Exact Z^2 scaling of pair production in the high-energy limit of heavy-ion collisions

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The two-center Dirac equation for an electron in the external electromagnetic field of two colliding heavy ions in the limit in which the ions are moving at the speed of light is exactly solved off the light fronts, and nonperturbative amplitudes are obtained for free electron-positron pair production. We find the condition for the applicability of this solution for large but finite collision energy, and use it to explain recent experimental results. The observed scaling of positron yields as the square of the projectile and target charges is a result of a cancellation of a nonperturbative charge dependence and holds as well for large coupling. Other observables would be sensitive to nonperturbative phases. [S0556-2813(99)03505-0]

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There is a very small number of problems in physics that yield to an exact solution. Remarkably, electromagnetic production of free electron-positron pairs in the high-energy limit of peripheral, heavy-ion collisions can be described by a two-center, time-dependent Dirac equation which can be solved off the light fronts exactly and in closed form [1]. In this paper, we place this exact solution in its physical context and discuss its implications for recent experiments performed at CERN's SPS [2] and possible future experiments at new facilities such as BNL's RHIC and CERN's LHC. (For relevant reviews of this field, see Ref. [3].)

Perturbative calculations have been held as suspect at high energies and for the heaviest projectiles, e.g., because the coupling constant is not small ($Z\alpha \sim 0.6$) [2–5]. It is therefore surprising that positron yields observed from pair production in peripheral collisions of Pb^{82+} ions at 33 TeV on a Au target scale as $Z_T^2 Z_P^2$ and that the observed positron-momentum distributions display an overall good agreement with leading-order perturbation-theory calculations [2]. The exact, nonperturbative solution presented here explains these effects and is also consistent with the observed enhancement of the positron yields at small positron momentum. For future experiments, we indicate what observables would show complete agreement with second-order perturbation theory and what other observables should be measured in order to detect nonperturbative effects.

The relativistic scattering problem of an electron in the external field of two pointlike charges (ions), moving on parallel, straight-line trajectories in opposite directions at speeds which approach the speed of light, and at an impact parameter $2\vec{b}$, reduces in the high-energy limit to

$$i\frac{\partial}{\partial t}|\Psi(\vec{r},t)\rangle = [\hat{H}_0 + \hat{W}_A(t) + \hat{W}_B(t)]|\Psi(\vec{r},t)\rangle, \qquad (1)$$

$$\hat{H}_0 \equiv -i\check{\alpha}\cdot\vec{\nabla} + \check{\gamma}^0,$$

$$\lim_{\gamma\to\infty}\hat{W}_A = (\mathrm{I}_4 - \check{\alpha}_z)Z_A\alpha\,\delta(t-z)\ln\left[\frac{(\vec{r}_\perp - \vec{b})^2}{b^2}\right],$$

$$\lim_{\gamma\to\infty}\hat{W}_B = (\mathrm{I}_4 + \check{\alpha}_z)Z_B\alpha\,\delta(t+z)\ln\left[\frac{(\vec{r}_\perp + \vec{b})^2}{b^2}\right],$$

where $\gamma \equiv 1/\sqrt{1-\beta^2}$, and $\beta \equiv v/c$ is the speed of the charges Z_A and Z_B. (γ in the collider frame is related to γ_T in the target frame by $\gamma_T = 2\gamma^2 - 1$.) Equation (1) is written in the collider frame, with natural units ($c=1$, $m_e=1$, $\hbar=1$), α is the fine-structure constant, and $\check{\alpha}$ and $\check{\gamma}^\mu$ are Dirac matrices in the Dirac representation.

For Eq. (1) to apply, one assumes first that the ions are sufficiently energetic and massive so that the deviation from straight-line trajectories can be neglected [3]. Second, one assumes peripheral collisions without nuclear interactions. Purely electromagnetic events can be distinguished experimentally from nuclear events by observing the full-energy projectile ion after the collision in coincidence with the produced electrons or positrons [2]. One also assumes that the ions are moving at the speed of light. Indeed, for the recent experiments at CERN [2], $\beta \approx 0.99$, and for future experiments possible at RHIC, $\beta \approx 0.9999$. The asymptotic solutions to the two-center Dirac equation for an electron distant from both ions (i.e., $z \neq \pm t$) factor in the $\beta \to 1$ limit into a product of a Dirac plane wave and a space-time-dependent phase factor which depends on the ions' charge [6]. Considering only such asymptotic channels off the light fronts, a phase (or gauge) transformation may be performed to a new (short-range) representation in which the asymptotic dynamics, arising from the interaction of the electron with the distant ions, is included into the states [6,8]. In this representation, the interaction of the electron with each ion is of short-range form in the longitudinal direction, and the asymptotic solutions are undistorted Dirac plane waves. Finally, to obtain the δ-function form of the interaction, one assumes that γ is much larger than the transverse distances from each ion, $|\vec{r}_\perp \pm \vec{b}|$, and the impact parameter $2b$ [1,6–8]. Contributions from large transverse distances are negligible for free-pair production as long as the momentum transfer is not too small, i.e., when Eq. (8) below applies.

Equation (1) displays a unique electromagnetic interaction, which, for each ion, factors into a logarithmic function of the transverse coordinate and a δ function of a light-front coordinate, $\tau_{\pm}\equiv(z\pm t)/2$, describing an electromagnetic shock on the light fronts. Anywhere but on the light fronts, Eq. (1) reduces to the free Dirac equation. The Dirac plane waves $\{|\phi_p(\vec{r},t)\rangle=\exp(-iE_pt)\exp(i\vec{r}\cdot\vec{p})|u_p\rangle\}$ which satisfy the free Dirac equation are each characterized by the quantum numbers $p\equiv\{\vec{p},\lambda_p,s_p\}$, with momentum $\vec{p}$, the sign of the energy $E_p=(-1)^{\lambda_p}\sqrt{p^2+1}$, and the spin $s_p=\pm$. Explicit forms for the four four-spinors $|u_p\rangle$ are given in Ref. [3], and $p_{\pm}\equiv p_z\pm E_p$.

The scattering problem for an electron which is not moving asymptotically at the rapidity of an ion is defined by Eq. (1) and by plane-wave asymptotic states:

$$\lim_{t\to-\infty}|\psi^{(j)}(\vec{r},t)\rangle=|\phi_j(\vec{r},t)\rangle,$$

$$\lim_{t\to+\infty}|\psi^{(j)}(\vec{r},t)\rangle=\sum_k A_k^{(j)}|\phi_k(\vec{r},t)\rangle, \tag{2}$$

where Σ_k stands for integration over $\vec{k}$ and summation over λ_k and s_k. We have obtained an exact, closed-form integral representation for the scattering amplitude $A_k^{(j)}$ [1]. First we have observed that, as the ions are approaching from infinity, no change occurs in the region of space between them ($|z|<|t|$) until $t=0$ when the two singular interaction planes collide. However, as each ion sweeps through space, they interact with each plane wave encountered, resulting in a superposition of plane waves after the ion passes. Each δ-function interaction induces a phase-shift discontinuity in the wave function across each light front [1,7,8]. A phase shift induced on a plane wave by the passage of a single ion is not sufficient to produce pairs, but as the two phase-shift planes collide at $t=0$, they interfere, and pairs are produced as a result. As the ions move apart ($t>0$), the solution in the space between them ($|z|<t$) is a new superposition of plane waves which is determined by the nontrivial boundary condition at the light fronts. We have calculated the transition amplitudes $A_k^{(j)}$ by integrating the flux of the conserved transition four-current which flows into this region across the light fronts. Two terms contribute to the amplitude corresponding to the two time orderings of the interaction of the electron with the two ions.

The transition amplitudes $A_k^{(j)}$ are represented in terms of the transverse-momentum-transfer distribution induced by a single ion, $Q_Z^{\vec{b}}(\vec{\kappa})$, which contains all the dynamics of the ion-electron interaction. When $\lambda_k=0$ and $\lambda_j=1$, $A_k^{(j)}$ is an amplitude for a transition from the negative continuum to the positive continuum, i.e., an amplitude for pair production. We have found [1]

$$A_k^{(j)}=\frac{i}{\pi}\int d\vec{p}_\perp\left\{\frac{\sigma_k^j(\vec{p}_\perp)Q_{Z_B}^{\vec{b}}(\vec{k}_\perp-\vec{p}_\perp)Q_{Z_A}^{\vec{b}}(\vec{j}_\perp-\vec{p}_\perp)}{p_\perp^2+1-j_-k_+}-\frac{\sigma_j^{k\dagger}(\vec{p}_\perp)Q_{Z_A}^{\vec{b}}(\vec{p}_\perp-\vec{k}_\perp)Q_{Z_B}^{\vec{b}}(\vec{p}_\perp-\vec{j}_\perp)}{p_\perp^2+1-j_+k_-}\right\}. \tag{3}$$

The spinor part is $\sigma_k^j(\vec{p}_\perp)\equiv(2\pi)^3\langle u_k|(I_4-\check{\alpha}_z)(\check{\alpha}\cdot\vec{p}_\perp+\check{\gamma}^0)(I_4+\check{\alpha}_z)|u_j\rangle$ and $Q_Z^{\vec{b}}(\vec{\kappa})$ is the Fourier transform of the phase shift at the light front,

$$Q_Z^{\vec{b}}(\vec{\kappa})\equiv\frac{1}{(2\pi)^2}\int d\vec{r}_\perp e^{i\vec{r}_\perp\cdot\vec{\kappa}}\left[\frac{(\vec{r}_\perp-\vec{b})^2}{b^2}\right]^{-i\alpha Z}=\frac{1}{2\pi}\frac{\exp(i\vec{\kappa}\cdot\vec{b})}{\kappa^2(b\kappa)^{-i2\alpha Z}}\int_{\xi>0}d\xi J_0(\xi)\xi^{1-i2\alpha Z}, \tag{4}$$

where Z is the charge of the corresponding ion, and $\vec{\kappa}$ is the transverse-momentum transfer. The integral over $\xi\equiv\kappa|\vec{r}_\perp-\vec{b}|$ in Eq. (4) should be regularized so as to avoid unphysical contributions from large, transverse distances, i.e., from $\xi>\kappa\gamma$. Several regularization schemes (for $\vec{\kappa}\neq0$) all gave [9]

$$Q_Z^{\vec{b}}(\vec{\kappa})\to\frac{-i\alpha Z}{\pi}\frac{\exp(i\vec{\kappa}\cdot\vec{b})}{\kappa^2}\left[\frac{\Gamma(-i\alpha Z)}{\Gamma(+i\alpha Z)}\left(\frac{b\kappa}{2}\right)^{+i2\alpha Z}\right]. \tag{5}$$

The exact amplitudes in the infinite γ limit are obtained by substituting the result of Eq. (5) in Eq. (3),

$$\begin{aligned}\lim_{\gamma\to\infty}A_k^{(j)}=&\left[\left(\frac{b}{2}\right)^{+i2\alpha(Z_A+Z_B)}\frac{\Gamma(-i\alpha Z_A)}{\Gamma(+i\alpha Z_A)}\frac{\Gamma(-i\alpha Z_B)}{\Gamma(+i\alpha Z_B)}\right]\\&\times\frac{i}{\pi^3}\alpha^2Z_AZ_B\int d\vec{p}_\perp(\vec{p}_\perp-\vec{k}_\perp)^{-2}\\&\times(\vec{p}_\perp-\vec{j}_\perp)^{-2}\left\{\frac{\sigma_k^j(\vec{p}_\perp)}{p_\perp^2+1-j_-k_+}e^{i\vec{b}\cdot(\vec{j}_\perp+\vec{k}_\perp-2\vec{p}_\perp)}\right.\\&\times[|\vec{p}_\perp-\vec{k}_\perp|^{i2\alpha Z_A}|\vec{p}_\perp-\vec{j}_\perp|^{i2\alpha Z_B}]\\&-\frac{\sigma_j^{k\dagger}(\vec{p}_\perp)}{p_\perp^2+1-j_+k_-}e^{-i\vec{b}\cdot(\vec{j}_\perp+\vec{k}_\perp-2\vec{p}_\perp)}\\&\left.\times[|\vec{p}_\perp-\vec{k}_\perp|^{i2\alpha Z_B}|\vec{p}_\perp-\vec{j}_\perp|^{i2\alpha Z_A}]\right\}.\end{aligned} \tag{6}$$

The branch-point singularities for the intermediate momentum $\vec{p}_\perp=\vec{k}_\perp$ or $\vec{j}_\perp$ are an artifact of using Eq. (5) for $\vec{\kappa}=0$. [Equation (3) has no such singularities.] An appropriate regularization at these points is assumed.

Equation (6) is nonperturbative and already includes the interaction to all orders in αZ. Yet its form is very similar to the high-energy limit of results obtained from the two-photon exchange diagrams of second-order perturbation theory [4] (which we shall call the perturbative result). As $\gamma\to\infty$, the only corrections to second-order perturbation theory calculations for free-pair production off the light fronts, including both higher orders and nonperturbative effects, are the *phases* in the square brackets. For small values of αZ, these phases tend to 1 and the perturbative limit is reproduced [1]. What are the observable nonperturbative effects for finite charges? The phase in the square brackets outside the integral over $\vec{p}_\perp$ has no physical implications, but the phases in the integrands may substantially alter the physical predictions. We find, for example, that the high-energy limit for $|A_k^{(j)}|^2$ differs, in general, from the perturbative result. On the other hand, using Eq. (6) to calculate the integrated observable $\int d(\overrightarrow{2b})|A_k^{(j)}|^2$, we get

$$\frac{4}{\pi^4}\alpha^4 Z_A^2 Z_B^2 \int d\vec{p}_\perp(\vec{p}_\perp-\vec{k}_\perp)^{-4}(\vec{p}_\perp-\vec{j}_\perp)^{-4} \times \left\{ \frac{|\sigma_k^j(\vec{p}_\perp)|^2}{(p_\perp^2+1-j_-k_+)^2}+\frac{|\sigma_k^j(\vec{p}_\perp)|^2}{(p_\perp^2+1-j_+k_-)^2} - 2\,\mathrm{Re}\frac{\sigma_k^j(\vec{p}_\perp)\sigma_j^k(\vec{j}_\perp+\vec{k}_\perp-\vec{p}_\perp)}{(p_\perp^2+1-j_-k_+)((\vec{j}_\perp+\vec{k}_\perp-\vec{p}_\perp)^2+1-j_+k_-)}\right\}, \tag{7}$$

which is identical to the perturbative result. The integration over the impact parameter results here in cancellation of the nonperturbative phases, as $\int d(\overrightarrow{2b})\exp(i\overrightarrow{2b}\cdot\vec{p}')=(2\pi)^2\delta(\vec{p}')$. We conclude that while some observables are sensitive to the nonperturbative phases, other observables are not, e.g., because these phases are averaged to 1 by an integration. In these cases, the results of the nonperturbative calculations, e.g., Eq. (7), would agree with the second-order perturbation theory calculations, regardless of the size of αZ.

Would our results apply in an actual experiment, where γ is finite? Equation (1) is *incorrect* for large $r_\perp$ (or large b) where it describes an interaction which continually increases in strength. Implicit in using Eq. (1) for large, finite γ is a nontrivial assumption that large, transverse distances do not contribute to pair production. In the recent experiments at CERN [2], $\gamma\approx 10$, while in possible future experiments at RHIC and LHC, $\gamma\approx 100$ and $\gamma\approx 3000$, respectively. For these values of γ, Eq. (1) is, respectively, limited to pairs produced at transverse distances much smaller than 10, 100, and 3000 Compton wavelengths away from the ions. This restriction is consistent with an experimental observation according to which the average length scale for pair production in relativistic heavy-ion collisions is one Compton wavelength [2]. Yet we are interested in distributions of observables, and one cannot exclude the possibility of pairs being produced at large, transverse distances from the ions if the transverse momentum transferred is sufficiently small. For Eq. (5) to be meaningful, the regulated integral of Eq. (4) must converge to the expression of Eq. (5) for ξ such that $|\vec{r}_\perp-\vec{b}|\ll\gamma$. The case of small coupling was previously studied [1]. The case of large αZ can be considered by the method of stationary phases. Expansion of Eq. (4) around the stationary point $\vec{r}_\perp-\vec{b}=2\alpha Z\vec{\kappa}/\kappa^2$ confirms Eq. (5) for this case. The procedure is consistent if the stationary point is located at small distances from the ion, i.e., if and only if

$$|\vec{\kappa}|\gg\frac{2\alpha Z}{\gamma}. \tag{8}$$

It is interesting to find that Eq. (8) is trivially satisfied in two very different limits: in the perturbative limit of $\alpha Z\to 0$ and in the high-energy limit of $\gamma\to\infty$.

Thus, the exact results, which we have first obtained for infinite γ, apply for finite γ as well, in an approximate manner with accuracy of the order γ^{-2}. The only restriction is of Eq. (8), i.e., that the transverse-momentum transfer is not too small. For pair production, it is a sufficient condition to assume that either the initial or final (i.e., positron or electron) transverse momenta are much larger than $2\alpha Z/\gamma$ where Z is the largest *free* charge involved in the collision. The argument goes as follows. There are three two-dimensional integration variables in Eq. (3). We first integrate over $\vec{p}_\perp$ to obtain simple combinations of the Bessel functions of the third kind, K_0 and K_1. We then use the condition that *one* of the two transverse momenta $\vec{j}_\perp$ or $\vec{k}_\perp$ is much larger than $2\alpha Z/\gamma$ to apply a stationary phase calculation to one of the coordinate integrations. If, on the other hand, one of the charges is screened (a target charge, for example), the integral with it converges and there is no need to restrict the momentum conjugate to it. The last integral, over the other coordinate-integration variable, converges due to the Bessel functions which drop exponentially for large values of their arguments. Having thus proved that contributions for the sixfold integral of Eq. (3) from large, transversal coordinates can be neglected, we can make the substitution of Eq. (5) and obtain Eq. (6). We emphasize that the convergence of the $\vec{p}_\perp$ integration to the Bessel functions occurs only for pair-production amplitudes for which $1-j_\pm k_\mp>0$, and is directly related to the mass gap between the two continua. It should be reconsidered for transitions within the same continuum.

We now consider the application of our results to the discussion of recent, pioneering experiments on pair production performed at CERN's SPS [2]. These experiments measured momentum spectra of positrons emitted from pair production in peripheral collisions of 33-TeV Pb^{82+} ions ($\gamma_T=168$) and 6.4-TeV S^{16+} ions ($\gamma_T=212$) with various targets [i.e., $(CH_2)_x$, Al, Pd, and Au]. The charge dependence of the positron yield was reported with excellent precision. The target-charge dependence for the sulfur projectile is $Z_T^{1.99\pm0.02}$ and for the lead projectile is $Z_T^{2.03\pm0.03}$, both within $\sim 1\%$ agreement with the prediction of perturbation theory. The projectile-charge dependence was observed to be $Z_P^{2.0\pm0.1}$, also in very good agreement with perturbation theory. The positron-momentum distributions for sulfur and lead projectiles are compared by scaling each spectrum by Z_P^2, and by scaling the sulfur data from $\gamma_T=212$ to $\gamma_T=168$ as $\ln^3\gamma_T$, as predicted by perturbation theory. The scaled distributions are observed to be approximately the same and to agree reasonably well with two-photon perturbation theory (see the discussion in [2]), except for enhancements for the lead projectile at very low (<2 MeV/c) and high (between 8 MeV/c and 12 MeV/c) momenta. The authors of Ref. [2] note that the variation of the scaled momentum distribution with the projectile charge, and not the target charge, is unexplained. Positron momenta observed in these experiments are restricted to the range $1-12$ MeV/c. This range excludes positrons produced on the light fronts, which would have momenta corresponding to the velocity of the projectile ion ($j \sim \beta\gamma_T mc$) or the target ion ($j \sim 0$).

The observed $Z_T^2 Z_P^2$ charge dependence of the single-positron yields, even for very large charges, is consistent with the charge dependence we have obtained for the non-perturbative, high-energy limit [see Eq. (7)]. It agrees with perturbation theory, but is not a perturbative effect. Our theoretical prediction of a Z^2 dependence of the cross section for pairs produced off the light fronts in the high-energy limit implies that multiple-pair production in very-high-energy collisions cannot be readily inferred from a measurement of the charge dependence of the total positron yield off the light fronts [2]. We suggest that the two regions of excess cross section observed in the experiment have a common origin: an enhancement over perturbation theory for small values of the transverse-momentum transfer, for which Eq. (4) diverges. We found that agreement with the perturbative result is restricted by Eq. (8). For pairs produced isotropically in the collider frame, this restriction, formulated for the transverse momentum, translates to a restriction on the total positron momentum: $j_\perp \sim j_z \sim j \gg 2Z_P\alpha/\gamma$. Taking $\gg$ to be a factor of 10, we then predict, in very good agreement with the observed scaled spectra, that the perturbative result for the positron-momentum distributions is valid for $j > 0.4$ MeV/c for the sulfur data and $j > 2$ MeV/c for the lead data. The excess cross section observed at high momentum for the lead projectile is consistent with an enhancement in the cross section at low transverse momentum in the collider frame after a relativistic transformation to the target frame is applied. The absence of an observed target-charge dependence for the scaled distributions is most likely attributable to screening by the atomic electrons [10].

In conclusion, we have shown that the exact, nonperturbative solution and the two-photon-exchange diagrams of second-order perturbation theory give exactly the same results in the high-energy limit for free-pair production yields off the light fronts integrated over the impact parameter. For large, but finite, collision energies the same conclusion holds, but now in an approximate sense and only for cases where the transverse momenta transferred from the ions to the electron are much larger than $2\alpha Z/\gamma$. The leading-order perturbative calculations *for this observable* therefore give correct results not only at the perturbative limit of $\alpha Z \ll 1$ but also in the high-energy limit of $\gamma \gg 1$. This explains recent experimental results according to which production rates scale as $Z_P^2 Z_T^2$, even for large charges. New nonperturbative effects could be detected by measuring observables different from the integrated, inclusive production rate that was measured in these experiments. The exact amplitudes of Eq. (6) include nonperturbative phases which may have an observable effect, e.g., if one does not integrate over the impact parameter $2\vec{b}$. We expect these phases to strongly influence the theoretical predictions for correlations and multiple-pair production. Several issues deserve further study. These include pair production at small transverse-momentum transfer and bound-free pair production for which one should obtain the solution *on* the light fronts themselves. For other calculations, Eq. (6), as well as the physical picture that led to it, is likely to become a useful theoretical tool [11].

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[9] G.N. Watson, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, London, 1962). Arguments presented on pp. 385, 393, and 401 can be used for an exponential, a Gaussian, and a Bessel function regularization, respectively.

[10] Screening begins at the *K*-shell radius (a_K) and is assumed complete at the Thomas-Fermi radius ($a_{\rm TF}$) (see also [13]). As either a_K or $a_{\rm TF}$ may be larger than γ, more work is required for detailed comparison. However, since the divergence in Eq. (4) is slow (logarithmic) and the charge dependence of $a_{\rm TF} \sim 1/(Z_T^{1/3}\alpha)$ is weak, the target-charge dependence of the scaled momentum distributions for fixed-target experiments will be very weak.

[11] After we completed this work, two manuscripts appeared in the Los Alamos reprint archive discussing the high-energy limit of pair-production amplitudes in heavy-ion collisions [14,15]. These manuscripts offer additional insight and perspective.

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