Possibility of Spontaneous Parity Violation in Hot QCD

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We argue that for QCD in the limit of a large number of colors, the axial U(1) symmetry of massless quarks is effectively restored at the deconfining phase transition. If this transition is of second order, metastable states in which parity is spontaneously broken can appear in the hadronic phase. These metastable states have dramatic signatures, including enhanced production of η and η' mesons, which can decay through parity violating decay processes such as $\eta \to \pi^0\pi^0$, and global parity odd asymmetries for charged pions. [S0031-9007(98)06613-7]

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It may be possible to observe the phase transition(s) from hadronic to quark and gluon degrees of freedom through the collisions of heavy nuclei at ultrarelativistic energies. In the region of central rapidity, the relevant phase transitions are those at nonzero temperature; these phase transitions can be studied by numerical simulations of lattice gauge theory. At present, simulations indicate that for three colors coupled to light quarks, there is at most one phase transition, controlled by the chiral dynamics of the light quarks [1]. The order of the phase transition in QCD, in which two flavors are very light, and one flavor not too heavy (up, down, and strange), is still unsettled.

The nature of the chiral phase transition depends crucially upon the dynamics of the axial U(1) symmetry of the light quarks [2,3]. Notably, for two massless flavors, if the axial U(1) symmetry is not restored about the chiral phase transition, then the transition can be of second order; if it is restored, the transition may be driven first order by fluctuations.

There are two approaches to understanding the dynamical breaking of the axial U(1) symmetry. The first assumes that the dominant fluctuations are semiclassical instantons [4–7]. The second is based upon the large N limit of an SU(N) gauge theory [8–15], and assumes that the dominant fluctuations are not semiclassical, but quantum.

At zero temperature, both approaches give a reasonably successful phenomenology for the η' mass and related processes. In this Letter we show that these two approaches give radically different predictions at nonzero temperature. In instanton models of the hadronic vacuum [4], the topological susceptibility is essentially constant below the phase transition, and drops off only *above* the phase transition. We argue that at large N, the topological susceptibility essentially vanishes *at* the phase transition. If the deconfining phase transition is of second order, then the axial U(1) symmetry is dynamically restored as the phase transition is approached from below. Under this assumption, using a nonlinear sigma model [11–15] we show that metastable states with spontaneous parity violation arise in the hadronic phase, and would produce striking experimental signatures.

The large N limit of SU(N) gauge theories is believed to be a reasonable approximation even for $N = 3$ [8]. We assume that confinement holds for all N, with the masses of mesons and glueballs of order one as $N \to \infty$; interactions between mesons and/or glueballs are suppressed by powers of $1/N$.

Holding the number of quark flavors fixed as $N \to \infty$, at large N the $\sim N^2$ gluons dominate the $\sim N$ quarks. Taking the degeneracy of hadronic bound states to be of order one, the gluonic free energy changes from $\sim N^0$ in the hadronic phase, to $\sim N^2$ in the deconfined phase. Thus the gluonic part of the free energy can be used to define the temperature of the transition, at $T = T_d \sim N^0$ [16,17]. We further assume that any other transitions in the theory also occur at T_d. Given the huge change in the free energy, any other possibility seems baroque, at best.

In the pure glue theory, the topological susceptibility $\lambda_{\rm YM}(T) \equiv \partial^2 F(\theta, T)/\partial\theta^2 = \int d^4x \, \langle Q(x)Q(0)\rangle$, where $F(\theta, T)$ is the free energy, and the θ parameter is conjugate to the integral of the topological charge density, $Q(x) = (g^2/32\pi^2)\,\mathrm{tr}(G_{\alpha\beta}\tilde{G}^{\alpha\beta})$. At zero temperature, the free energy reduces to the energy $F(\theta, 0) = E(\theta)$.

Because $Q(x) = \partial_\alpha K^\alpha$, where K^α is the (gauge variant) topological current, $\lambda_{\rm YM}(T)$ vanishes order by order in perturbation theory. At zero temperature, Witten suggested that quantum fluctuations generate a nonzero value, $\lambda_{\rm YM}(0) \sim N^0$ [9]. At high temperature, the theory is asymptotically free and so weakly coupled, with electric fluctuations suppressed by Debye screening. Thus at high temperature, $\lambda_{\rm YM}(T)$ is unequivocally calculable by semiclassical means, using instantons [5,6]. With g^2 the gauge coupling, the instanton action is $8\pi^2/g^2$; as g^2N is held fixed when $N \to \infty$, $\lambda_{\rm YM}(T) \sim \exp(-aN)$, $a = 8\pi^2/g^2N$. This naive picture was verified by Affleck in a soluble asymptotically free theory, the CP^N model in 1 + 1 dimensions [5].

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Thus for gauge theories, $\lambda_{\rm YM}(T)$ changes from $\sim N^0$ at low temperatures, to $\sim\exp(-aN)$—which at large N is essentially zero—at high temperature. Appealing to simplicity, we assume that this change happens at the deconfining transition, $\lambda_{\rm YM}(T) \sim 0$ for $T > T_d$ [5,14].

How $\lambda_{\rm YM}(T)$ vanishes as $T \to T_d^-$ depends upon the order of the deconfining phase transition. If the deconfining phase transition is of first order, then since all interactions at large N are suppressed by $1/N$, presumably $\lambda_{\rm YM}(T) = \lambda_{\rm YM}(0)$ for $T < T_d$.

While there is some evidence that the deconfining phase transition is of first order for all $N \geq 4$, this conclusion may be premature [18]. Following the conjecture of [19], we henceforth assume that the deconfining phase transition is of second order at large N. This requires that the phase transition is driven by a Hagedorn spectrum [17]. Adding $N_f \geq 2$ flavors of massless quarks, we assume that chiral symmetry is spontaneously broken at zero temperature [8], and that the deconfining phase transition forces chiral symmetry restoration at $T = T_d$. Since we shall argue that $\lambda(T) \to 0$ as $T \to T_d^-$, the chiral transition is driven first order by fluctuations [2,3]. At large N, however, since the mesonic couplings start out small, $\sim 1/N$, fluctuations can only drive the chiral transition first order within a narrow critical region, $\sim 1/N$. Thus the latent heat for the chiral transition is $\sim N^0$, and relative to the $\sim N^2$ for the gluons in the deconfining phase, it is a very weakly first order transition. Consequently, we let quantities associated with the chiral transition vary with temperature, but only as in mean field theory. Notably, the pion decay constant f_π, which is $\sim\sqrt{N}$, is assumed to decrease as $f_\pi(T) \sim (T_d - T)^{1/2}$ when $T \to T_d$.

While the pure glue theory depends upon θ, the addition of massless quarks must cancel any θ dependence. Witten showed that this happens at large N by the appearance of a light meson, the η' [9]. We generalize this to nonzero temperature, to estimate how $\lambda_{\rm YM}(T)$ and the η' mass vanish as $T \to T_d^-$.

At large N, at zero temperature any gauge invariant correlator is saturated by the exchange of single glueballs and mesons [8]. Normally this changes in a thermal bath, due to scattering off states in the thermal distribution. The hadronic phase of large N QCD, however, is "cold": chiral symmetry is restored not at a scale set by the pion decay constant $f_\pi \sim \sqrt{N}$ (as in, say, the sigma model at large N), but at a much lower temperature, given by the deconfining transition $T_d \sim N^0$. Thus we can use the same type of arguments as at zero temperature, simply allowing any quantity which enters to be temperature dependent.

We start by following Veneziano [10], and define $\lambda_{\eta'}$ as the form factor between the topological current and the η' meson, $\langle 0|K^\alpha|\eta'\rangle = i(\sqrt{N_f/N}\,)p^\alpha \lambda_{\eta'}(T)$, with p^α the momentum of the η' meson. This form factor is precisely analogous to the coupling of π^0 to two photons. Following [20], to one loop order in a constituent quark model, the (anomalous) coupling of the η' to two, or indeed any finite number of gluons, vanishes as chiral symmetry is restored, like $\lambda_{\eta'}(T) \sim f_\pi(T) \sim (T_d - T)^{1/2}$ as $T \to T_d^-$. Using Veneziano's relation, $m^2_{\eta'}(T) = N_f \lambda^2_{\eta'}(T)/N$, we find the η' mass vanishes as $m^2_{\eta'}(T) \sim (T_d - T)$ when $T \to T_d^-$.

We now use Witten's formula [9] for the η' mass, $m^2_{\eta'}(T) = 4N_f \lambda_{\rm YM}(T)/f^2_\pi(T)$. This relation shows that $\lambda_{\rm YM}(T)$ and $f_\pi(T)$ must vanish at the same point, but not much else. Since Veneziano's formula tells us how $m^2_{\eta'}(T)$ vanishes, though, we can use this to deduce how the free energy depends upon θ about T_d:

$$F(\theta, T) \sim (1 + c\theta^2)(T_d - T)^{2-\alpha}, \qquad T \to T_d^-, \quad (1)$$

for some positive constant c, $|\theta| < \pi$. That the θ dependence is only quadratic is characteristic of large N [9]. Then $\lambda_{\rm YM}(T) = \partial^2 F(\theta, T)/\partial\theta^2 \sim (T_d - T)^{2-\alpha}$ and $m^2_{\eta'}(T) \sim (T_d - T)^{1-\alpha}$ as $T \to T_d^-$. Because the critical exponent $\alpha \neq 0$, this does not quite agree with our estimate using Veneziano's formula. We trust (1), since the calculations of [20] are only at one loop, and so basically mean field. Even so, as $\alpha \sim -0.013$ [2], this difference is small, and does not alter the conclusion that $\lambda_{\rm YM}(T)$ and $m_{\eta'}(T)$ vanish at the phase transition, with T_d and α independent of θ.

Previously, Affleck [5] and also Davis and Matheson [14] argued that $\lambda_{\rm YM}(T)$ vanishes when $T > T_d$; our contribution is to estimate how it vanishes as $T \to T_d^-$ if the deconfining phase transition is of second order. If the deconfining transition is of first order, then as the hadronic phase is cold, presumably $\lambda_{\rm YM}(T) = \lambda_{\rm YM}(0)$ for $T < T_d$, at which point it drops discontinuously to zero. (For an alternate view, with $T_d \neq T_\chi$, see Meggiolaro [15].)

An effective nonlinear sigma model which incorporates the breaking of the axial U(1) symmetry can be constructed [12–15]. With U a U(N_f) matrix satisfying $U^\dagger U = 1$, the potential for U is

$$V(U) = \frac{f_\pi^2}{2}\{\mathrm{tr}[M(U + U^\dagger)] - a(\mathrm{tr}\ln U - \theta)^2\}; \quad (2)$$

M is the quark mass matrix. When $M = 0$, $m^2_{\eta'} \sim a$, so $a \sim \lambda^2_{\eta'}/N$. (Our $a = a/N$ in Refs. [11–15].)

Instanton processes are often modeled using a linear sigma model with a field Φ, by introducing a term $\sim e^{i\theta}\det(\Phi)$ [2–4]; this term is well behaved in both the low and high temperature phases. In contrast, in (2) the term $\sim(\mathrm{tr}\ln U)^2$, which breaks the axial U(1) symmetry, makes sense only in the low temperature phase, and is singular if the vacuum expectation value (VEV) of U vanishes. At large N, however, everything fits together: since $a(T) \to 0$ as $T \to T_d^-$, there is simply no such term in the high temperature phase. Moreover, at large N, even at zero temperature $\sim\det(\Phi)$ must be dropped, since it is inconsistent with the θ dependence [13].

Taking $M_{ij} = \mu_i^2 \delta^{ij}$, any VEV of U can be assumed to be diagonal, $U_{ij} = e^{i\phi_i}\delta^{ij}$; then

$$V(\phi_i) = f_\pi^2 \left[-\sum_i \mu_i^2 \cos(\phi_i) + \frac{a}{2}\left(\sum_i \phi_i - \theta\right)^2 \right], \tag{3}$$

which is minimized for $\mu_i^2 \sin(\phi_i) + a(\sum \phi_i - \theta) = 0$. Note that as $\sum \phi_i$ arises from tr ln U, it is defined modulo 2π.

All of the parameters in (3) are temperature dependent. When $a = 0$, the Goldstone boson masses $\mu_i^2 \sim m_i \langle \overline{q}q \rangle / f_\pi^2$, with m_i the current quark masses, and $\langle \overline{q}q \rangle$ the chiral order parameter. When $M \neq 0$, there is no true critical point, but we can use mean field theory to estimate that as $T \to T_d^-$, f_π and a decrease, while the μ_i all uniformly increase: $f_\pi^2(T) \sim a(T) \sim (T_d - T)$, $\langle \overline{q}q \rangle \sim (T_d - T)^{1/2}$, and $\mu_i^2(T) \sim m_i/(T_d - T)^{1/2}$. The solutions are independent of f_π, and depend only upon the ratio a/μ_i^2. In mean field theory this ratio is independent of flavor, and scales as $a(T)/\mu_i^2(T) \sim (T_d - T)^{3/2}$.

Several authors have studied how the VEV's for the ϕ_i change at zero temperature when $\theta \neq 0$ [11–13]. Instead, we consider $\theta = 0$, and follow Witten [13] to investigate metastable solutions at small a. Related metastable states have been discussed by Shifman [21].

For a single flavor, the vacua are at $\phi = 0, \pm 2\pi$, etc. By balancing $\mu^2 \sin(\phi)$ against $a\phi$, however, for small a/μ^2 it is easy to show that there are other solutions with $\phi \neq 0$. These solutions have higher potential energy, and so are local but not global minima. Numerically, we find that the first metastable state occurs when $a < a_{\rm cr}$, with $a_{\rm cr}/\mu^2 \sim 0.217$, and $\phi_c \sim 4.493$; the ϕ field is massless about ϕ_c. As $a \to 0$, $\phi \to 2\pi$, which is equivalent to $\phi = 0$. There is an infinite tower of metastable states; we consider only that with lowest energy.

These metastable states are like regions with nonzero θ, and so spontaneously break CP symmetry. Under charge conjugation, $\phi \to +\phi$, while under parity, $\phi \to -\phi$. Although there is a solution at $-\phi$, when $-\phi$ does not differ from $+\phi$ by a shift of 2π, parity is spontaneously violated. This does not conflict with Vafa and Witten [22], who showed that at $\theta = 0$, parity is not spontaneously violated in the QCD vacuum. Their theorem generalizes to the thermodynamic minimum at nonzero temperature (although not at nonzero quark density), but it does not constrain metastable states.

The appearance of metastable states when $N_f \geq 2$ is somewhat subtle. To illustrate the basic point, we consider a two flavor model in which $\mu_1^2 = 0$ and $\mu_2^2 = \mu^2$. If the two flavors decoupled [13], one might guess the solution $\phi_1 = 0$ and ϕ_2 as for one flavor. The equation of motion for ϕ_1, however, forces $\phi_1 + \phi_2 = 0$: when $\mu_1^2 = 0$, for any value of a there is only the trivial solution, $\phi_1 = \phi_2 = 0$, modulo 2π.

This example shows that there are no metastable states if any quark mass vanishes. This is natural: After all, there is also no θ dependence when any quark mass vanishes, and these metastable states are similar to regions with nonzero θ. Analogously, when all quarks have nonzero mass, metastable states appear only when a is small relative to the *lightest* quark mass. Thus in QCD, for metastable states to occur a must be small relative not to the strange quark mass, but to the up and down quark masses. For this reason, $a(T)$ must become *very* small near the phase transition.

The potential of (3) can be used to obtain a qualitative estimate. Let m_u, m_d, and m_s be the masses of the up, down, and strange quarks. The charged pions and the kaons are unaffected by the anomaly; with $m_{\pi^\pm}^2 \sim m_u + m_d$ and $m_K^2 \sim m_{u,d} + m_s$, and assuming that $m_u = m_d/2$, $m_{\pi^\pm} = 140$ MeV and $m_K \sim 496$ MeV give $\mu_1^2 = (114\ \text{MeV})^2$, $\mu_2^2 = (161\ \text{MeV})^2$, and $\mu_3^2 = (687\ \text{MeV})^2$. We take Veneziano's [10] value of $a = (492\ \text{MeV})^2$, and numerically diagonalize the mass matrix in (3) to obtain $m_{\pi^0} \sim 139$ MeV, $m_\eta \sim 501$ MeV, and $m_{\eta'} \sim 983$ MeV. This is reasonably close to the experimental values of $m_\eta \sim 548$ MeV, and $m_{\eta'} \sim 958$ MeV.

Taking the ratios of the μ_i^2 as fixed, and varying a/μ_1^2, we studied numerically the appearance of the lowest energy metastable state. For the sake of discussion, we take the zero temperature μ_i^2; thus only the ratios of masses are believable, with all true masses larger by some uniform factor, $\sim \mu_i^2(T)/\mu_i^2(0) \sim (T_d - T)^{-1/2}$. The masses of the π^0, η, and η' are read off by diagonalizing the mass squared matrix obtained from (3). The masses of the charged pions are $m_{\pi^\pm}^2 = \mu_1^2 \cos(\phi_1) + \mu_2^2 \cos(\phi_2)$. We ignore changes in the kaon masses; as ϕ_3 is small, their masses do not change much.

We find that there is a metastable solution when $a/\mu_1^2 < 0.2467$, but it is unstable in the π^0 direction unless $a < a_{\rm cr}$, $a_{\rm cr}/\mu_1^2 \sim 0.2403$. At $a_{\rm cr}$, $\phi_1 \sim 4.47$, $\phi_2 \sim -0.524$, and $\phi_3 \sim -0.028$; the π^0 is massless at $a_{\rm cr}$, while $m_{\pi^\pm} \sim 106$ MeV, $m_\eta \sim 150$ MeV, and $m_{\eta'} \sim 687$ MeV. As $a \to 0$, the metastable state becomes equivalent to the vacuum, as $\phi_1 \to 2\pi$, ϕ_2 and $\phi_3 \to 0$. At $a = 0$, $m_{\pi^0} \sim 114$ MeV, $m_{\pi^\pm} \sim 140$ MeV, $m_\eta \sim 161$ MeV, and $m_{\eta'} \sim 687$ MeV. In the thin wall approximation [23], the decay rate of the metastable state is $\Gamma \propto \exp(-F_c/T)$, where $F_c \sim (32\sqrt{2}/3)\,(\mu_1^3 f_\pi^2/a^2)$.

Putting in the zero temperature values, in order for metastable states to occur, near T_d the ratio of a/μ_1^2 must be about 1% of its value at zero temperature. It is not clear if this is possible in QCD, but of course this estimate is manifestly model dependent. Since $F_c \sim 1/a^2$, at small a the metastable states live a very long time; thus in heavy ion collisions, metastable states do not decay by bubble nucleation; instead, as the hot phase cools, the value of a changes dynamically, and the metastable state rolls smoothly into the true vacuum.

When a becomes very small, there are several features common to both the ground state and the metastable states. First, the neutral Goldstone bosons are eigenstates

not of SU(3), but of flavor [2,11]: at $a = 0$, $\pi^0 \sim \overline{u}u$, $\eta \sim \overline{d}d$, and $\eta' \sim \overline{s}s$. This generates maximal isospin violation: the neutral pion is lighter than the charged pions, and so produced more readily. This effect is much stronger for the metastable states, since the π^0's are massless at $a_{\rm cr}$, and so very light for $a \sim a_{\rm cr}$. Similarly, the η and η' also become light in both phases; this is especially true for the η, as it sheds all of its strangeness. It is not clear how much lighter the η' becomes, given the overall increasing mass scale of $\mu_i^2(T)$. Light η and η' mesons are produced more readily [24], and can be observed either directly, through $\gamma\gamma$ decays [24], or indirectly, through pion Bose-Einstein correlations [25].

There are two types of experimental signatures special to the formation of a parity violating phase. The first is that decays normally forbidden by parity are allowed (for a related phenomenon, see [26]). Kinematically, $\eta \to \pi^+\pi^-$ is not allowed, but $\eta \to \pi^0\pi^0$ is. The processes $\eta' \to \pi^+\pi^-$ and $\eta' \to \pi^0\pi^0$ are also allowed; however, as the η' is almost pure $\overline{s}s$, this is suppressed by $\sim m_u/m_s$.

There are also global variables which are sensitive to the dynamics of a parity violating phase. It can be shown that the interactions of charged pions differ if there are regions with ϕ_1 and $\phi_2 \neq 0$, which change in either space or time. This is similar to the propagation of charged particles in a background magnetic field: an e^+e^- pair, produced back to back, are both deflected in the same direction by a magnetic field. A parity odd asymmetry could be observed by summing over all $\pi^+\pi^-$ pairs in a given event,

$$\mathcal{P} = \sum_{\pi^+\pi^-} \frac{[\vec{P}_{\pi^+} \times \vec{P}_{\pi^-}] \cdot \vec{z}}{|\vec{P}_{\pi^+}||\vec{P}_{\pi^-}|}; \qquad (4)$$

$\vec{z}$ is the beam axis of the collision, and $\vec{P}_{\pi^\pm}$ are the pion momenta. $\mathcal{P}$ is like handedness in jet physics [27].

These metastable domains might be of cosmological interest. A region with $\phi_i \neq 0$ implies $\alpha_s G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \sim \lambda_{\rm YM}(T)\sum \phi_i$ [11]; likewise, the coupling to electromagnetism should also generate $\alpha \vec{E} \cdot \vec{B} \sim \alpha_s G_{\alpha\beta}\widetilde{G}^{\alpha\beta}$. Thus if the entire Universe fell into such a metastable domain, it would generate a nonzero value for a cosmological magnetic field at the time of the QCD phase transition [28].

To summarize, in the limit of large N the topological susceptibility (essentially) vanishes in the deconfined phase. If the deconfining transition is of first order, then the susceptibility is constant in the hadronic phase; if of second order, the susceptibility vanishes as $T \to T_d$ (1). If the latter happens, metastable states in which parity is spontaneously broken can appear, although in a nonlinear sigma model, one must be very close to the phase transition for them to occur. It is not clear if the large N expansion is a good guide to this physics when $N = 3$; certainly at $N = 3$, the susceptibility will be nonzero in the high temperature phase. Nevertheless, the large N expansion provides a qualitative guide against which other models can be tested.

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