Geodesic Normal Coordinates

For any reaction hypersurface S a coordinate system may be defined for which g_{ri} vanishes for $i \neq r$ and for which g_{rr} is a constant:[22] A coordinate system q^i ($i=1$ to n, $i \neq r$) is first defined on the surface in mass-weighted space. The coordinates of any point off this surface are then defined by drawing the geodesic through the point, such that the geodesic cuts the hypersurface orthogonally. The q^i for $i \neq r$ are then assigned the same values as those occurring at the intersection of the geodesic and the hypersurface. The value for q^r is set equal to the arc length along this geodesic from the hypersurface to the point. Hence, $ds^2 = g_{rr}(dq^r)^2 = (dq^r)^2$ along this geodesic in mass-weighted space. The line element in this space is:

$$ds^2 = \sum_{i,j \neq r} g_{ij}\, dq^i dq^j + (dq^r)^2 \tag{57}$$

Correspondingly, it can be shown, g^{ri} vanishes for $i \neq r$, and g^{rr} equals unity. The kinetic energy then has the following simple form.

$$T = \tfrac{1}{2} \sum_{i,j \neq r} g^{ij}\, p_i\, p_j + \frac{p_r^{\,2}}{2} \tag{58}$$

(p_i equals $\partial[(ds/dt)^2/2] / \partial \dot{q}^i$). If the definition of q^r is modified so that ds^2 equals $\mu\,(dq^r)^2$ along the geodesic, where μ is some constant then the coefficient of p_r^2 would be $1/2\,\mu$ instead.