Estimating Electron Proton Instability Thresholds
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1 Introduction

Very fast, high frequency, transverse instabilities have been observed in the Los Alamos PSR[1, 2, 3] and the AGS Booster[4].

- instability can "hold off" for 100 μs
- e-folding time $\sim$ 10 turns.
- 50% beam loss in $\sim$ 20 μs.
- if due to $Z_\perp$ then $Re(Z_\perp) \sim 10\text{M}\Omega/\text{m}$, and broadband
- ω_c strong function of tune/threshold current.
- $\omega_c = \omega_c(t)$ during instability

Could these be due to trapped electrons?[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. For round coasting beams the coupled equations of motion are

$$\begin{aligned} \ddot{Y}_p &= -\omega_\beta^2 Y_p + \Omega_p^2 (Y_e - Y_p) \\ \ddot{Y}_e &= -\omega_e^2 (Y_e - Y_p) \end{aligned}$$

with frequencies

$$\Omega_e^2 = \frac{e\lambda_p}{2\pi a^2 \epsilon_0 m_e} \qquad \Omega_p^2 = \frac{f m_e}{\gamma m_p}\Omega_e^2$$

where λ_p is the proton line density and $f = \lambda_e/\lambda_p$.

Data from the AGS Booster

machine parameters

parameter	Booster Study	PSR
circumference	$2\pi R = 202$m	90.2m
kinetic energy	200MeV	797MeV
rms frequency spread	$\approx$ 300Hz	$\approx$ 20kHz at 18kV
nominal betatron tunes	$Q_x = 4.8, Q_y = 4.95$	$Q_x = 3.16, Q_y = 2.14$
beam pipe radius	$b = 5$cm	$b = 5$cm
injected beam radius	$\approx$ 3cm	$\approx$ 3cm
nominal chromaticity	$Q'_x = -3$, $Q'_y = -1$	$Q'_x = -4$, $Q'_y = -2$
sextupoles off	$Q'_x = -7.5, Q'_y = -2.6$	same
rf voltage ($h = 1$)	0V (60 kV nominal)	$\leq$ 18kV
linac RF frequency	200MHz	400MHz
injected pulse length	200 to 450μs	500μs
revolution period	1207ns	358ns

Diagnostics:

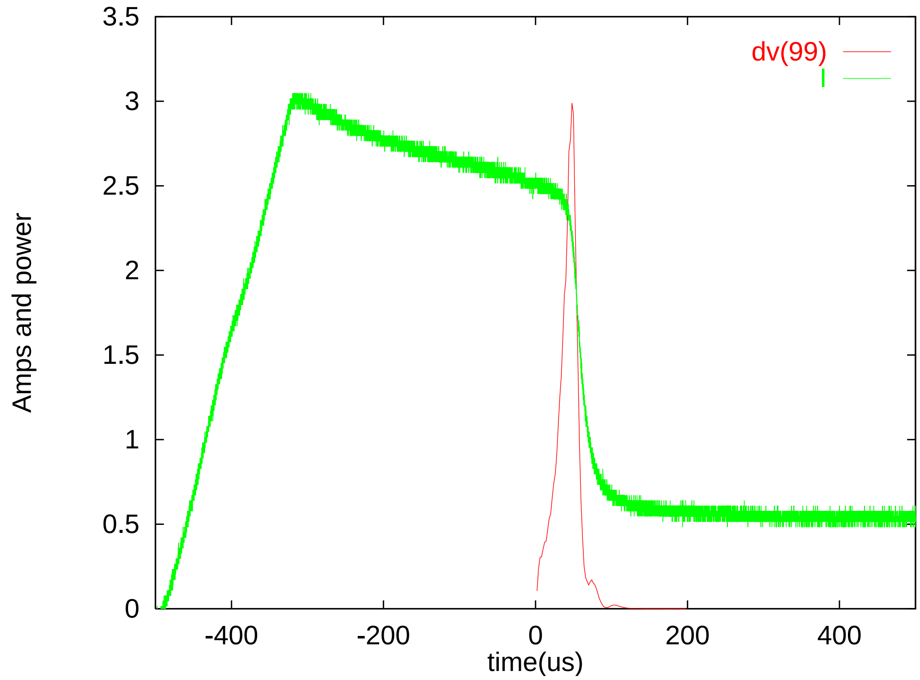
- current transformer, $0 \rightarrow 100$ kHz
- wall current monitor $1 \rightarrow 200$ MHz
- horizontal and vertical split can capacitive BPMs $1 \rightarrow 200$ MHz

BPMs were sampled at 1GHz. Sum and difference good to $\tau = 1$ ns. Checked FFTs, Mountain ranges, narrow band power P_n.

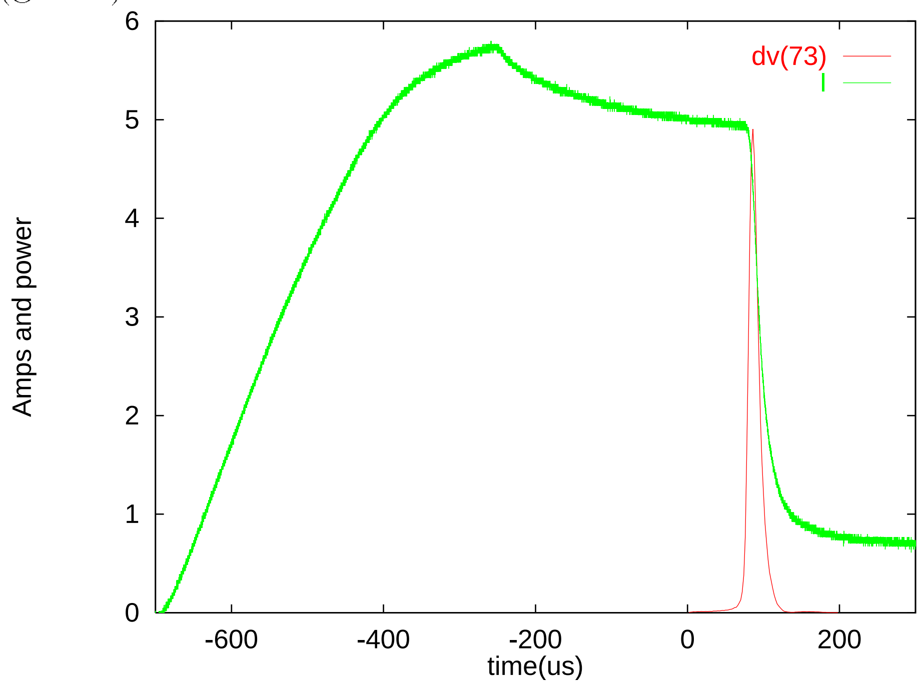
$$F_{n+1} = (\cos(\tilde{\omega}\tau)F_n - \sin(\tilde{\omega}\tau)G_n)\, e^{-\alpha\tau} + S_n \tag{1}$$

$$G_{n+1} = (\sin(\tilde{\omega}\tau)F_n + \cos(\tilde{\omega}\tau)G_n)\, e^{-\alpha\tau} \tag{2}$$

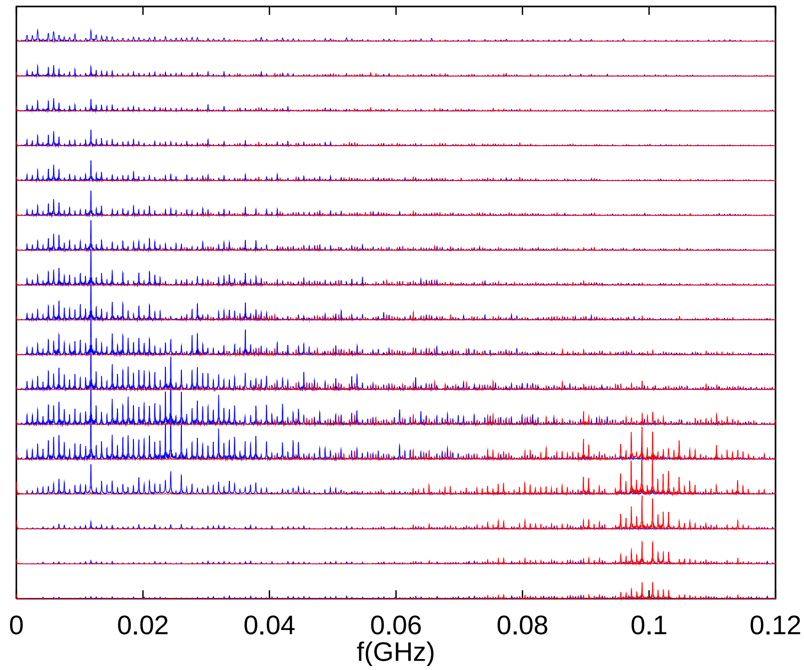
$$P_{n+1} = e^{-\tau/\tau_0} P_n + G_n^2 \tag{3}$$



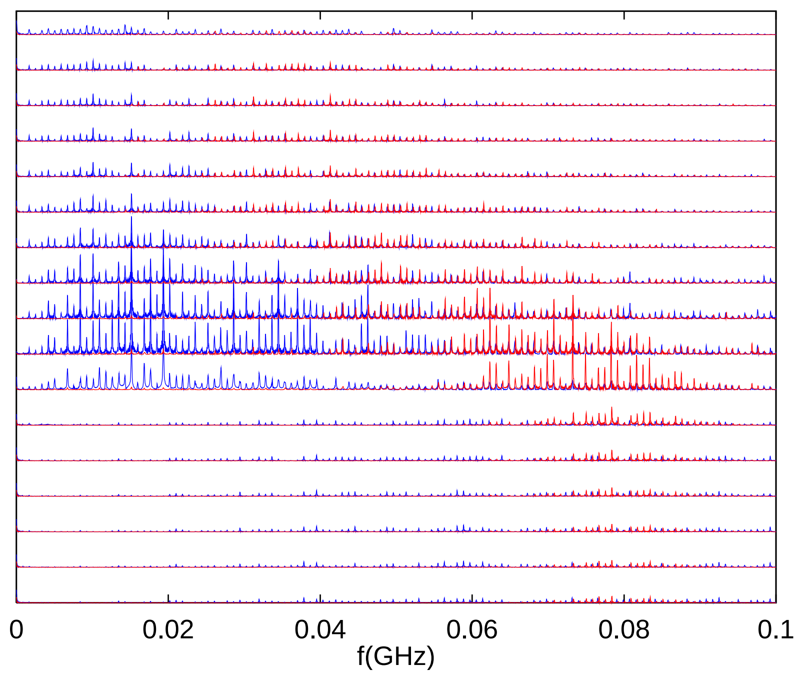
$Q_x = 4.75$, $Q_y = 4.50$, sextupoles off
power in narrow band vertical difference (red), and beam current (green).



$Q_x = 4.80$, $Q_y = 4.95$, sextupoles off
power in narrow band vertical difference (red), and beam current (green).

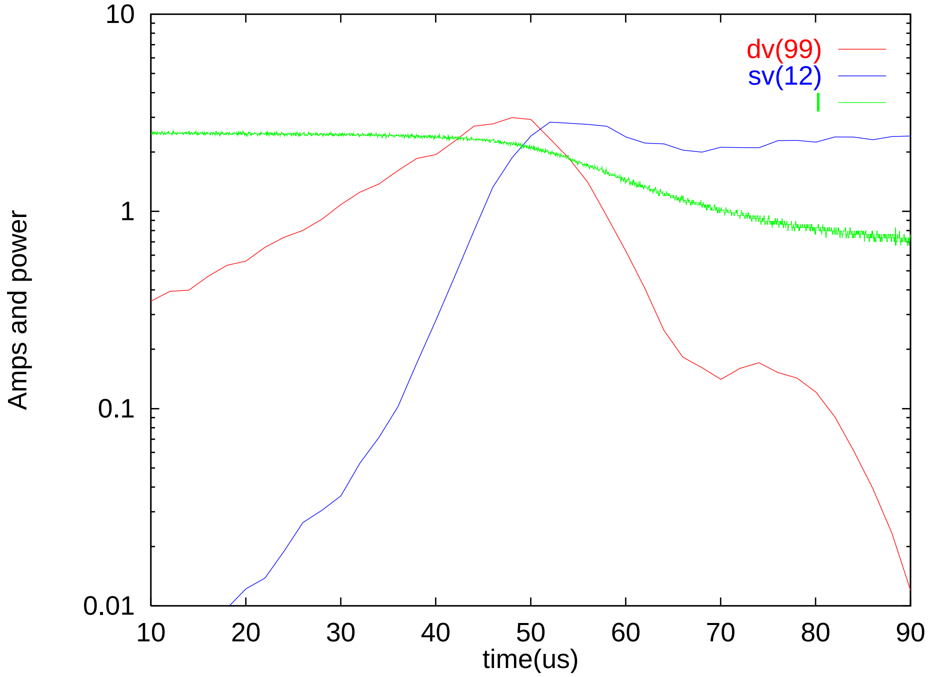


Spectral amplitude of vertical sum (blue) and difference (red).
$Q_x = 4.75$, $Q_y = 4.5$, sextupoles off



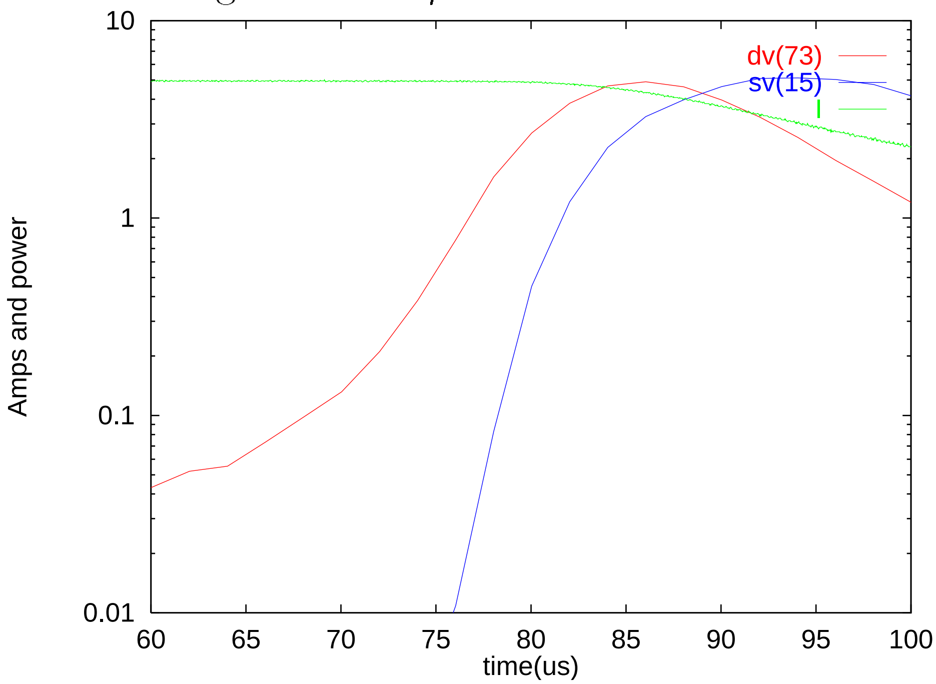
Spectral amplitude of vertical sum (blue) and difference (red).
$Q_x = 4.8$, $Q_y = 4.95$, sextupoles off
FFTs used ten turns of data (12μs between traces).

Narrow band signals



$Q_x = 4.75$, $Q_y = 4.5$, sextupoles off

Net smearing time $\approx 2\ \mu$s.



$Q_x = 4.8$, $Q_y = 4.95$, sextupoles off

Impedance estimate

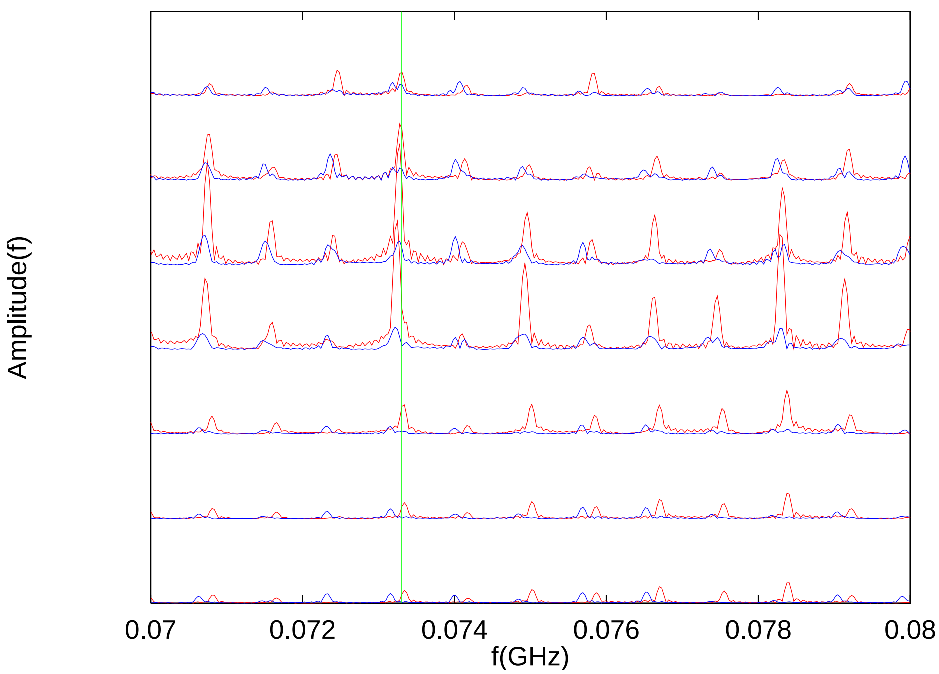
Transverse growth rate of a cold coasting beam,

$$Im(\Omega) = \frac{qcI_{peak}Re(Z_\perp)}{4\pi E_0 Q_\beta}, \tag{4}$$

For $Q_y = 4.5$, e-folding time of 11.4μs implies $Re(Z_\perp) = 5.4\mathrm{M}\Omega/\mathrm{m}$.
Many unstable lines implies broad band.
Coherent transverse space charge impedance with $\beta\gamma = 0.69$.

$$-i\frac{RZ_0}{\beta^2\gamma^2 b^2} = -i8.4\mathrm{M}\Omega/\mathrm{m}.$$

For $Q_y = 4.95$, $d\log P/dt$, peaks at 350/ms.
If $Z_\perp$ then $Re(Z_\perp) = 8.8\mathrm{M}\Omega/\mathrm{m}$.
High reslolution of second case



The vertical line is at 73.3MHz. The nearest vertical peak shifts down by 90 kHz $= 0.11 f_{rev}$ during the instability. Electron focusing?

Simple threshold estimate assumes

- Space Charge Tune shift $\Delta Q_{sc} \gg$ others, same for ep and $Z_\perp$
- Relevant Betatron sideband Frequency $\approx$ electron bounce frequency $f_{rev} Q_e$
- Coasting beam theshold

Threshold condition for semi-circular momentum distribution [7]

$$2\Delta Q_{sc,max} \lesssim |\eta| Q_e \left|\frac{\Delta p}{p}\right|_{HW@B} \tag{5}$$

For bunched beams take momentum spread from rf

$$|\eta| \beta^2 \frac{E_T}{q} \left|\frac{\Delta p}{p}\right|^2 = \frac{V_{rf}}{\pi}(1 - \cos\hat{\phi}) \tag{6}$$

For fixed transverse beam size there is a linear relationship between threshold intensity and gap voltage.
Setting $I_{avg} = I_{peak}$, $V_{rf} \approx 2V_{true}$

Macek's plot.

Assume the instability is due to electrons.

For coasting beams near threshold the dispersion relation gives.

$$Y_e/Y_p \sim Q_e \gg 1$$

A simple bunched beam model gives a similar result.

Assume the proton centroid at a fixed position oscillates at the electron bounce frequency.

$$y_p = \hat{y}_p e^{-i\omega_e t}$$

Take electron force due to protons

$$\ddot{y}_e + \omega_e^2(y_e - y_p) = 0$$

$$y_e(0) = 0 \to y_e(t) \approx \frac{i\omega_e t}{2} y_p$$

Since $\omega_e \tau_b \sim Q_e$ get a similar result.

So, Strong Secondary Emission is necessary for fast loss (TiN).

Coasting beam models have been studied, fractional neutralization is the major unknown. For bunched beams assume a large source of electrons as the bunch passes (PSR data).

They repel each other and the cloud expands.

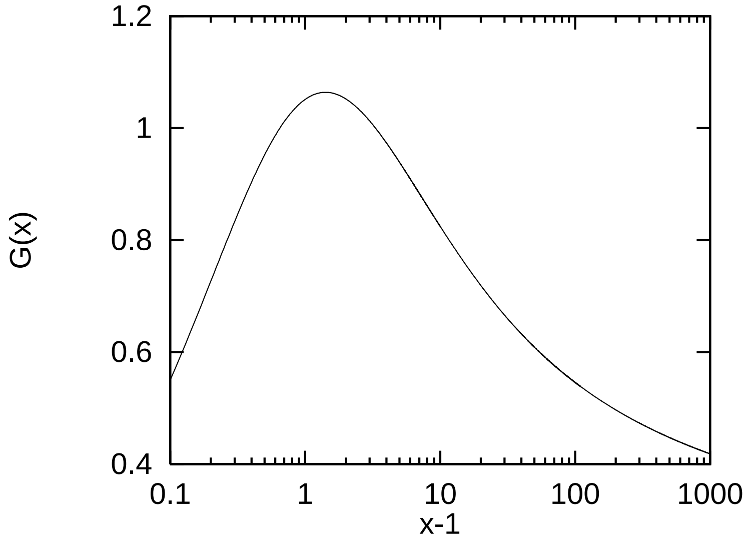
Take a uniform initial density, n_0 with negligible velocity.

$$m_e \frac{d^2 r}{dt^2} = \frac{e\lambda_e(r)}{2\pi\epsilon_0 r} \qquad (7)$$

density remains uniform during expansion

Define $T(r_0)$ = time when e^- starting at r_0 reach $r = b$, the wall.

$$T(r_0) = bG(b/r_0)\sqrt{\frac{2\epsilon_0 m_e}{e^2 n_0 r_0^2}}, \qquad G(x) = \frac{1}{x}\int_1^x \frac{dy}{\sqrt{\ln y}} \qquad (8)$$



The electron charge per meter at time T after bunch passage is

$$e\pi n_0 r_0^2 = \left(\frac{bG}{T}\right)^2 \frac{2\pi\epsilon_0 m_e}{e} = 28\mu\mathrm{C/m}\left(\frac{bG}{cT}\right)^2 \tag{9}$$

Density after gap of duration T depends on the intital density only through G. b/T similar in PSR and SNS.

Bunched Beam Threshold Simulations. Same algorithms as [12].

- Take electron density from gap length ($G = 1$).
- Initial electron amplitude = 0 (capture by beam potential).
- Linear tranverse centroid force law

  pseudo wake potential (eigenmodes).
- Linear space charge forces in proton beam (destabilizing!)
- linear rf restoring force (simplify)
- Want to find the *threshold*, nonlinear beyond.

Ideal equations of motion

Longitudinal: $\tau(\theta) = \omega_0 t - \theta$, where ω_0 is the angular revolution frequency, t is time and θ is azimuth.

$$\frac{d^2\tau}{d\theta^2} = \frac{dv}{d\theta} = -Q_s^2\tau = -\frac{dU(\tau)}{d\tau}.$$

Transverse:

$$\begin{aligned}\frac{d^2x}{d\theta^2} &= -Q_x(v)^2 x + C_{sc}\rho(\theta,\tau)(x - < x(\theta,\tau) >) \\ &+ \quad C_{ep} y_e(\tau)\end{aligned}$$

Space charge forces are proportional to

$$C_{sc} \approx 2Q_x \Delta Q_{sc}/\rho_{max}$$

where

$$\Delta Q_{sc} = |\text{max sc tune shift}|$$

The electron centroid is calculated once per turn at $\theta = 0$ using

$$\frac{d^2 y_e}{d^2\tau} = Q_e^2(\theta,\tau)\left[< x(\theta,\tau) > -y_e(\tau)\right]$$

The equations can be simulated using macro-particles

$$\frac{d^2\tau_k}{d\theta^2} = -Q_s^2\tau_k, \qquad k = 1, 2, \ldots N \sim 10^4$$

$$\frac{d^2 x_k}{d\theta^2} = -Q_x^2 x_k + \frac{C_{sc}}{N}\sum_{j=1}^{N}(x_k - x_j)\lambda(\tau_k - \tau_j) + C_{ep} y_e(\tau_k)$$

Update τ_ks once per turn with a simple rotation. For x_k and $p_k \equiv dx_k/d\theta$ use a transfer matrix followed by space charge $M \gtrsim 4Q_x$ times per turn.

$$F_{sc,k} = \hat{C}_{sc}\sum_{j=1}^{N}(x_k - x_j)\lambda(\tau_k - \tau_j)$$

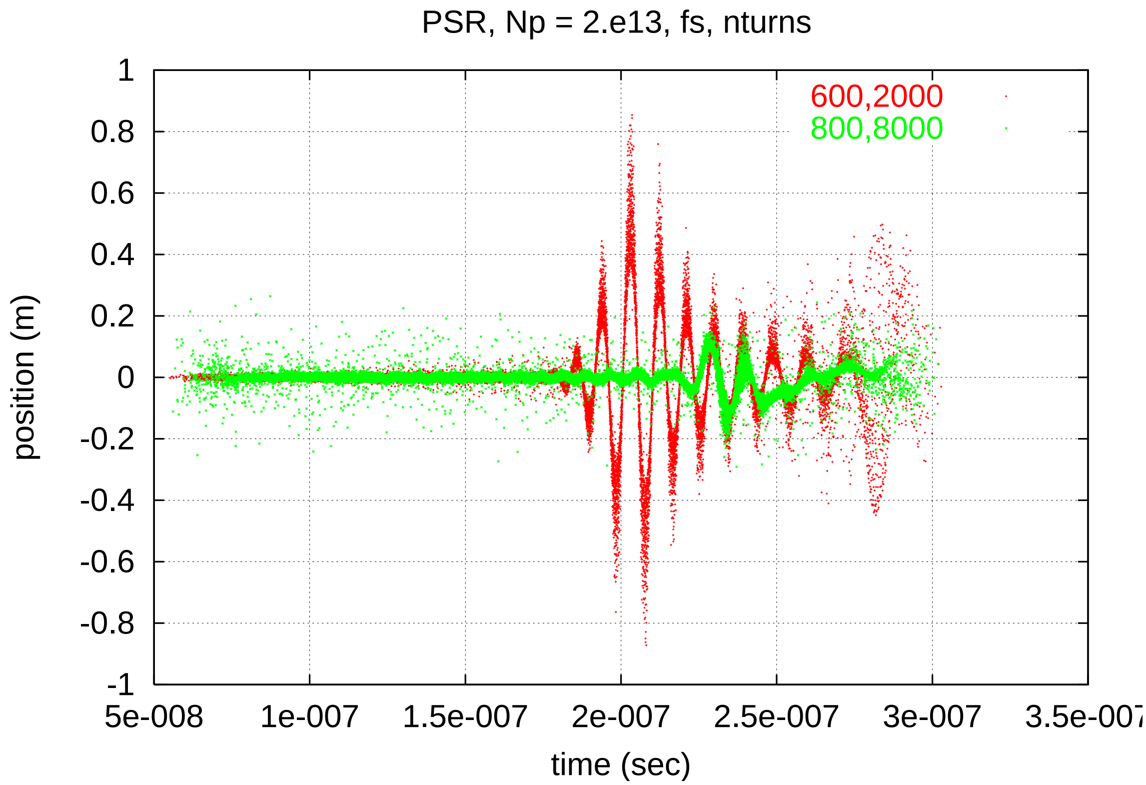
For nice $\lambda(\tau)$ the space charge sums can be done in $O(N \log N)$ operations. Details can be found in [12].
The kick from electrons is applied once per turn

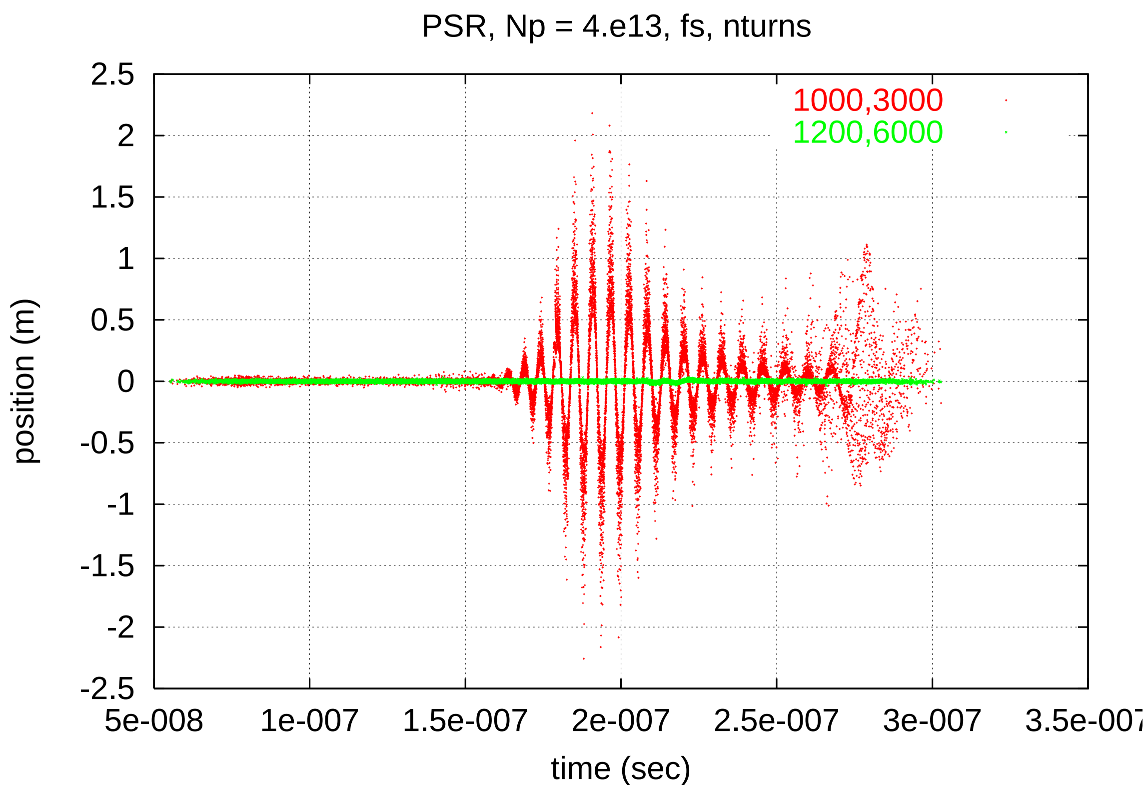
$$F_{ep,k} = \hat{C}_{ep}[y_e(\tau_k) - \bar{x}(\tau_k)]$$

with $y_e(0) = \dot{y}_e(0) = 0$ and a numerical solution of

$$\frac{d^2 y_e}{d^2\tau} = \hat{Q}_e^2 \sum_{j=1}^{N}(x_j - y_e(\tau))\lambda(\tau_j - \tau)$$

In practice there are 3 important numerical parameters.
1) the number of macro-particles, N
2) the smoothing length used for the space charge and electron forces
3) the number of space charge updates per turn, M (less important)


PSR, Np = 2.e13, fs, nturns
600,2000
800,8000
position (m)
time (sec)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
5e-008
1e-007
1.5e-007
2e-007
2.5e-007
3e-007
3.5e-007


PSR, Np = 4.e13, fs, nturns
1000,3000
1200,6000
position (m)
time (sec)
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5e-008
1e-007
1.5e-007
2e-007
2.5e-007
3e-007
3.5e-007

Factors leading to increased growth rate

- increasing intensity
- increasing Z_{sc} [13]

  for $Z_{sc,i} \gg Z_{sc,c}$ (beam radius $\ll$ pipe radius)

  $Z_{sc,i} - Z_{sc,c}$ is primary factor
- increasing chromaticity

  below transition, $\xi < 0$ stabilizes

  seems stronger than coasting beam estimate suggests
- reducing f_{synch} (gap volts)
- reducing gap length (more electrons)

For 30k macro-particles and a 1 ns smoothing length

intensity	$f_{synch} Hz$	growth rate ms^{-1}
6×10^{13}	1600	3.5
4×10^{13}	1600	1.3
2×10^{13}	1600	< 0.5
4×10^{13}	800	10
2×10^{13}	800	3.5
1×10^{13}	800	1.2

From Macek's plot get
$f_{synch} = 900$Hz (6 kV) for 2×10^{13}
$f_{synch} = 1500$Hz (16 kV) for 4×10^{13}

Conclusions and Questions

- Impedance driven instability is hard to believe
- ep simulations have reasonable agreement with PSR data

    correct order of magnitude

    correct variation with machine parameters

    How far from continuum limit?
- Are SNS simulations appropriate yet?

    coasting beam suggests factor of 4 safety margin

    psychology

References

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