U.S. Department of the Interior
U.S. Geological Survey

USER GUIDE FOR THE PULSE PROGRAM

Open-File Report 02-455

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By A.T. Rutledge

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PREFACE

This document is a user manual for the PULSE program. The program can be downloaded from the following Web site:

http://water.usgs.gov/ogw/pulse/

This document supersedes an earlier version that was available at this site in 2002. Program users will need to download this manual, the program, and several additional files that are also included at this Web site. These files are named in the Introduction. Included are three programs that were compiled to run on a personal computer that uses one of the Microsoft Windows operating systems.

CONTENTS

FIGURES

FIGURES (continued)

CONVERSION FACTORS

Multiply	By	To obtain
inch (in.)	25.4	millimeter
inch per day (in/d)	25.4	millimeter per day
inch per year (in/yr)	25.4	millimeter per year
foot (ft)	0.3048	meter
square mile (mi^2)	2.590	square kilometer
foot squared per second (ft^2/s)	0.09290	meter squared per second
foot squared per day (ft^2/d)	0.09290	meter squared per day
cubic foot per second (ft^3/s)	0.02832	cubic meter per second

USER GUIDE FOR THE PULSE PROGRAM

By A.T. Rutledge

OVERVIEW

This manual describes the use of the PULSE computer program for analysis of streamflow records. The specific instructions included here and the computer files that accompany this manual require streamflow data in a format that can be obtained from U.S. Geological Survey (USGS) sites on the World Wide Web. The program is compiled to run on a personal computer that uses a Microsoft Windows-based operating system. This manual provides instructions for use of Microsoft Excel for plotting hydrographs, though users may choose to use other software for plotting.

The program calculates a hydrograph of ground-water discharge to a stream on the basis of user-specified recharge to the water table. Two different formulations allow recharge to be treated as instantaneous quantities or as gradual rates. The process of ground-water evapotranspiration can be approximated as a negative gradual recharge.

The PULSE program is intended for analyzing a ground-water-flow system that is characterized by diffuse areal recharge to the water table and ground-water discharge to a stream. Program use can be appropriate if all or most ground water in the basin discharges to the stream and if a streamflow-gaging station at the downstream end of the basin measures all or most outflow. Ground-water pumpage and the regulation and diversion of streamflow should be negligible. More information about the application of the method is included in Rutledge, 1997, pages 2-3.

The program can be used in conjunction with ground-water-level data. If a well is open to the surficial aquifer, observed water-level rises in the well can be used to evaluate the timing of recharge. Such evaluation is most effective if there are numerous water-level observation wells in the basin. Water levels in observation wells can also be used to evaluate the rate of ground-water discharge estimated by the PULSE program. The results of such an evaluation may be problematic, however, because the relation between ground-water level and ground-water discharge may not be unique.

Departures from the linear model of recession occur because of areal variation in transmissivity and because of the longitudinal component of ground-water flow (parallel to the stream). If the PULSE program is used to estimate ground-water recharge, the recession index should not be obtained from periods of extreme low flow, and the calibration process should include plotting flow on the linear scale in addition to plotting flow on the log scale.

INTRODUCTION

The PULSE program is intended for analyzing a ground-water-flow system that is characterized by diffuse areal recharge to the water table and ground-water discharge to a stream. Program use can be appropriate if all or most ground water in the basin discharges to the stream and if a streamflow-gaging station at the downstream end of the basin measures all or most outflow. Ground-water pumpage and regulation and diversion of streamflow should be negligible. More information about the application of the method is included in Rutledge, 1997, pages 2-3. Assumptions required for applicability of the PULSE program are similar to those of the RORA program (Rutledge, 1998). For more information about the applicability of the latter program, see Rutledge (2000, pages 10-11).

The specific instructions included here and the computer files that accompany this manual require streamflow data in a format that can be obtained from U.S. Geological Survey (USGS) sites on the World Wide Web. Although this report relies heavily on the content of an earlier report (Rutledge, 1997) for thorough explanation of the method, additional material is included that was not in that report. The section "Supplemental Information about the PULSE Program" includes discussion of ground-water levels and departures from the linear model of recession.

Several computer files are needed to execute the PULSE program and the other programs that are described in this report. The names of these files and programs are given below, with a brief description of each:

Executable applications

screen.exe -- This program will read a streamflow-data file and display the period of record.
prep.exe -- This program will read a streamflow-data file and create the *pulseIN.txt* file.
pulse.exe -- The PULSE program.

Streamflow-data file

Indian.txt -- Streamflow-data file for Indian Creek near Troy, Alabama.

Other program input files

station.txt -- A file in which the drainage area is entered (read by the prep application).
index.txt -- A file in which the recession index can be entered for each station.
pulseIN.txt -- The input file to the pulse application.

Files that can be copied to the *pulseIN.txt* file

fig4.txt -- This example input file will generate the hydrograph on figure 4.
fig5.txt -- This example input file will generate the hydrograph on figure 5.

Fortran source codes (*screen.f*, *prep.f*, and *pulse.f*) are provided but may not be needed by the program user. This manual is on a Web site from which the user can obtain the three computer programs as executable files (*screen.exe, prep.exe*, and *pulse.exe*) and as fortran source codes (*screen.f, prep.f*, and *pulse.f*). Throughout the rest of this manual, names of programs are written with capital letters with no extension (for example, SCREEN, PREP, and PULSE). Names of specific files are generally written in italics (example, *Indian.txt*).

MATHEMATICAL MODEL

This manual does not include a complete explanation of the theoretical basis of the program (provided in Rutledge, 1997); however, brief explanation of the mathematical model is provided. Depending on model-input information designated by the program user, the model uses one of two equations to calculate ground-water discharge over time. The first equation describes ground-water discharge after an instantaneous recharge amount:

$$Q = \frac{1.866AR_i}{K} \times \sum_{m=1,3,5}^{\infty} e^{(-0.933m^2\pi^2t)/(4K)} \tag{1}$$

where Q is total basin ground-water discharge; A is basin drainage area; R_i is instantaneous recharge depth; K is recession index (explained in a later section); and t is time elapsed after the instantaneous recharge.

The second equation describes ground-water discharge caused by an instantaneous recharge amount followed by a gradual recharge rate:

$$Q = R_gA + 2R_gA \times \sum_{m=1,3,5}^{\infty} \left[\frac{0.933R_i}{R_gK} - \frac{4}{\pi^2m^2} \right] \times e^{(-0.933m^2\pi^2t)/(4K)} \tag{2}$$

where R_g is the gradual recharge rate. Equation 2 can be used to simulate the effect of gradual recharge in the absence of instantaneous recharge by specifying R_i to be zero. Gradual recharge will begin at t=zero and will continue infinitely. In many applications, the program user will need to terminate the gradual recharge rate at a later time. Termination is accomplished by executing equation 2 a second time, applying the principle of superposition. A later section in this report (Using PULSE to Generate a Hydrograph of Ground-Water Discharge) describes and illustrates an example.

PREPARING TO USE THE PULSE PROGRAM

This section describes tasks the program user may need to complete before using PULSE to analyze a streamflow record other than the one included with this report. If the program is used only to generate a hydrograph of ground-water discharge on the basis of hypothetical basin characteristics and recharge, the user can skip this section and proceed to page 10. The first-time user who is experimenting with the streamflow data file included with this report can skip this section and proceed to page 13 (although the sub-section about the SCREEN program may be helpful).

Obtaining Streamflow-Data Files

A streamflow-data file can be obtained from http://water.usgs.gov/nwis/discharge. From this Web site, choose from one or more station-selection criteria. For example, if the site of interest is Indian Creek near Troy, Alabama; yet the site-identification number is unknown, the selection criteria might be *State* and *Site Name*. In this case, proceed to the option to select a site, enter *Indian* under site name, then select Alabama from the pull-down menu. After these options, select the particular station of interest. Under *Choose Output Format,* select a range of dates, then select *Tab-separated data*. Then select to save as a file, designate the directory where the data are to be written on the user's computer, and specify a file name. Using 12 characters or less, name the file with an abbreviated station name and a *.txt* file extension. In this case, the file name might be designated as *Indian.txt*. Under *save as file type* specify *plain text*. An example streamflow-data file, obtained as described, is illustrated in figure 1.

The "SCREEN" Program

Before analyzing a streamflow-data file, the user might execute the SCREEN program to display the period of record. To execute the program, click on the application or enter the command "screen" in the MS-DOS command window. Designate the streamflow-data file *Indian.txt*. The output file *screenout.txt* should look like figure 2. This example illustrates that data are available from October 1958 to September 1968 and from October 1970 to September 1986.

```
#
# U.S. Geological Survey
# National Water Information System
# Retrieved: 2001-08-29 12:57:29 EDT
#
# ---------------------WARNING---------------------
# The data you have obtained from this automated
# U.S. Geological Survey database have not received
# Director's approval and as such are provisional
# and subject to revision.  The data are released
# on the condition that neither the USGS nor the
# United States Government may be held liable for
# any damages resulting from its use.
#
# This file contains published daily mean streamflow data.
#
# This information includes the following fields:
#
#  agency_cd   Agency Code
#  site_no     USGS station number
#  dv_dt       date of daily mean streamflow
#  dv_va       daily mean streamflow value, in cubic-feet per-second
#  dv_cd       daily mean streamflow value qualification code
#
# Sites in this file include:
#  USGS 02371200 INDIAN CREEK NEAR TROY AL
#
#
agency_cd     site_no        dv_dt  dv_va  dv_cd
5s     15s    10d     12n    3s
USGS   02371200       1958-10-01    1.5
USGS   02371200       1958-10-02    1.9
USGS   02371200       1958-10-03    2.4
USGS   02371200       1958-10-04    5.1
USGS   02371200       1958-10-05    3.6
USGS   02371200       1958-10-06    2.4
USGS   02371200       1958-10-07    1.9
USGS   02371200       1958-10-08    1.8
USGS   02371200       1958-10-09    1.8
USGS   02371200       1958-10-10    1.7
USGS   02371200       1958-10-11    1.4
USGS   02371200       1958-10-12    1.1
USGS   02371200       1958-10-13    1.1
USGS   02371200       1958-10-14    1.1
USGS   02371200       1958-10-15    1.2
USGS   02371200       1958-10-16    1.4
USGS   02371200       1958-10-17    1.6
USGS   02371200       1958-10-18    1.9
USGS   02371200       1958-10-19    2.4
USGS   02371200       1958-10-20    2.5
```

Figure 1. Example streamflow-data file obtained from a U.S. Geological Survey Web site. (This example is truncated from the data file *Indian.txt*, which is available with this report.)

```
READING FILE NAMED Indian.txt
FIRST YEAR IN RECORD =  1958
 LAST YEAR IN RECORD =  1986
                 MONTH
 YEAR   J F M A M J J A S O N D
 1958   X X X X X X X X X . . .
 1959   . . . . . . . . . . . .
 1960   . . . . . . . . . . . .
 1961   . . . . . . . . . . . .
 1962   . . . . . . . . . . . .
 1963   . . . . . . . . . . . .
 1964   . . . . . . . . . . . .
 1965   . . . . . . . . . . . .
 1966   . . . . . . . . . . . .
 1967   . . . . . . . . . . . .
 1968   . . . . . . . . . X X X
 1969   X X X X X X X X X X X X
 1970   X X X X X X X X X . . .
 1971   . . . . . . . . . . . .
 1972   . . . . . . . . . . . .
 1973   . . . . . . . . . . . .
 1974   . . . . . . . . . . . .
 1975   . . . . . . . . . . . .
 1976   . . . . . . . . . . . .
 1977   . . . . . . . . . . . .
 1978   . . . . . . . . . . . .
 1979   . . . . . . . . . . . .
 1980   . . . . . . . . . . . .
 1981   . . . . . . . . . . . .
 1982   . . . . . . . . . . . .
 1983   . . . . . . . . . . . .
 1984   . . . . . . . . . . . .
 1985   . . . . . . . . . . . .
 1986   . . . . . . . . . X X X

 COMPLETE RECORD = .      INCOMPLETE = X
```

Figure 2. Example display of a streamflow record, showing periods of complete record (a number for each day in the month) and periods of incomplete record. (This display is generated on file *screenout.txt* by the program SCREEN, and on the monitor by PREP. Streamflow-data file is *Indian.txt*.)

The “PREP” Program

The PULSE program is supplied with an auxiliary program, PREP, which may be executed before PULSE. The PREP program will read a streamflow-data file, and place initial estimates of recharge (and other variables) into a file named *pulseIN.txt* that is read by the PULSE program. When PREP is executed, it will generate a graphical display of the period of record similar to the display generated by the SCREEN program (fig. 2). Although PULSE could be used without PREP, the use of the auxiliary program is recommended as a time-saving measure. Specific instructions for running PREP are included in a later section in this report.

If PREP is run in the MS-DOS window, expand the window size if the streamflow record is long. On some operating systems, this can be accomplished by clicking on the MS-DOS icon at the upper left corner of the window, selecting properties, then layout, then increasing window height or the screen buffer height.

Drainage Area

The PULSE program allows the user to derive estimates of ground-water recharge and ground-water discharge that are expressed in units of length (inches) and in units of specific discharge (inches per month). To make these calculations, the program needs the drainage area. If the program is used to generate a hydrograph of ground-water discharge on the basis of hypothetical basin characteristics and recharge, enter drainage area in the *pulseIN.txt* file before executing PULSE. (Examples of *pulseIN.txt* are shown on pages 11 and 12.) If the program is used to generate a hydrograph of ground-water discharge that represents part of a streamflow hydrograph, the PREP program might be executed before executing PULSE. When PREP is executed, it will automatically generate *pulseIN.txt*. If PREP is used, there should be one line written to the *station.txt* file for each streamflow-data file. Each line in *station.txt* must include the name of the streamflow-data file and the drainage area. The tabulation below is an example of *station.txt*, showing entries for five streamflow-gaging stations. These entries can be used as a guide for the format of additional entries.

```
File "station.txt"
This file is read by programs PREP, RECESS, RORA, and PART, to
obtain the drainage area.  Note: This file should have ten header
lines.  The streamflow file name should be 12 characters or less.
-------------------------------------------------------------------
            Drainage
 Name of      area    The space below, after drainage area, is
 streamflow  (Square  for optional information that is not read
 file         miles)  by the programs.  This is free-form.
---------------------- --------------------------------------------
Indian.txt      8.88   02371200  Indian Creek near Troy Alabama
Holiday.txt     8.53   02038850  Holiday Cr near Andersonville, VA
BigHill.txt    37.00             Big Hill Cr near Cherryv, Kansas
LitAndro.txt   73.50   01057000  Lt Androscoggin R nr S.Paris, ME
Sewee.txt     117.00             Sewee Cr nr Decatur TN
```

Drainage area can be obtained from USGS databases, Web sites, or data books. It can be measured from topographic maps or by use of a geographic information system and a digital elevation model.

In many cases, the ground-water contributing area can be considered to be equivalent to the surface-water drainage area of the streamflow-gaging station. In some applications, the program user may have information that indicates the ground-water contributing area is significantly different from the surface-water drainage area. In this case, the former might be designated in the *pulseIN.txt* or *station.txt* file.

The Recession Index

The PULSE program requires an estimate of the recession index. There are many methods for obtaining this variable; the basic procedure is to draw, on a streamflow hydrograph plotted on semi-log graph paper, a straight line through data that represent linear recession (fig. 3), then designate the recession index as the slope in days per log cycle. According to the Rorabaugh model of ground-water discharge to a stream, the recession will appear linear on such a graph when the ground-water-head profile becomes stable (Rorabaugh, 1964).

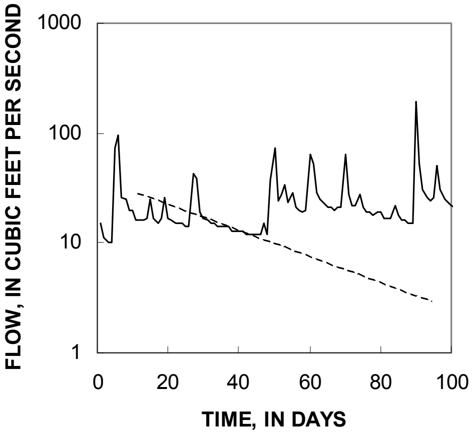


Figure 3. The basic procedure for determining the recession index is to draw a straight line through streamflow data during periods that show linear recession on the semilog hydrograph. (Streamflow record is Indian Creek near Troy, Alabama, first 100 days of calendar year 1962.)

Although derivation of the index from streamflow data is convenient, the index can be calculated as a lumped parameter that represents aquifer properties:

$$K = \frac{0.933a^2S}{T} \qquad (3)$$

where a is the distance from the stream to the hydrologic divide; S is storage coefficient; and T is transmissivity. Equation 3 is derived from Rorabaugh and Simons (1966, p. 12). It is noteworthy that the recession index, as defined above and quantified in days per log cycle, is one way of describing recession behavior. Another formulation utilizes a dimensionless recession constant that is always less than unity (Singh and Stall, 1971), which should not be confused with the recession index used as input to the PULSE model. For additional discussion of methods for quantifying recession, see Snyder (1939); Knisel (1963); Toebes and Strang (1964); Bevans (1986); Nathan and McMahon (1990); and Rutledge (2000). A later section in this report includes discussion of program use when there are departures from the linear model of recession.

If the PULSE program is used to generate a hydrograph of ground-water discharge for a hypothetical basin, the recession index can be designated in the input file, *pulseIN.txt*. Examples of *pulseIN.txt* are on pages 11 and 12. If a streamflow dataset is analyzed, it may be convenient to execute the PREP program (which will automatically generate *pulseIN.txt*) before running PULSE. In this case, the recession index might be designated in *index.txt* for each streamflow-data file. The following tabulation is an example of an *index.txt* file.

```
File "index.txt" -- The recession index is entered on this file
manually or by running the RECESS program.  This file "index.txt"
can be read by the PREP program (before running PULSE) and it can
be read by RORA.  Note -- this file should have ten header lines.
-----------------------------------------------------------------
              Recession
Name of         index
streamflow    (days per
file          log cycle)
------------------------
Indian.txt     100.00
Holiday.txt     80.00
Sewee.txt       65.00
LitAndro.txt    60.00
BigHill.txt     32.00
```

The PREP program will obtain the recession index from *index.txt.* It will then transfer this value to the *pulseIN.txt* file. The *index.txt* file that is included with this report shows entries for five streamflow-data files. These entries can be used as a guide for the format of additional entries. The recession index can be designated interactively when the PREP program is executed.

RUNNING THE PROGRAMS

This section describes the use of the PULSE program. The first sub-section describes how to generate a hydrograph of ground-water discharge on the basis of hypothetical basin characteristics and recharge. The next sub-section describes how to generate a hydrograph of ground-water discharge that represents the ground-water component of a streamflow hydrograph.

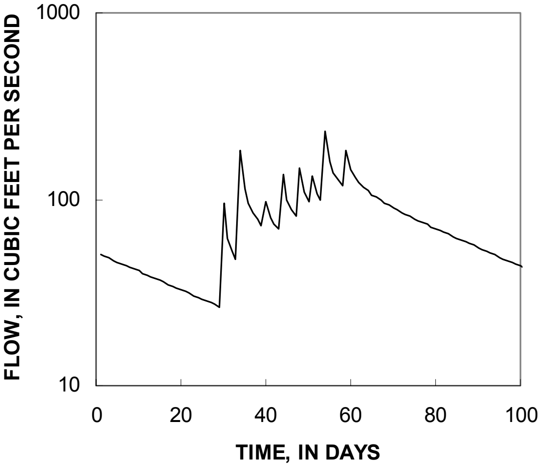
Using PULSE to Generate a Hydrograph of Ground-Water Discharge

To gain familiarity with the program, the user should start by generating a hydrograph of ground-water discharge for hypothetical basin characteristics and recharge. An example of such a hydrograph is shown in figure 4. The PULSE program will read model-input data from the *pulseIN.txt* file. To assure the correct input data for this figure, copy file *fig4.txt* to *pulseIN.txt*. The *pulseIN.txt* file should then be identical to the tabulation in figure 4 (next page).

Start the program by clicking the PULSE application (*pulse.exe*) in Windows or entering *pulse* in the MS-DOS command window. When asked if this corresponds to a streamflow record, enter the letter *n* (for “no”). Various methods can be used to chart results, which are written to a file named *pulseout.txt*. In Excel, click the *open* option, direct Excel to the folder (directory) where the file is located, browse for “text” files, then open the *pulseout.txt* file. The data should be read in columns of fixed width. When the data are displayed in the spreadsheet, click on columns A and C (holding the control key), then click the charting icon. Select to generate an “xy scatter” plot, then designate a plot that shows lines between points. To generate a graph that can be compared with figure 4, designate the range of x to be 0-100, and a logarithmic y scale from 10 to 1,000.

The program user might use this simulation as a starting point, to experiment with various other recharge events. Experimentation might include changing the recession index, changing the drainage area, modifying the timing or magnitude of recharge, and removing or adding recharge. After saving the modified *pulseIN.txt* file, repeat the procedures in the previous paragraph to generate the hydrograph.

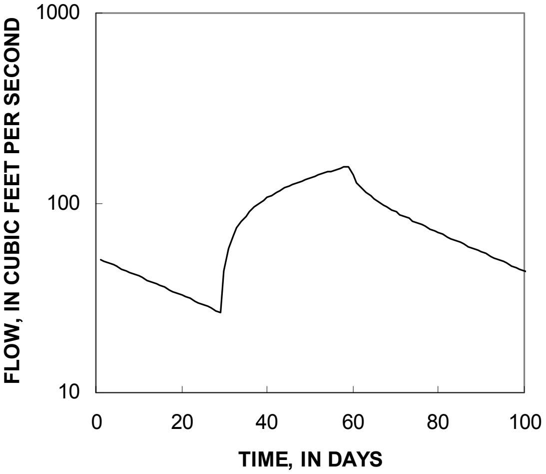
When the PULSE program is executed as described here, the *pulseout.txt* file will include three columns. Column A is the day, column B is the “baseline ground-water discharge” that would have occurred if no recharge was designated, and column C is the simulated ground-water discharge calculated by PULSE. Column B represents the ground-water discharge resulting from recharge events prior to the simulation. In most applications, graphical output will be generated using columns A and C, and column B will not be used.



```
Recession index (days/log cycle)              100.00
Drainage area in square miles                  60.00
Ground-water discharge on first day (cfs)      50.00
Events on this hour of the day                     1
Julian date of first day                           1
Number of recharge events                          8
         Instant-
          aneous   Gradual
 Day of  Recharge  Recharge
 year    (inches) (inch/day)
 -------------------------
  30       0.50      0.00
  34       1.00      0.00
  40       0.20      0.00
  44       0.50      0.00
  48       0.50      0.00
  51       0.30      0.00
  54       1.00      0.00
  59       0.50      0.00
```

Figure 4. Hydrograph of ground-water discharge calculated by the PULSE program for a series of instantaneous recharge events, and program input file (*pulseIN.txt*) used to generate this hydrograph. (Total recharge is 4.5 inches.)

If the *pulseIN.txt* file has been modified, the program user can once again generate the hydrograph on figure 4 by copying the *fig4.txt* file to *pulseIN.txt*. The hydrograph in figure 4 shows the results of a simulation of eight instantaneous recharge events that occur over a 30-day period. Another input data file, *fig5.txt,* can be used to simulate a gradual recharge event lasting 30 days (fig. 5). Simulations such as this one require entries on two lines in the *pulseIN.txt* file (illustrated on fig. 5). One line will initiate recharge (a positive number in the gradual recharge column), and the other line will terminate recharge (a negative number in the same column, with the same absolute value). For the simulation illustrated in figure 5, these are +0.15 inch per day (in/d) beginning on day 30, and –0.15 in/d beginning on day 60 (see file *fig5.txt*). In this simulation, all recharge is gradual, thus the column designating instantaneous recharge gives zeros. The total recharge is the same in figures 4 and 5.



```
Recession index (days/log cycle)            100.00
Drainage area in square miles                60.00
Ground-water discharge on first day (cfs)    50.00
Events on this hour of the day                   1
Julian date of first day                         1
Number of recharge events                        2
        Instant-
         aneous   Gradual
Day of  Recharge  Recharge
 year   (inches) (inch/day)
-------------------------
  30       0.00       0.15
  60       0.00      -0.15
```

Figure 5. Hydrograph of ground-water discharge calculated by the PULSE program for a gradual recharge event, and program input file (*pulseIN.txt*) used to generate this hydrograph. (Total recharge is 4.5 inches.)

Running PREP and PULSE to Generate a Hydrograph of Ground-Water Discharge for a Streamflow Hydrograph

This section describes how to estimate ground-water recharge and discharge on the basis of a streamflow dataset. An auxiliary program, PREP, is provided that makes initial estimates of model-input variables. After a first run with that model-input dataset, use the process described below to modify the *pulseIN.txt* file until a calibration is achieved. More information about the process is in Rutledge (1997, p. 14-25).

This and subsequent paragraphs describe the application of the model to Indian Creek near Troy, Alabama, for the calendar year 1963. To analyze a streamflow record, start the PREP program (by clicking on the application in Windows or by keying “prep” in the MS-DOS window). After entering “*y”* to continue, enter the streamflow file name *Indian.txt*, specify that one year is being analyzed (1963), and that the recession index is to be read from the *index.txt* file. If the user has not modified *index.txt*, the program will assign the recession index to be 100 days per log cycle (from *index.txt*).

When the PREP program is executed as described above, it will generate a new *pulseIN.txt* file. This file gives initial estimates of recharge (and other variables) for running PULSE to simulate the hydrograph of ground-water discharge for Indian Creek near Troy, Alabama, for the calendar year 1963. After executing PULSE, various types of computer graphics software can be used to observe the hydrograph of ground-water discharge and streamflow. In Excel, click the *open* option, direct Excel to the folder (directory) where the file is located, browse for “text” files, then open the *pulseout.txt* file. The data should be read in columns of fixed width. When the data are displayed in the spreadsheet, click on columns A, C, and D (holding the control key) (these columns represent day, simulated ground-water discharge, and streamflow), then click the charting icon. Select to generate an “xy scatter” plot, then designate a plot that shows lines between points. To generate a graph that can be compared with figure 6, designate the range of x to be 0-60 and a logarithmic y scale from 1 to 100. The graph generated should be similar to figure 6.

At this point, the simulation represents initial estimates that were calculated by the PREP program. The user can improve the simulation by using a repetitive process of (1) observing needs for improvement on the hydrograph, (2) modifying the *pulseIN.txt* file accordingly, (3) executing PULSE, and (4) generating the hydrograph (Rutledge, 1997, p. 14-15). Modifications in the input file should proceed forward with time. For example, the first adjustment might be a reduction in the initial ground-water discharge (third line in *pulseIN.txt*) because of anomalies before day 11 (fig. 6). After this adjustment, the user might experiment with reduction in recharge for the event on day 11 because of anomalies in days 14-17 on the hydrograph. Figure 7 indicates improved simulation after adjustments such as these. A variety of other simulations have been shown for this dataset by Rutledge (1997, figs. 13-16).

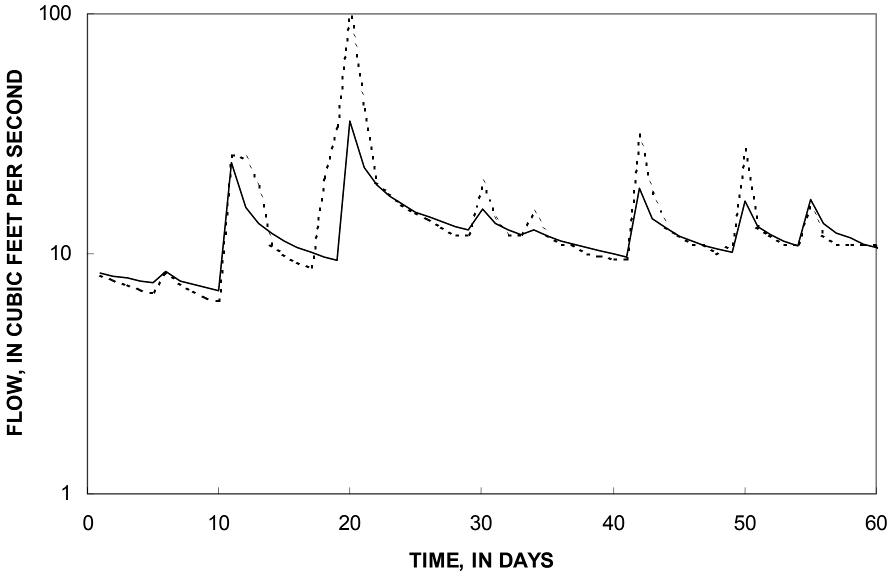


Figure 6. Ground-water discharge calculated by the PULSE program (solid curve) and measured streamflow (dotted curve) for Indian Creek near Troy, Alabama, for the first 60 days of calendar year 1963, based on initial estimates of recharge determined by the PREP program.

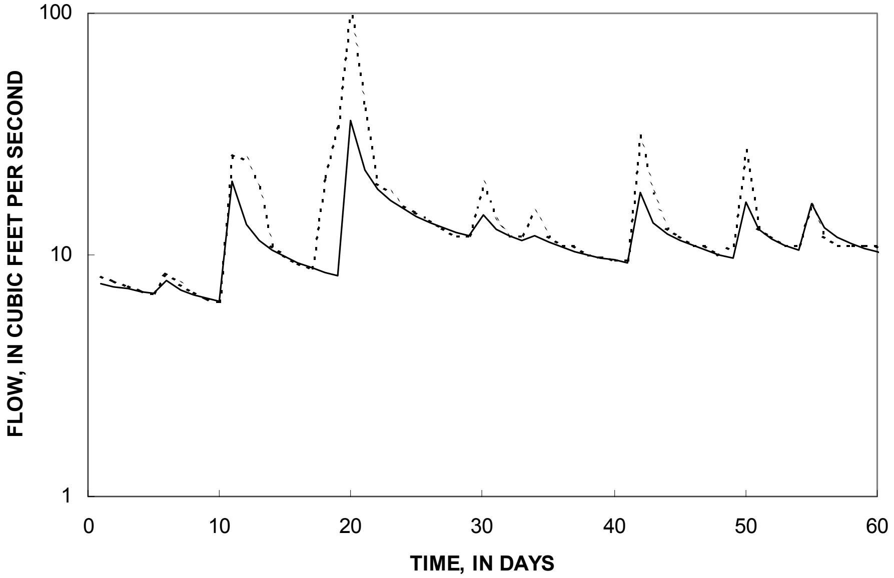


Figure 7. Ground-water discharge calculated by the PULSE program (solid curve) and measured streamflow (dotted curve) for Indian Creek near Troy, Alabama, for the first 60 days of calendar year 1963, after calibration.

The PULSE program can allow for the effect of ground-water evapotranspiration (GWET) as a gradual negative recharge. Figure 8 shows the simulation described previously, with an expanded time scale to include warmer months. In some parts of the hydrograph, ground-water discharge is above the streamflow hydrograph, possibly because of the need to simulate GWET (Daniel, 1976, p. 362). To allow for GWET, add a gradual "negative recharge" in the *pulseIN.txt* file as follows: Day 60, gradual recharge = -0.006 in/d; Day 170, gradual recharge = +0.006 in/d. As noted in the previous section, a gradual recharge event of finite duration requires two entries in the *pulseIN.txt* file, one to initiate the event, and the other to terminate the event. In this case, the event is negative, so it must be terminated with a positive of the same absolute value. Figure 9 shows the result of the simulation when allowing for GWET. Calibration included adjustments in recharge late in the simulation.

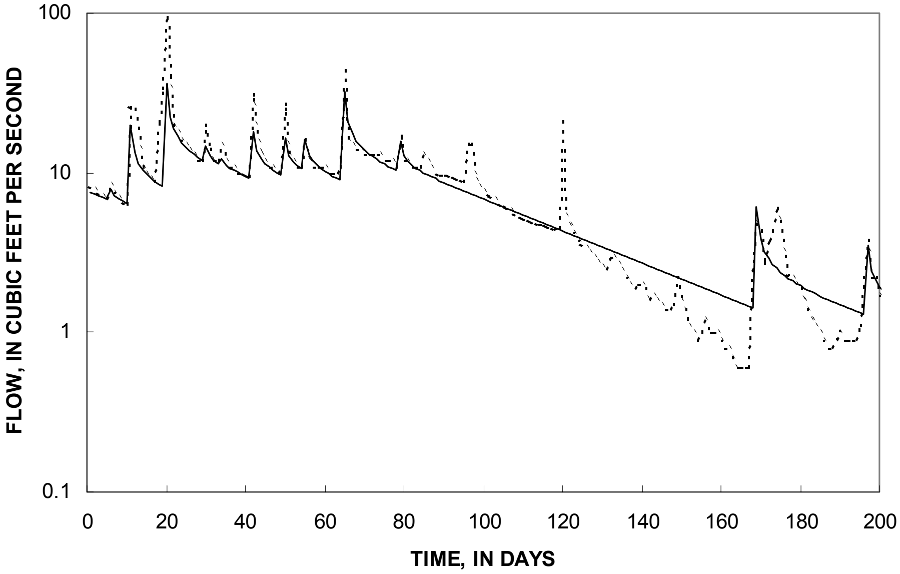


Figure 8. Ground-water discharge calculated by the PULSE program (solid curve) and measured streamflow (dotted curve) for Indian Creek near Troy Alabama, for the first 200 days of calendar year 1963, before simulating the effect of ground-water evapotranspiration.

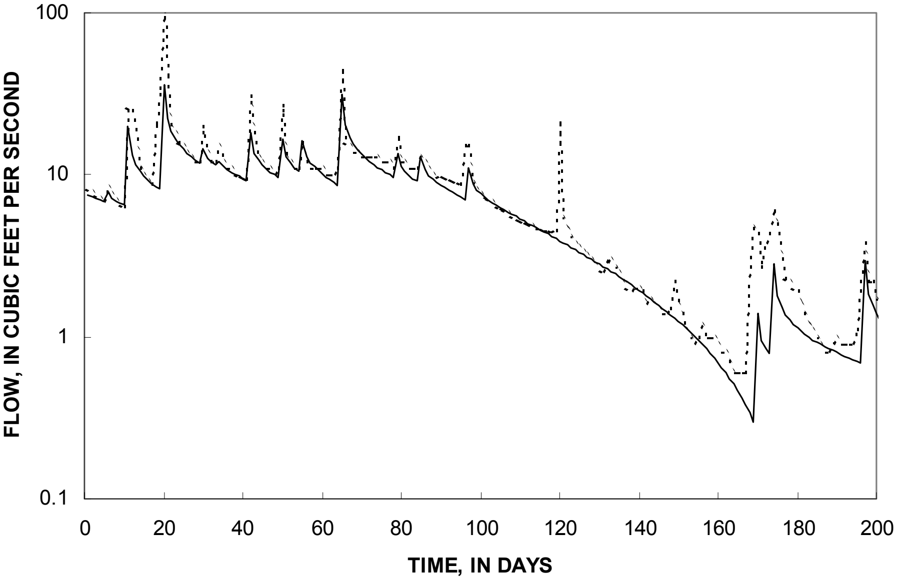
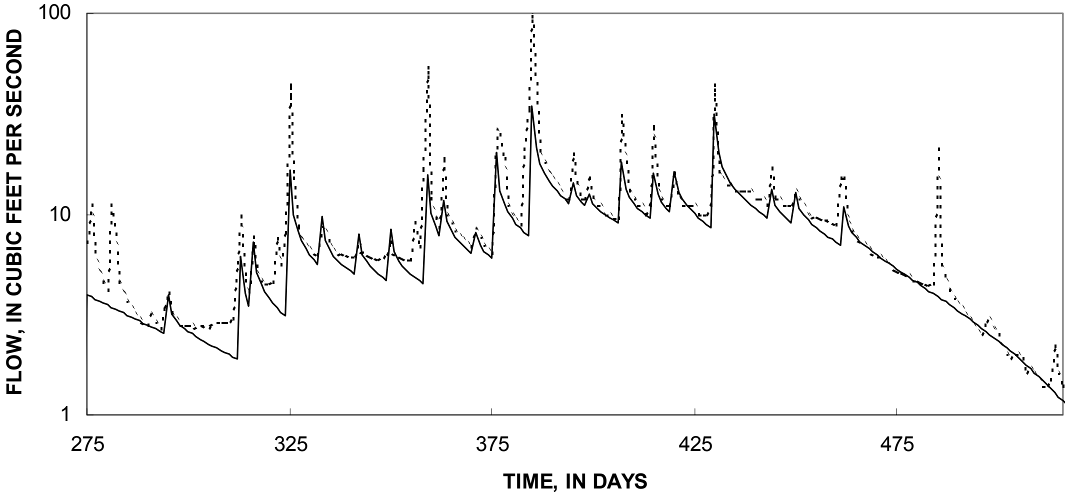


Figure 9. Ground-water discharge calculated by the PULSE program (solid curve) and measured streamflow (dotted curve) for Indian Creek near Troy, Alabama, for the first 200 days of calendar year 1963, after calibration and the simulation of ground-water evapotranspiration.

Another application spans the "recharge season" that can occur primarily in the cooler months, beginning late in one calendar year and continuing into the next calendar year. The program user specifies the time of interest as 2 years (two calendar years) when executing PREP and PULSE. The analysis might be restricted to a period that begins with low flow and ends with low flow, so that water balances can be calculated without the need to allow for changes in ground-water storage. In this case (fig. 10), the period is October 1, 1962, to May 31, 1963. The Julian date of the first day of the simulation is designated 274 (line 5 of *pulseIN.txt*), and the ground-water discharge on the first day of the simulation (line 3 of *pulseIN.txt*) should be designated on the basis of the streamflow record near the beginning of the time of interest. If there is negligible direct-surface runoff at the beginning of the period of interest, then this designation can be readily apparent from the streamflow hydrograph. In this application, there is direct surface runoff at this time, so the ground-water discharge designated at the beginning of the simulation is inferred to be some quantity less than streamflow. Eliminate recharge events that are automatically generated by the PREP program on days before day 274 and after day 516 (the latter is May 31, 1963). These steps are taken to simplify water-balance calculations. The tabulation in figure 10 is the input file *pulseIN.txt* that was used to generate the hydrograph. In this particular simulation, GWET equal to 0.006 in/d is initiated but not terminated. A simulation further into 1963 would require termination of GWET, similar to figure 9.



```
Recession index (days/log cycle)               100.00
Drainage area in square miles                    8.88
Ground-water discharge on first day (cfs)        4.00
Events on this hour of the day                      1
Julian date of first day                          274
Number of recharge events                          22
        Instant-
         aneous   Gradual
Day of  Recharge  Recharge
 year   (inches) (inch/day)
-------------------------
 295       0.07      0.00
 313       0.21      0.00
 316       0.20      0.00
 325       0.66      0.00
 333       0.21      0.00
 342       0.15      0.00
 350       0.19      0.00
 359       0.55      0.00
 363       0.21      0.00
 371       0.09      0.00
 376       0.70      0.00
 385       1.32      0.00
 395       0.16      0.00
 399       0.10      0.00
 407       0.45      0.00
 415       0.33      0.00
 420       0.32      0.00
 425       0.00    -0.006
 430       1.13      0.00
 444       0.20      0.00
 450       0.20      0.00
 462       0.20      0.00
```

Figure 10. Ground-water discharge calculated by the PULSE program (solid curve) and measured streamflow (dotted curve) for Indian Creek near Troy, Alabama, for the period October 1, 1962, to May 31, 1963.

The *pulsesum.txt* file, which is generated by PULSE, can be used to account for monthly recharge and discharge over a simulation period. From the simulation illustrated in figure 10, PULSE generates the *pulsesum.txt* file illustrated in figure 11. The first half of the file is a restatement of program-input data from the *pulseIN.txt* file. Monthly recharge resulting from instantaneous recharge events can be calculated from the amounts given in the second column of the tabulation. For example, the total recharge in October 1962 is calculated as 0.07 inches because the first recharge event is the only one occurring in October. The total for November is the sum of 0.21, 0.20, 0.66, and 0.21 inches because these recharge amounts occur on days in that month (days = 313, 316, 325, and 333).

Monthly ground-water discharge is calculated by the program and tabulated in the *pulsesum.txt* file next to monthly streamflow (fig. 11). Because the period of interest began on October 1, no recharge was simulated before this month (month 10), so the ground-water discharge in months 1-9 is zero and should be ignored. Because no recharge was simulated after May 1963, the ground-water discharge after month 17 should also be ignored. (It is noteworthy that some ground-water discharge is calculated after month 17 because of recharge during the period of interest.) Model-simulated ground-water discharge is 0.386 inch in October, 0.637 inch in November, and so on.

Monthly ground-water recharge and ground-water discharge for this simulation are illustrated in figure 12. The total ground-water recharge is 7.65 inches and total ground-water discharge (to the stream) is 7.39 inches. One other component of the water balance is GWET. As noted in the tabulations in figures 10 and 11, GWET was simulated as a “negative recharge” equal to 0.006 in/d, beginning on April 1 and continuing through the end of the period of interest (the end of May). The total GWET simulated during the period of interest is therefore 0.37 inch. In this simulation, the total inflow of water into the system (recharge) is close to the total outflow from the system (ground-water discharge to the stream plus GWET). This balance occurs because, as noted earlier, the simulation period begins and ends with a period of low ground-water discharge (figure 10). (It is noteworthy that quantities are given here with significant figures for the purpose of evaluating water-balance calculations, and that the precision of estimates of ground-water recharge, discharge, and GWET is much less than indicated by these values.)

The monthly rates illustrated in figure 12 indicate the variation in timing of ground-water recharge and ground-water discharge. Recharge shows greater variability than discharge, and trends in the latter tend to lag behind trends in the former. For example, the largest rate of recharge is in January, while the largest rates of discharge are in January, February, and March.

Most recharge in this simulation occurs in the form of instantaneous events (see tabulation on figure 10). The program user might experiment with a combination of instantaneous and gradual recharge. For example, the three small instantaneous amounts on days 333, 342, and 350 might be replaced with a gradual recharge event, which might provide a more realistic hydrograph of ground-water discharge during this period.

```
This is file "pulsesum.txt," from program pulse:
------------------------------------------------

Input variables:

Recession index, in days per log cycle  100.000000
Drainage area, in square miles.......   8.88000011
Flow on first day, in cfs ...........   4.00000000
Events occur on this hour of the day..  1
First day is this day of the year ....  274
Number of events ....................   22


Recharge events:

          INSTANT-
           ANEOUS   GRADUAL
  Day of   INPUT     INPUT
   year   (inches)   (in/day)
 ---------------------------
   295       0.07   0.00000
   313       0.21   0.00000
   316       0.20   0.00000
   325       0.66   0.00000
   333       0.21   0.00000
   342       0.15   0.00000
   350       0.19   0.00000
   359       0.55   0.00000
   363       0.21   0.00000
   371       0.09   0.00000
   376       0.70   0.00000
   385       1.32   0.00000
   395       0.16   0.00000
   399       0.10   0.00000
   407       0.45   0.00000
   415       0.33   0.00000
   420       0.32   0.00000
   425       0.00  -0.00600
   430       1.13   0.00000
   444       0.20   0.00000
   450       0.20   0.00000
   462       0.20   0.00000

The following quantities are in inches.

Instantaneous recharge in first year =   2.4500
Instantaneous recharge in second year =  5.2000
```

```
Monthly ground-water discharge and
streamflow, in inches:

           GW      Stream-
  Mon  Discharge   flow
 ----------------------------------
    1    0.000    2.978
    2    0.000    2.316
    3    0.000    3.861
    4    0.000    3.313
    5    0.000    1.225
    6    0.000    0.755
    7    0.000    0.445
    8    0.000    0.419
    9    0.000    0.532
   10    0.386    0.560
   11    0.637    0.900
   12    0.868    1.206
   13    1.507    2.256
   14    1.359    1.524
   15    1.544    1.702
   16    0.787    0.948
   17    0.302    0.328
   18    0.055    0.255
   19    0.000    0.187
   20    0.000    0.211
   21    0.000    0.191
   22    0.000    0.183
   23    0.000    0.404
   24    0.000    0.832
--------------------------
Total    7.443   27.531

 The following quantities are in inches.


        GW discharge in first year =    1.8907
        GW discharge in second year =   5.5522

  The baseline ground-water discharge that would
  have occurred in the absence of the events
  simulated here:  0.727477431
```

Figure 11. The output file *pulsesum.txt* generated by the PULSE program in the simulation illustrated in figure 10.

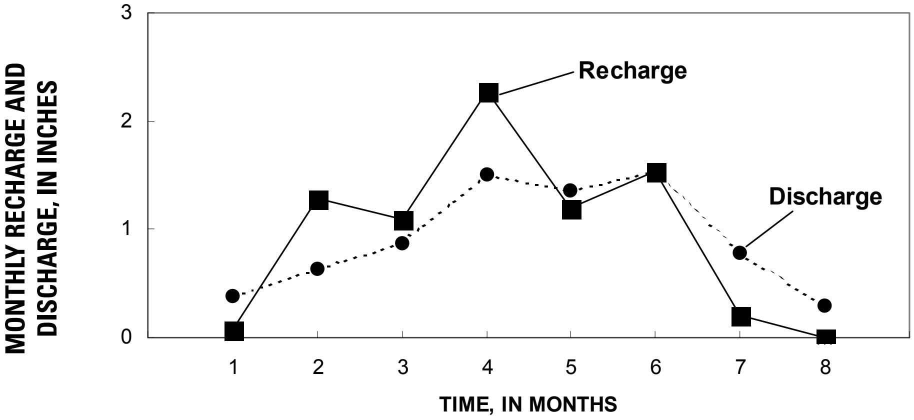


Figure 12. Monthly ground-water recharge and monthly ground-water discharge for the simulation illustrated in figures 10 and 11. (Month number 1 is October 1962.)

SUPPLEMENTAL INFORMATION ABOUT THE PULSE PROGRAM

This section covers topics that were not covered in the original report about the PULSE program (Rutledge, 1997). These include the use of ground-water levels and departures from the linear model of recession.

Ground-Water-Level Records and the Time of Recharge

The PULSE program can be used when streamflow is the only available dataset. Additional information such as ground-water levels can be used to evaluate the timing of recharge. The following paragraphs describe how to use the observed rises in water levels in observation wells within the basin. The following items should be considered before making such determinations.

(1) The observation well should be open to a surficial aquifer that is hydraulically connected to the stream and not to a deep confined aquifer that is isolated from the stream.
(2) Although the well may be open to a zone of the aquifer near the water table, the response to water-table fluctuation may be delayed for various reasons, such as poor hydraulic connection between the aquifer and the well.
(3) Data collected at a well represent a small area, but recharge estimates associated with the PULSE model should represent average conditions over the entire basin that is contributing to flow.

The ideal condition for using ground-water-level records with PULSE would be one in which several observation wells are hydraulically connected to zones near the water table, and each well is equipped with instrumentation for recording water levels (daily or more frequently). Analysis from limited ground-water-level data may not appreciably improve estimates. For example, data from only one well in a basin may be of limited value if recharge is not uniform in the basin.

An example dataset (fig. 13) shows ground-water levels in three wells within or near the drainage basin of Reedy Creek near Vineland, Florida, during calendar year 1995, referenced to the January 1, 1995, water level (O'Reilly, 1998). Data are processed here to show daily water-level rises that are caused by recharge (fig. 14). The water-level rise for a particular day is calculated as the difference between water level on that day and the water level on the previous day, and converting to zero if this difference is negative.

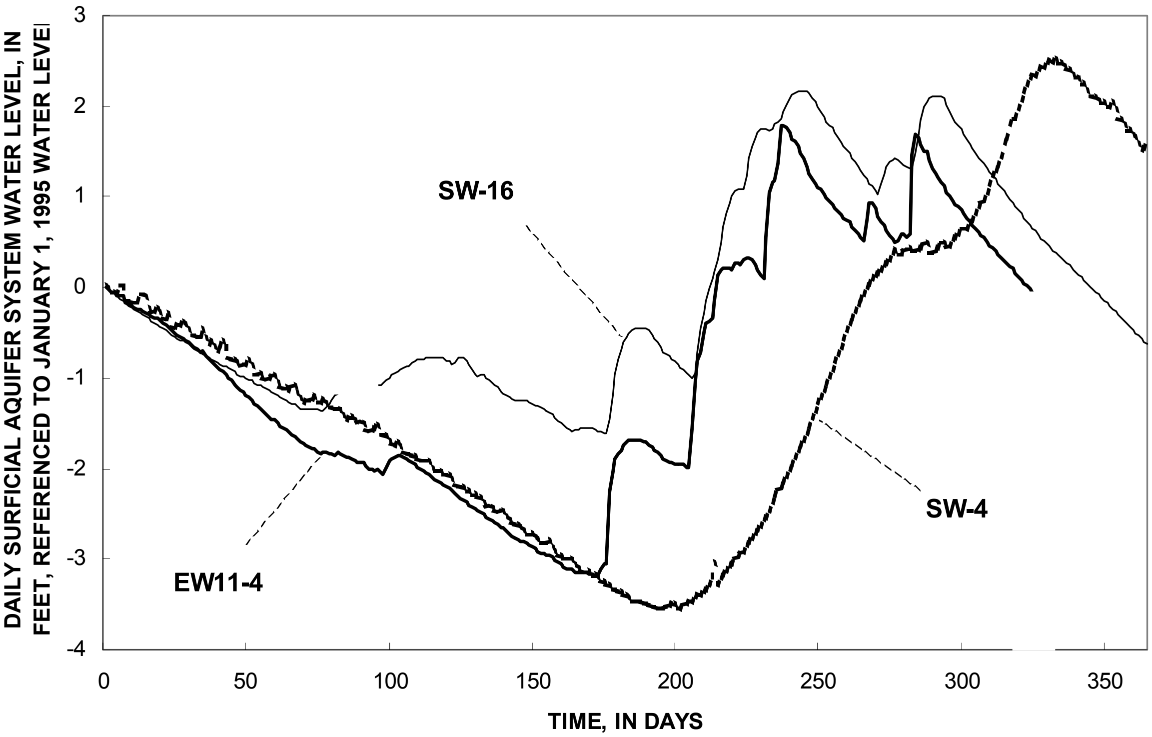


Figure 13. Water levels in three wells in or near the drainage basin of Reedy Creek near Vineland, Florida, in calendar year 1995. (From O'Reilly, 1998).

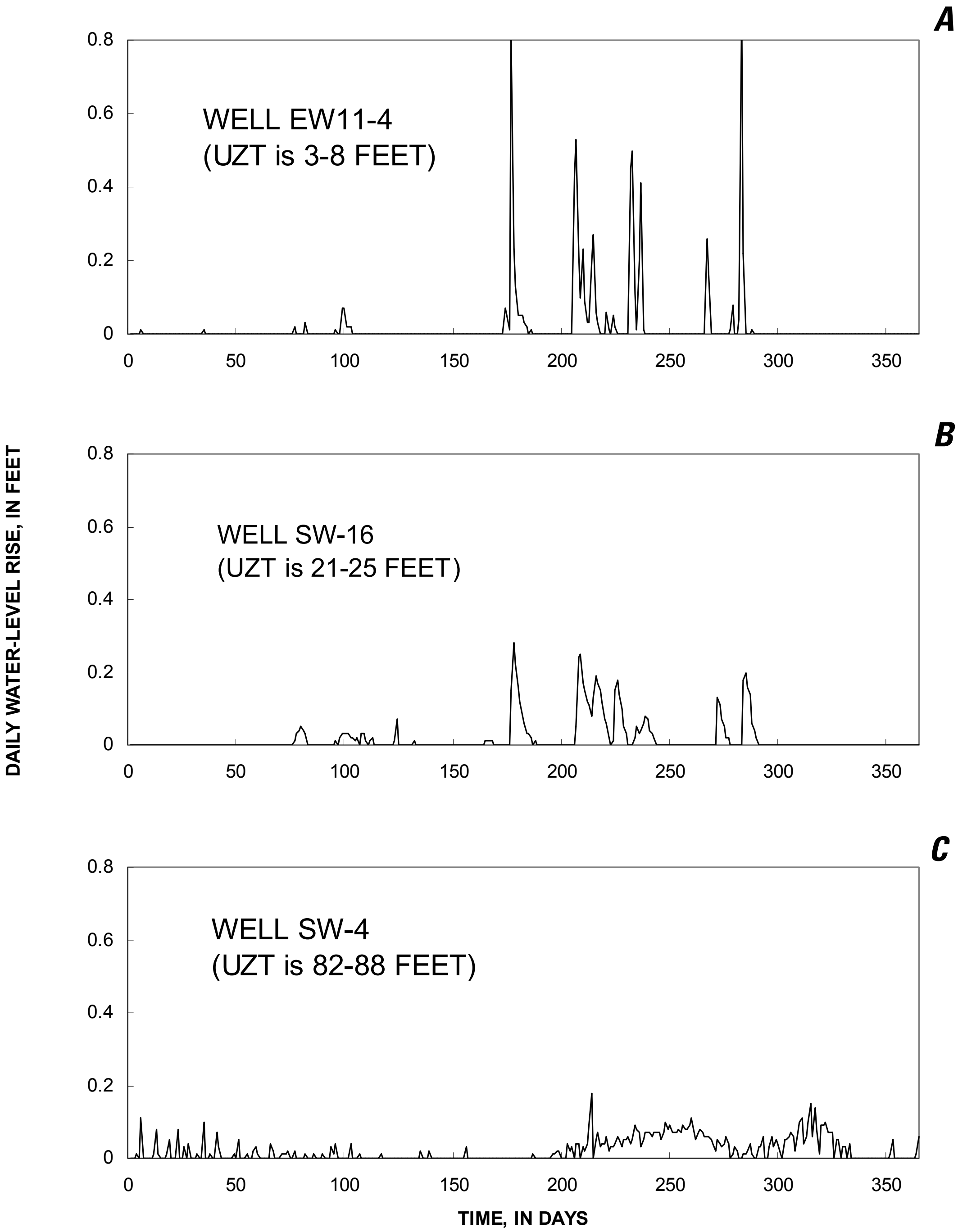


Figure 14. Daily water-level rise in three wells in or near the drainage basin of Reedy Creek near Vineland, Florida, in the calendar year 1995. (Modified from O'Reilly, 1998. UZT is unsaturated-zone thickness. There is a data gap for well EW 11-4 in the last 40 days of the year.)

The PULSE model is simulating recharge as a uniform process throughout the area contributing to flow at the streamflow station; therefore, there are weaknesses in the data analysis from this streamflow station. These weaknesses are evident from the considerable variation in the timing of recharge among the sites (fig. 14), which apparently results from the variation in unsaturated-zone thickness (UZT). Although these weaknesses are noteworthy, analysis might be conducted on the basis of the observation that the average UZT in the basin is about 10 ft (A.M. O'Reilly, U.S. Geological Survey, written commun., 1999.) The response observed at EW11-4 and SW-16 may be considered as representative of conditions that prevail throughout the basin (discrete recharge events) because the UZT at these sites brackets the basin average.

The PULSE model is used to estimate ground-water recharge and ground-water discharge to the stream, for a particular part of the streamflow hydrograph (fig. 15). A calibration process was used, similar to the process described earlier, except that recharge was designated primarily during periods of significant upward movements of ground-water level in EW11-4 (fig. 15*A*). Although the timing of recharge (fig. 15*B*) was constrained according to ground-water levels, the magnitude of each event was modified until the hydrograph of ground-water discharge was roughly equal to streamflow on days that can be considered primarily ground-water discharge (fig. 15*C*). Each event in this simulation (fig. 15*B*) was considered to be a gradual event lasting 3 days (roughly compatible with the duration of water-level rises shown in fig. 15*A*).

As noted, the use of one ground-water-level record to designate time of recharge may have limited value. For example, some recharge probably occurred in this basin during the storm on day 225. This recharge is apparent from the streamflow hydrograph (fig. 15*C*) and the significant upward movement of the water level at SW-16 (fig. 16). If data from only one well are used to identify the time of recharge in the basin, there can be errors; therefore, if using ground-water levels for these purposes, use data from numerous wells in the basin, if available.

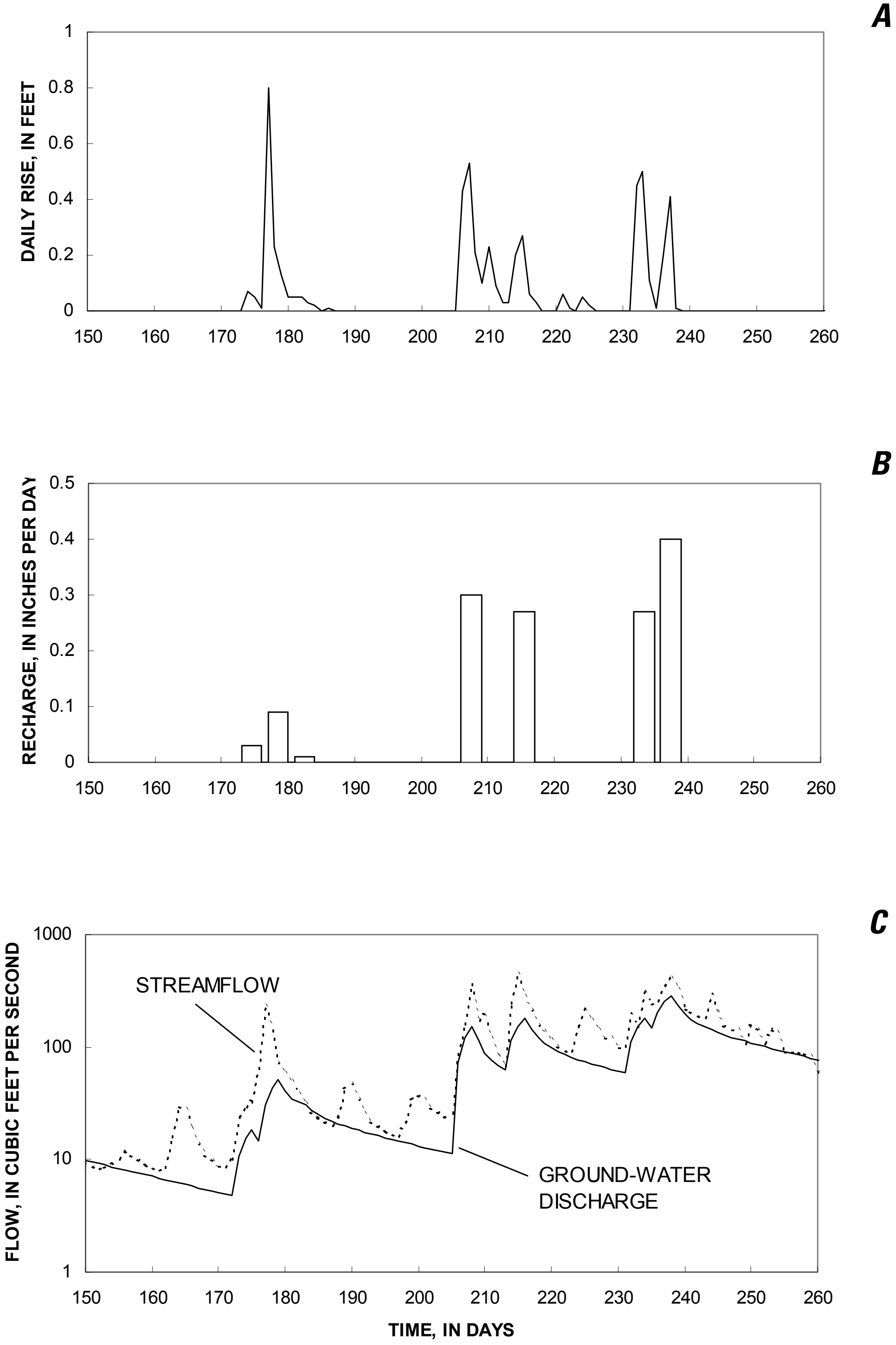


Figure 15. Daily water-level rise at well EW 11-4 (*A*) (modified from O'Reilly, 1998), recharge simulated using the PULSE program (*B*), and streamflow hydrograph for Reedy Creek near Vineland, Florida, with simulated ground-water discharge calculated by the PULSE program (*C*).

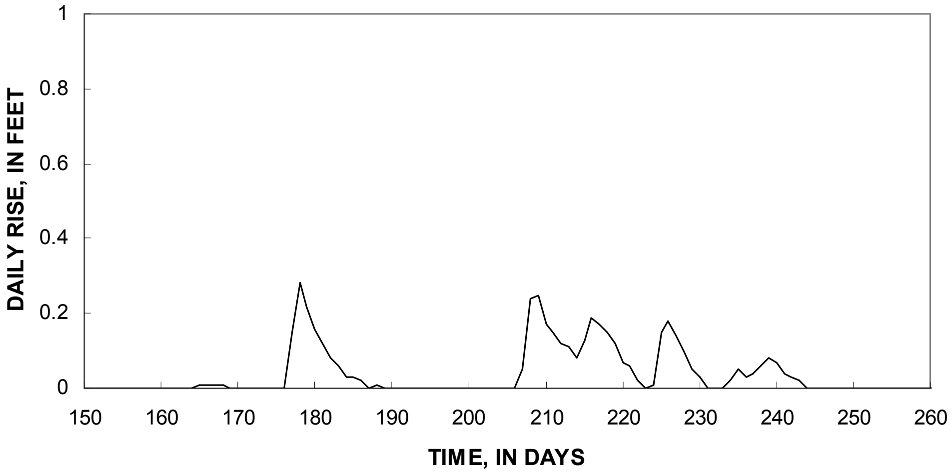


Figure 16. Daily water-level rise in well SW-16 (modified from O'Reilly, 1998).

Rises in ground-water levels (figs. 14 and 15*A*) indicate that most recharge can be restricted to small time increments in hydrologic settings with thin unsaturated zones. The recharge rate during these periods may be considerable. For example, the rate simulated during the four large recharge events in figure 15*B*, when expressed in cubic feet per second, may exceed streamflow during those events.

Although data from Reedy Creek near Vineland, Florida, are analyzed, characteristics of this basin may violate basic assumptions about the ground-water system that are required for the PULSE method to be valid. According to A.M. O'Reilly (U.S. Geological Survey, written commun., 1999), a considerable amount of the ground water in the basin leaves by way of flow to a deeper aquifer system, not by way of discharge to Reedy Creek. The PULSE program may be underestimating recharge to the water table. Nonetheless, the inferences stated previously about temporal distribution of recharge to the shallow aquifer may be reasonable.

The Relation between Ground-Water Level and Ground-Water Discharge

Ground-water-level data (L) collected within the basin might be used to evaluate the ground-water discharge (D) that is calculated using the PULSE program. For example, if L is larger during one particular period than it is during some other period, it might be expected that D should be larger during the first period than it is during the latter period. There can be limitations to such evaluation because the relation between D and L is not unique. A finite-difference flow model of a rectangular area that includes a line of drains along its length (fig. 17) can be used to demonstrate this relation. The area measures 8,000 ft by 2,000 ft (the latter represents the distance from the stream to the hydrologic divide), and all finite-difference cells measure 100 ft by 100 ft. Aquifer properties are uniform (transmissivity = 2,000 ft^2/d, storage coefficient = 0.05). The altitude of the drain is zero, and the conductance of each drain cell is 0.1 ft^2/s. Flow is induced by an initial uniform hydraulic head equal to 10 ft, and by 9 recharge events each uniformly inducing 1 inch of recharge to the model. MODFLOW-96 (Harbaugh and McDonald, 1996) is used to simulate the flow system, to calculate ground-water discharge to the line of drains, and to calculate ground-water level. A graph shows total flow to drains (D) and simulated L at the finite-difference cell that is in the middle of the model (fig. 18).

A unique relation between L and D appears to exist if results are selected from periods when there is linear recession on the semilog flow hydrograph (fig. 19). When all simulated times are considered, including during or immediately after recharge, the relation between L and D is notably not unique (fig. 20). These results demonstrate there will be limitations in an evaluation of PULSE-estimated ground-water discharge that is based solely on ground-water-level data, because a clear relation between L and D can be expected only during certain periods. These findings may extend to other interpretations that are based on ground-water-level data. For example, there can be errors in the “ground-water-level rating curve method” for estimating ground-water discharge, because this method requires a unique relation between L and D. The long-term variability in D may not be estimated with great confidence on the basis of the long-term variability of L at an observation well.

For more information, refer to Kraijenhoff van de Leur (1958). Analytical solutions can also be used to evaluate relations between L and D (Barlow and Moench, 1998).

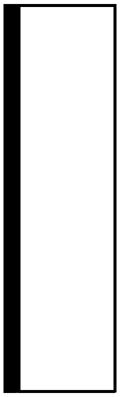


Figure 17. A finite-difference flow model. (The model area is a rectangle measuring 2,000 feet by 8,000 feet. The bold line on the left boundary represents a drain.)

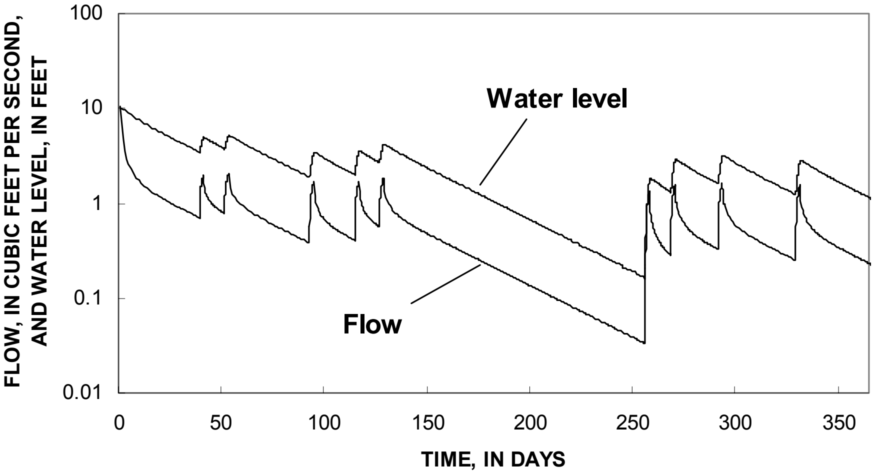


Figure 18. Simulated flow to drains and ground-water level, for the finite-difference simulation illustrated in figure 17. (Water levels are shown for the finite-difference cell in the middle of the model.)

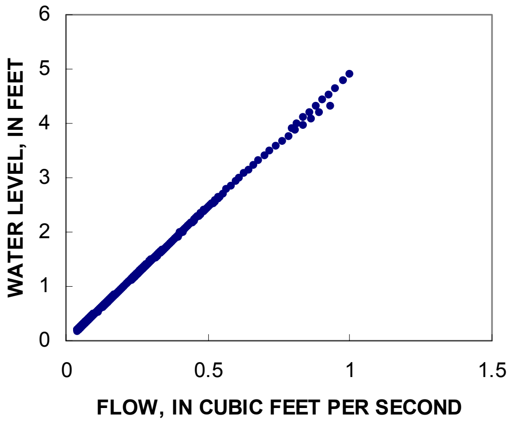


Figure 19. Relation between simulated ground-water discharge and water level for the finite difference simulation illustrated in figures 17 and 18. (This graph shows data from the model only during periods of linear recession on the semilog plot of flow: days 25 to 35, days 60 to 90, days 145 to 250, days 310 to 325, and days 350 to 365. Water levels are shown for the finite-difference cell in the middle of the model.)

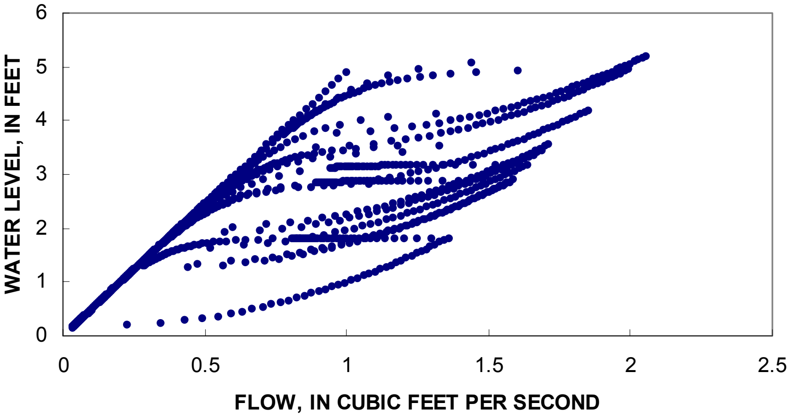


Figure 20. Relation between simulated ground-water discharge and water level for the finite-difference simulation illustrated in figure 17 and 18. (This graph shows all data from the model after day 25. Water levels are shown for the finite-difference cell in the middle of the model.)

Departures from the Linear Model of Recession

The formulations used by PULSE are derived from a cross-sectional flow model that calculates ground-water discharge per unit of stream length on the basis of designated transmissivity, storage coefficient, distance from the stream to the hydrologic divide, and increase in head caused by recharge in the ground-water system (Rorabaugh, 1964, equation 1). In effect, the program is based on simple geometry as illustrated in figure 21*A*. This finite-difference simulation has uniform transmissivity, storage coefficient, and distance from stream to hydrologic divide, so that its features are compatible with assumptions of the Rorabaugh model. A drain extends along the length of the model, so the direction of ground-water flow is one-dimensional. After a time has transpired since the last recharge event, the recession of ground-water discharge in simulation A will appear linear on the semilog plot, as is the case with the analytical model (Rorabaugh, 1964, fig. 1). The other simulations in figure 21 include features that cause departures from the linear model. These features include a zone of low transmissivity (fig. 21*B*) and a longitudinal component of flow (parallel to the stream) (fig. 21*C*). Natural flow systems may include such a component in the direction of the next higher-order stream. This condition is represented in simulation C as an additional drain at one end of the simulated area.

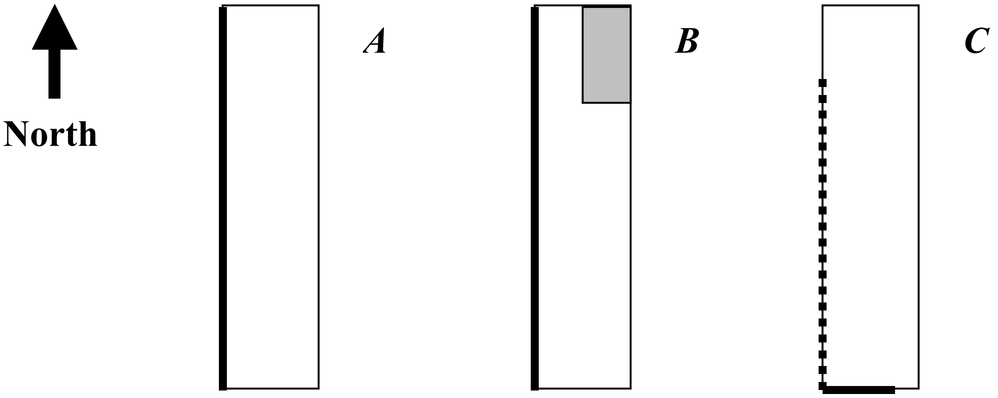


Figure 21. Three finite-difference simulations with (*A*) uniform hydrologic characteristics, (*B*) a zone of low transmissivity, and (*C*) a longitudinal component of flow. (Each measures 8,000 feet in the "north-south" direction and 2,000 feet in the "east-west" direction. The solid bar is a drain with altitude equal to zero. The dashed bar is a drain with altitude increasing from zero at the south end to 12.8 feet at its northernmost point. Storage coefficient is 0.05 in all simulations. Transmissivity is 2,000 feet squared per day in all locations except the grey area shown for simulation *B*, where it is 20 feet squared per day.)

MODFLOW-96 (Harbaugh and McDonald, 1996) is used to simulate flow in each case. Each model measures 8,000 ft in the north-south direction and 2,000 in the east-west direction. All finite-difference cells are squares measuring 100 ft on the side. The storage coefficient is 0.05 in all simulations, and transmissivity is 2,000 ft^2/d everywhere except the grey area shown for simulation B, where it is 20 ft^2/d. The altitude of all drains is zero, except for the drain in simulation C on the west side of the model. The altitude of this drain is zero at the south end and increases by 0.2 ft per 100 ft in the northern direction. The altitude of the northernmost drain is 12.8 ft. The conductance of each drain cell is 0.1 ft^2/s. Flow is initiated by designating hydraulic head above zero in each simulation. Head is 10 ft throughout the area in simulations A and B. Initial head is 16.4 ft in simulation C, so that the average amount by which head exceeds the altitude of the drain will be roughly similar to simulations A and B. All drain segments in the three simulations are intended to represent streams that receive ground-water discharge most of the time, with the exception of extreme drought. All simulations include nine recharge events, each lasting 2 days and each inducing 1 inch of recharge uniformly to the model. The resulting ground-water discharge is shown in figure 22.

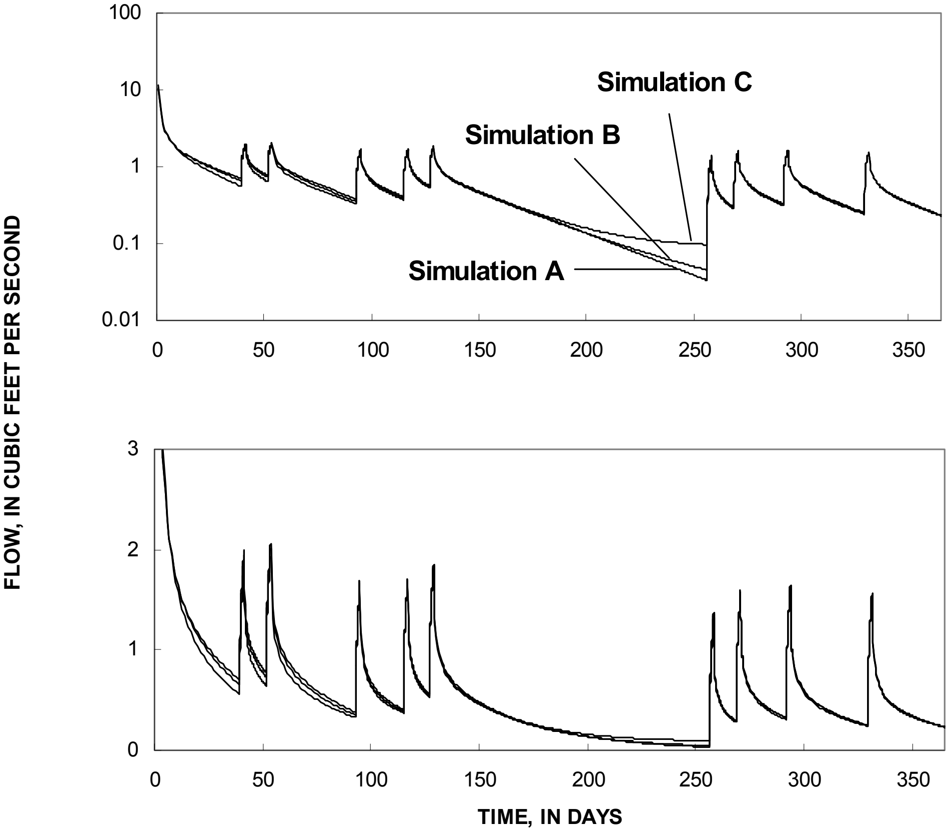


Figure 22. Hydrographs of ground-water discharge (log scale and linear scale) from the simulations illustrated in figure 21.

The three hydrographs are similar, with small departures evident especially after a considerable period has elapsed since the last recharge event. The log scale exaggerates the magnitude of these departures. If the PULSE program is used to estimate ground-water recharge for a streamflow record that exhibits departures from the linear model of recession, the following practices are suggested.

(1) The recession index, obtained from the log plot of flow, should not be obtained from periods of extreme low flow.
(2) The calibration process should include plotting flow on the linear scale in addition to plotting flow on the log scale. The quantitative significance of departures can be ascertained using the linear plot.

These suggestions apply if departures from the linear model of recession can be attributed to properties of the flow system illustrated in figures 21*B* and 21*C*. If anomalies result from other processes, the correct course of action may differ. If recession is nonlinear, the program will not accurately simulate ground-water discharge during periods of extreme low flow.

Results of these simulations might be used as a guide when equation 3 is used to obtain the recession index. The result is straightforward for simulation A, in which it is 93 days per log cycle (a=2,000 ft, S=0.05, T=2,000 ft^2/d). The value of “a,” however, may be open to various interpretations if there is longitudinal flow (21*C*). In this simulation, there is a longitudinal component of flow that extends 8,000 ft. Given the similarity of linear-scale hydrographs A and C in figure 22, it appears that the value of “a” that should be used in this equation is the one that corresponds to transverse flow if the purpose of the recession index is to use the Rorabaugh Model to estimate recharge.

For the simulation that includes the longitudinal flow component (fig. 21*C*), ground-water-level maps are shown at a time that is soon after a recharge event, and at a time following a long period of zero recharge (fig. 23). The longitudinal component is significant at both times, and dominant for the latter. Water-table-contour maps that exhibit longitudinal flow may lead to questions about the validity of the Rorabaugh flow model, which is based on transverse flow from the hydrologic divide to the stream. Nonetheless, it appears from previous discussion that the flow model can be valid in conditions of longitudinal flow.

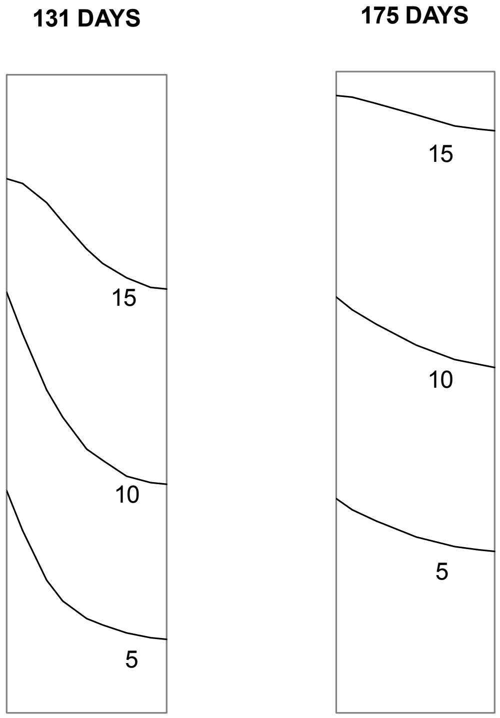


Figure 23. Ground-water levels in the finite-difference simulation illustrated in figure 21C, at 131 days and 175 days. (Numbered contours represent equal ground-water level, in feet.)

Given the basic assumptions of the Rorabaugh flow model, the recession of ground-water level (when expressed as vertical distance above the altitude of the outflow boundary) will be identical to the recession of streamflow, as long as the time considered is long after the last recharge event. This similarity is evident from the results of the finite-difference model shown in figure 18. It is noteworthy that the altitude of the outflow boundary (the drain) is specified as zero throughout its length in this simulation. When the recession of natural ground-water levels is considered, the altitude at the outflow boundary (the stream) may not be unique because of the slope of the stream channel; therefore, use caution when deriving the recession index from ground-water levels.

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