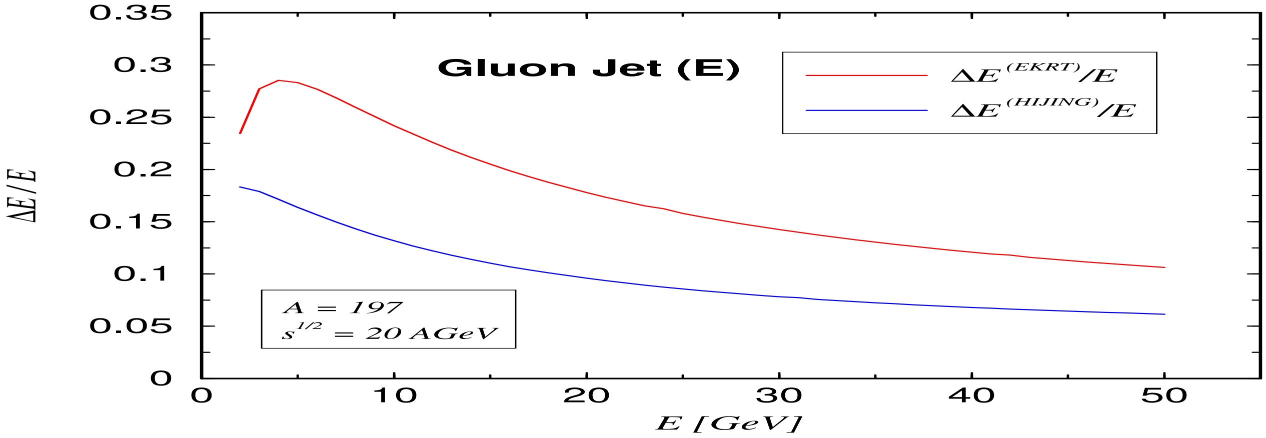
Realistic Calculations of the Radiative (Inelastic) Energy Loss

- $\mu^2 = 4\pi\alpha_s(Q^2), \;\; \sigma^{gg} = \frac{9\pi}{2}\frac{\alpha_s^2}{\mu^2}, \;\; \lambda_g(\tau) = \frac{1}{\sigma^{gg}(\tau)\rho(\tau)}$

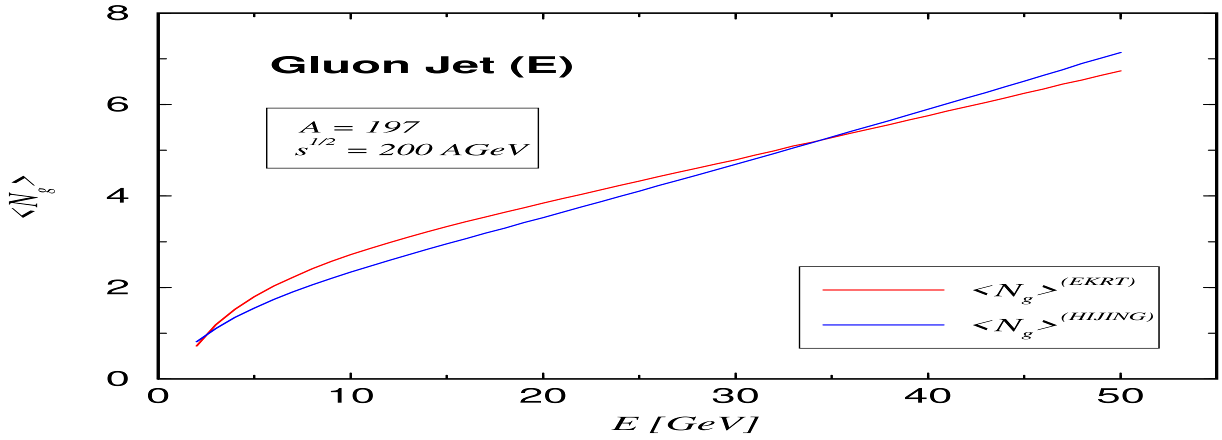
- Bjorken expansion ($C_{assympt.} = 2$)

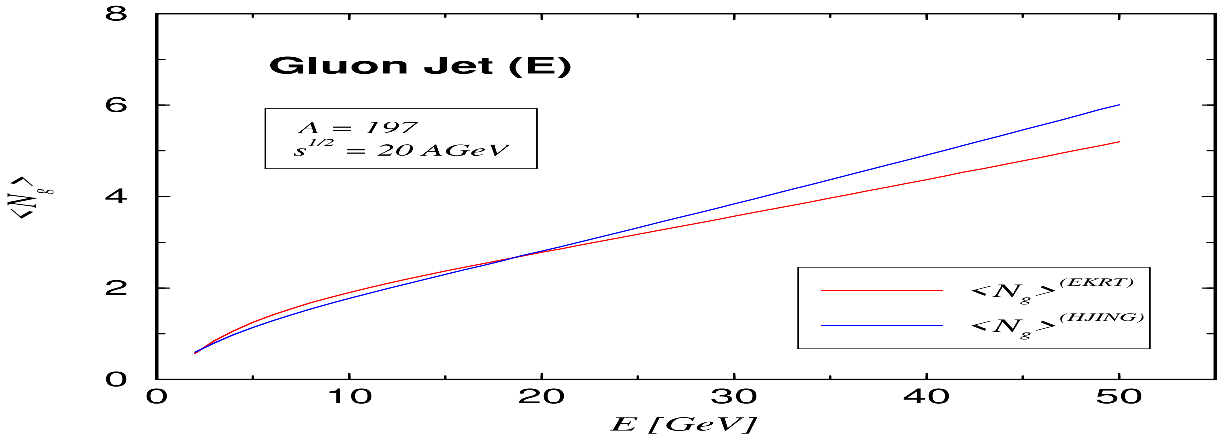
$$\rho_0\tau_0^{\alpha} = \rho\tau^{\alpha} \;\Rightarrow\; \Delta E^{expand.} = \frac{C_{eff.}}{2-\alpha}\Delta E^{stat.}(\tau_f\rho_f)$$





Mean Gluon Number $\langle N_g \rangle$ as a Function of Jet Energy (Q^2)





Typically in the range of $E \simeq p_\perp = 2 - 10\, GeV$ we find $\langle N_g \rangle \simeq 1 - 2$

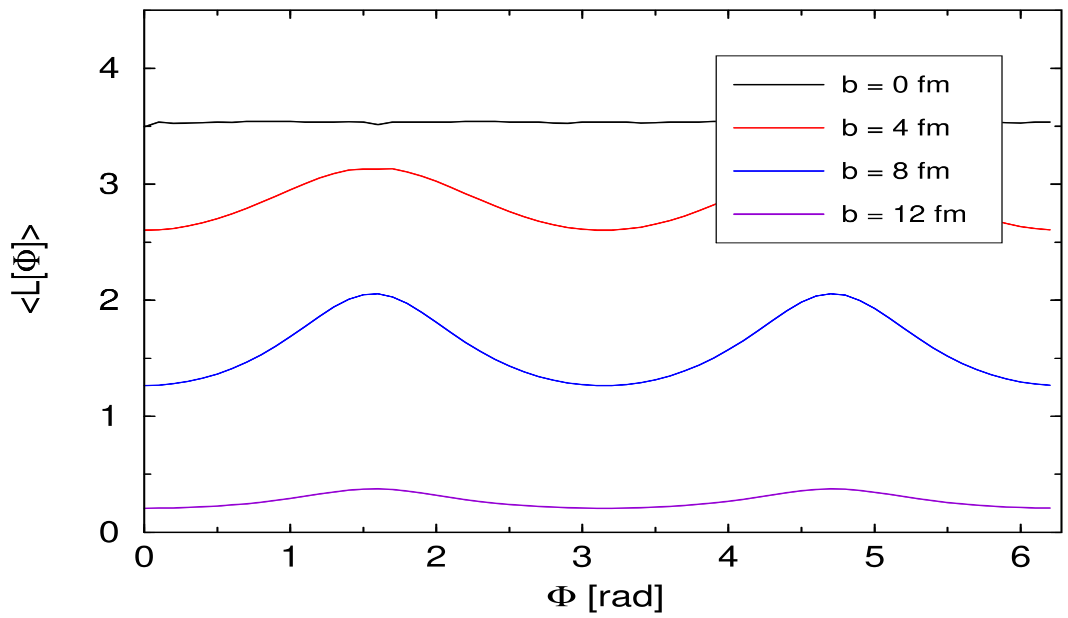
We can generate all distributions but we can simply use $\langle \omega_g \rangle = \frac{\Delta E}{\langle N_g \rangle}$

Typically $\langle \omega_g \rangle \simeq 0.9\, GeV - 1.7\, GeV$

The Effect of Spatial Anisotropy

The line integral $\int ds \cdots$ is normalized to the central density since $\Delta E \propto \mu^2/\lambda \propto \rho$. Note that $\langle \bar{L} \rangle_b \simeq 2.2\ fm$. This is the average - (Min. Bias)

$$\langle L(\varphi) \rangle = \frac{\int_0^\infty ds \int d^2\vec{r}\, t_A(r) t_B(|\vec{b}-\vec{r}|)\rho^2(\vec{r}+s\hat{n}(\varphi))/\rho^2(0)}{\int d^2\vec{r}\, t_A(r) t_B(|\vec{b}-\vec{r}|)}$$



A simple way of thinking about v_2 is that significant fraction of the partons are removed from the direction where $\langle L(\varphi) \rangle$ is big and redistributed well to the side. This flips the above Fig. into the familiar v_2

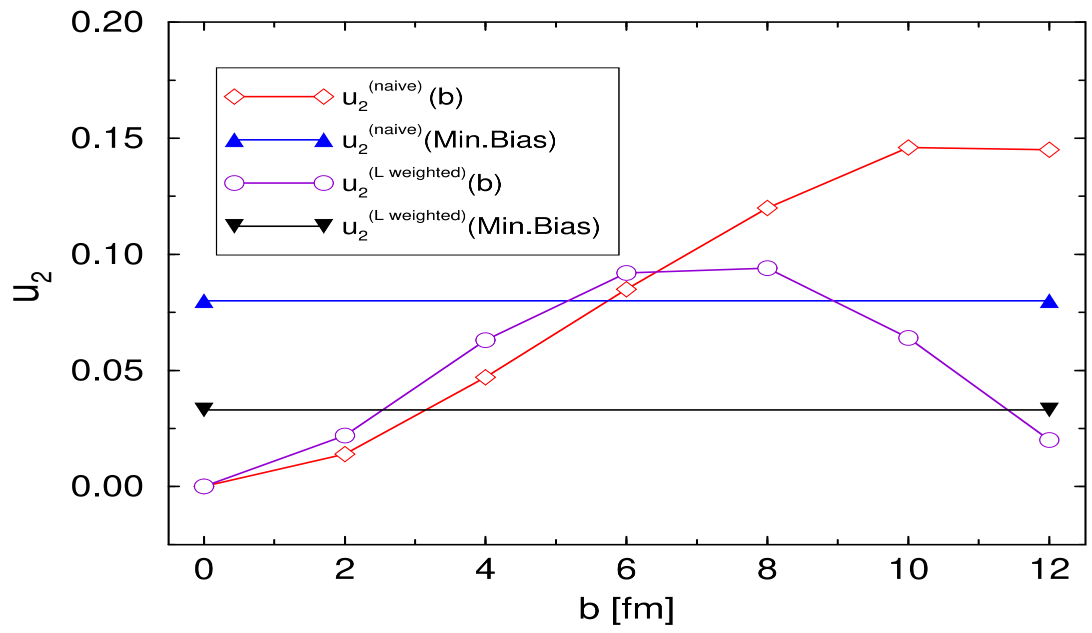
Centrality Dependence of the Elliptic Flow

Think of u_2 as the precursor of v_2 in most naive terms. Since energy loss depends on L we might weigh the naively extracted u_2 by $\langle L \rangle / \langle \bar{L} \rangle$

Why weigh by L and not by L^2? Recall the Bjorken scenario

$$\rho_f L = \rho_0 \tau_0 = \frac{dN}{dy} \frac{1}{\pi R^2}$$

A number fixed from the initial conditions - e.g. EKRT or HIJING



The STAR data is between the two curves. The distribution effectively looks close to hard sphere

The Shape of v_2 as a Signature of the Dimensionality of the Expansion

$\alpha \in [0,1]$ - velocity of longitudinal expansion
$\beta \in [0,2]$ - engages transverse expansion

From the previous Fig. one can conclude that it is likely case # 2

$$\rho = \rho_0 \left(\frac{\tau_0}{\tau}\right)^{\alpha} \left(\frac{R}{R + v_\perp \tau}\right)^{\beta}$$

If you consider the different orderings of $\{\tau_0, L, R\}$ then

- $\alpha \approx 0,\ \tau < L \quad \Rightarrow \quad \Delta E \propto \frac{dN}{dy}\frac{1}{\pi R^2}\frac{L^2-\tau_0^2}{\tau_0}$
- $\alpha \approx 1,\ \tau < L \quad \Rightarrow \quad \Delta E \propto \frac{dN}{dy}\frac{2}{\pi R^2}(L-\tau_0)$
- $\tau > L,\ \beta = 1 \quad \Rightarrow \quad \Delta E \propto \frac{dN}{dy}\frac{2}{\pi R^2}\frac{R}{v_\perp}\log\frac{L}{\tau_0}$
- $\tau > L,\ \beta = 2 \quad \Rightarrow \quad \Delta E \propto \frac{dN}{dy}\frac{2}{\pi R^2}\frac{R^2}{v_\perp^2}\left(\frac{1}{\tau_0} - \frac{1}{L}\right)$

$p_\perp$ Dependence of v_2

The distribution of high transverse momentum jets is typically

$$\frac{d^2 N_J}{dp_\perp^2} \simeq \frac{C}{|p_\perp|^a}, \quad a \simeq 6-8, \quad p_\perp > p_0$$

Assuming that the gluons emit in a narrow cone almost collinearly to the original gluon one gets

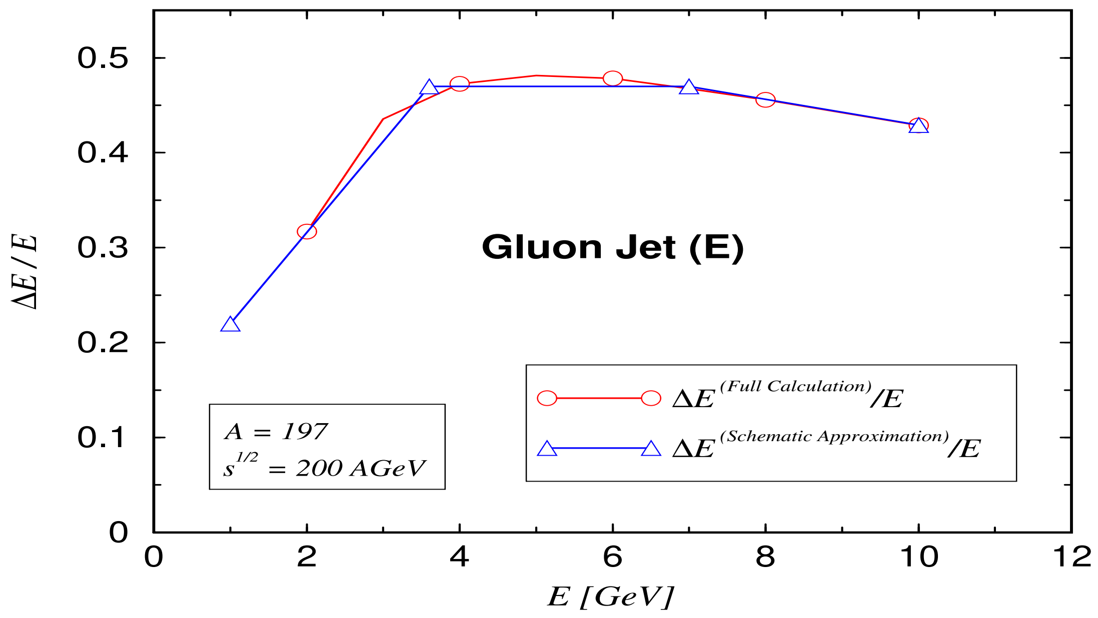
$$N_{Had.}(\phi) \;=\; \int_{p_\pi}^{\infty} dp_\perp \, \frac{dN}{dp_\perp} \left[D\left(\frac{p_\pi}{p_\perp - \Delta E(\phi)} \right) + \langle n_g(\phi) \rangle \, D\left(\frac{p_\pi}{\langle \omega_g(\phi) \rangle} \right) \right]$$

From the Field-Feynman parameterization a qualitatively correct $D(z)$ is given by

$$D(z) = \frac{(d+1)}{z}(1-z)^d, \quad \int_0^1 dz \; zD(z) = 1$$

In the simplest and naive approach take $d = 0$

Qualitative Behavior of v_2



The bulk effect comes predominantly from the kinematically allowed gluons of lowest energy. Recall that the $p_\perp$ distributions of jets fall very fast.

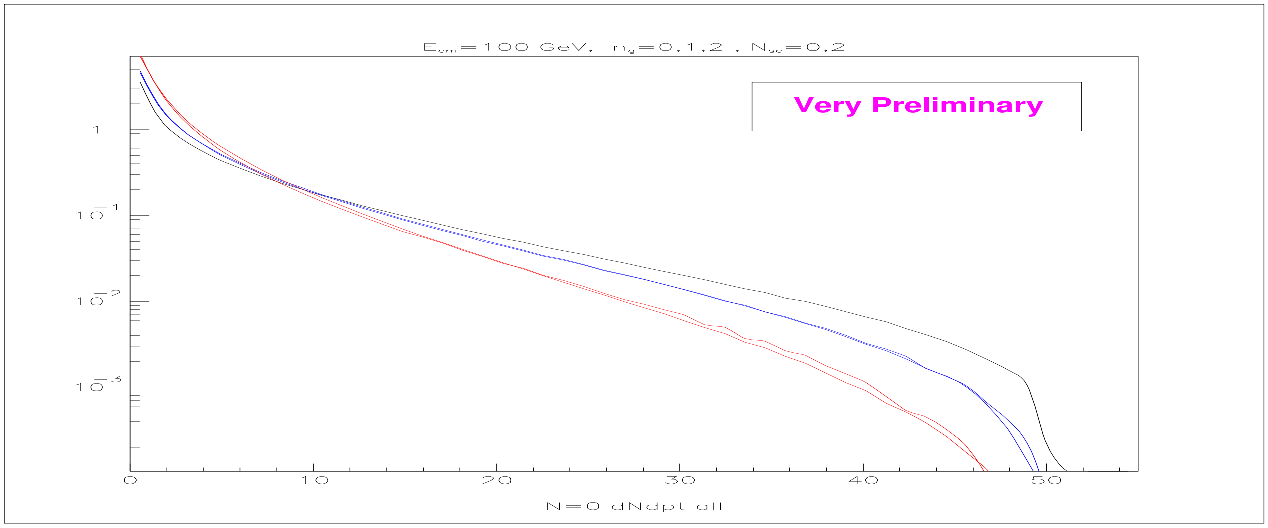
- $p_\pi \in (m_\pi, E_{(1)})$, $\quad N_{Had.}(\phi) \propto \left(1 - p_\pi \frac{c_1 L(\phi)}{L_0}\right)^{(a-1)}$

- $p_\pi \in (E_{(1)}, E_{(2)})$, $\quad N_{Had.}(\phi) \propto \left(1 - \frac{c_2 L(\phi)}{L_0}\right)^{(a-1)}$

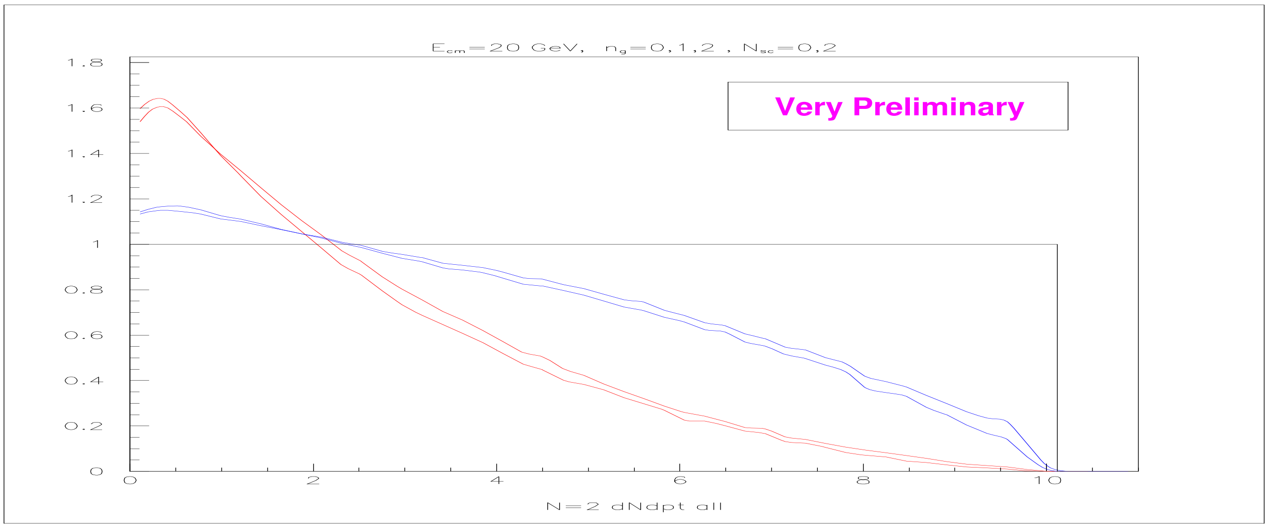
- $p_\pi \in (E_{(2)}, \infty)$, $\quad N_{Had.}(\phi) \propto \left(1 - \frac{\log p_\pi}{p_\pi} \frac{c_3 L(\phi)}{L_0}\right)^{(a-1)}$

Other Hadronic Observables at RHIC Energies

High $P_\perp$ tail shifts in hadronic spectra – Jet quenching evidence (Lund JETSET model)



The effect of N_g vs. N_{scat} on the hadronic spectra – Surprising insensitivity to the detailed gluon distribution



Monte Carlo simulation

Conclusions

- Hard processes play an important role at RHIC and LHC . An estimate is 50% - 50% for hard - soft physics

- The non-abelian analog of the Landau-Pomeranchuk-Migdal effect for the hard gluon spectrum and leads to $\Delta E^{(ind)} \propto L^2$. However one should note that in the expanding case this is effectively reduced back to linear

- The jet energy loss approach qualitatively accounts for the b and $p_\perp$ dependences of v_2

- This is work in progress and has to be refined before it becomes quantitative