Paper

UNCERTAINTY AND VARIABILITY IN UPDATED ESTIMATES OF POTENTIAL DOSE AND RISK AT A U.S. NUCLEAR TEST SITE—BIKINI ATOLL

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Abstract—**Uncertainty and interindividual variability were assessed in estimated doses for a rehabilitation scenario for Bikini Island at Bikini Atoll, in which the top 40 cm of soil would be removed in the housing and village area, and the rest of the island would be treated with potassium fertilizer, prior to an assumed resettlement date of 1999. Doses were estimated for ingested ^{137}Cs and ^{90}Sr, external gamma-exposure, and inhalation+ingestion of ^{241}Am + $^{239+240}$Pu. Two dietary scenarios were considered: imported foods are available (IA); imported foods are unavailable with only local foods consumed (IUA). After ~5 y of Bikini residence under either IA or IUA assumptions, upper and lower 95% confidence limits on interindividual variability in calculated dose were estimated to lie within a ~threefold factor of its in population-average value; upper and lower 95% confidence limits on uncertainty in calculated dose were estimated to lie within a ~twofold factor of its expected value. For reference, the expected values of population-average dose at age 70 y were estimated to be 16 and 52 mSv under IA and IUA dietary assumptions, respectively. Assuming that 200 Bikini resettlers would be exposed to local foods (under both IA and IUA assumptions), the maximum 1-y dose received by any Bikini resident is most likely to be approximately 2 and 8 mSv under the IA and IUA assumptions, respectively. Under the most likely dietary scenario, involving access to imported foods, this analysis indicates that it is most likely that no additional cancer fatalities (above those normally expected) would arise from the increased radiation exposures considered.**

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Key words: cesium; Marshall Islands; fallout; dose assessment

INTRODUCTION

THIS PAPER supplements updated dose assessments for Bikini Island at Bikini Atoll conducted by Robison et al. (1994a, 1995, 1997), which address doses estimated under two resettlement options: (1) current conditions assuming no environmental remediation, and (2) resettlement after soil removal in the housing and village area, and potassium treatment of the rest of the island. The present detailed analysis of uncertainty and interindividual variability in estimated doses to potential Bikini resettlers focuses only on resettlement option (2), under the two dietary scenarios considered by Robison et al. (1995, 1997), referred to as IA (imported foods will be available and will comprise 60% of the diet) and IUA (local foods only—considered unlikely; see Robison et al. 1995, 1997). Estimated dose is typically a function of distributed quantities reflecting either uncertainty (lack of knowledge concerning "the true" value of a variate) or interindividual variability (or simply "variability," referring to heterogeneity in true variate values pertaining to different people at risk). Consequently, predicted dose typically involves joint uncertainty and interindividual variability (JUV). This paper illustrates an application of analytic and Monte Carlo methods for JUV analysis pertaining to estimated fallout-related doses to hypothetical Bikini resettlers. Specifically, 70-y and maximum 1-y doses to hypothetical Bikini resettlers are calculated, as described below, using analytic and Monte Carlo procedures to characterize JUV in estimated dose as a function of distributed input variates involved.

METHODS

Dose models

If dose variability is simply treated as dose uncertainty, the latter is constrained to refer only to an individual selected at random from the exposed population and not to any specific (e.g., relatively highly exposed) individual(s) who may be of particular concern. To characterize JUV in estimated dose, appropriate methods must therefore be used to distinguish and treat these attributes systematically as each or both pertain to each input variate (Bogen and Spear 1987; Nazaroff et al. 1987; IAEA 1989; Bogen 1990, 1995; NRC 1994). We used such methods to recalculate dose to potential Bikini residents as a function of several distributed input variates. Uncertainty and variability were characterized for predicted total integrated doses arising from (1) external gamma-ray exposure, (2) ^{241}Am and $^{239+240}$Pu inhalation and ingestion, (3) ^{90}Sr ingestion, and (4) ^{137}Cs ingestion. Expected values of the relatively minor source-specific doses (1–3) were all calculated using the

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same ICRP models (Leggett 1986; ICRP 1988, 1990, 1991) employed by Robison et al. (1995, 1997) to calculate adult doses from these sources, except for one modification accounting for greater absorption of ingested ^{90}Sr in children (discussed below). To facilitate JUV analysis of total integral dose, the dominant source of potential radiological exposure on Bikini, ^{137}Cs ingestion (see Robison et al. 1995, 1997), was treated somewhat differently. Specifically, the one-compartment ICRP (1990) model for ingested ^{137}Cs was replaced by the following structurally equivalent model:

$$q_{ij}(t_i) = FBR_{ij} \exp(-\lambda t_i) \quad \text{at any time } t_i, \quad 0 \le t_i \le t, \tag{1}$$

$$q'_{ij}(u) = -(\beta K + \lambda) q_i(u) \quad \text{for any time } u, \quad t_i \le u \le t, \tag{2}$$

$$q_{ij}(u) = BFR_{ij} \exp(-\lambda t_i \exp[-(\beta K + \lambda)u] \quad \text{for any time } u, \quad t_i \le u \le t, \tag{3}$$

where $q_{ij}(u)$ is the activity (in Bq kg^{-1} body weight of ^{137}Cs) in the whole body at any time u following ingestion of an activity R_{ij} (in Bq kg^{-1} body weight) of ^{137}Cs contained in a food item of type j at time t_i; B represents a dietary-dose-model bias (i.e., a dose-estimation uncertainty factor) associated with R_{ij}, prime (′) denotes differentiation with respect to time, λ is the radiological decay rate of ^{137}Cs, $K = \mathrm{Ln}(2)H^{-1}$ is the biological loss rate of ^{137}Cs from the dominant "slow" metabolic compartment of a reference adult (see ICRP 1990), F is the fraction of ingested dose entering this slow metabolic compartment, and β is a factor representing uncertainty associated with H. Henceforth, angle brackets, $\langle\ \rangle$, are used to denote mathematical expectation only with respect to uncertainty, and an overbar is used to denote expectation only with respect to interindividual variability (Bogen 1995).

Daily intakes R_{ij} in Bq kg^{-1} d^{-1} of ^{137}Cs, as well as corresponding intakes of ^{90}Sr, in local food items of type j were assumed to be obtained from independent random samples of such items collected n_j days per year from among the possible selections of the type available on Bikini. The corresponding cumulative dose $D(t)$ from all exposure sources was estimated as

$$D(t) = D_x(t) + D_{\mathrm{AmPu}}(t) + D_{\mathrm{Sr}}(t) + \int_0^t \sum_j \sum_{i=1}^{n_j} \frac{365}{n_j} c q_{ij}(u)\, du, \tag{4}$$

where $D_x(t)$ is the external-gamma dose modeled as interindividually variable (and not uncertain), $D_{\mathrm{AmPu}}(t)$ is the unmodified ICRP-model estimate of total Am+Pu inhalation+ingestion dose (modeled as neither uncertain nor variable, in view of its relatively minor role), $D_{\mathrm{Sr}}(t)$ is the dose due to ^{90}Sr ingestion (modeled as both uncertain and interindividually variable, similar to the approach taken for ^{137}Cs—see Appendix), and c is a unit-conversion constant. Eqn (4) was evaluated using a combination of analytic and Monte-Carlo methods detailed in the Appendix.

Parameter distributions

Using the angle-bracket and overbar notation discussed above, specific assumptions regarding distributions for each variable and/or uncertain parameter appearing or implied in eqn (4) are discussed individually below. These assumptions are summarized in Table 1.

External gamma dose. $\overline{D_x(t)}$ was modeled using the assumptions of Robison et al. (1995, 1997) for average daily occupancies and exposure rates in house and beach/lagoon areas (12 h × 0.1z), house-surrounding and village areas (9 h × 0.2z), and island-interior areas ($\bar{T}$ × 19z), with $\bar{T}$ = 3 h and z = 0.0717 pC kg^{-1}s^{-1} which imply a time-weighted average exposure rate of 0.18 pC kg^{-1} s^{-1}. Variability in mean daily time T (h) spent in the island interior (the principal source of gamma dose) was assumed to be triangularly distributed over a range of 1 to 5 h. Thus, it was assumed that, $D_x(t) = X_\gamma \overline{D_x(t)}$, where the exposure-variability factor X_γ is triangularly distributed over the range 1 ± (19/30).

Metabolic factors for ^{137}Cs. Variability in the fraction, F, of ingested ^{137}Cs input to the dominant biological compartment was assumed to be uniformly distributed between an uncertain lower bound ranging between 0.71 and 0.89 and an upper bound of 1. Thus, uncertainty in $\bar{F}$ was assumed to be uniformly distributed within ± 5% of an assumed expected value of 0.9, and variability of $\langle F \rangle$ was assumed to be uniformly distributed between 0.8 and 1. These assumptions approximately characterize the empirical data on the value of F obtained for 17 individuals reported by Schwartz and Dunning (1982).

Interindividual variability in the biological half-time, H, of the dominant slow compartment was modeled as lognormally distributed based on the data pertaining to 23 Marshallese males indicating a median of 115 d and a geometric standard deviation (SD_g) of 1.23, as shown in Figure 4 of Robison et al. (1995). For the present analysis, however, it was assumed that $\bar{H}$ = 110 d and that SD_g = 1.32 for H, based, respectively, on the ICRP (1979) reference mean value (used earlier) and on data reviewed by Schwartz and Dunning (1982) indicating slightly greater variability associated with the parameter among 53 individuals from whom measurements were available. A geometric mean (GM) value of H (105.9 d) consistent with the values selected for and SD_g was obtained using the method of moments. Uncertainty pertaining to H was represented by the independent factor β assumed to be uniformly distributed (between 0.9 and 1.107), such that the true value of H pertaining to any specific individual was taken to lie within 10% of the expected value for that individual.

Table 1. Parameters used in analysis of uncertainty/variability in estimated dose to hypothetical bikini residents.

Parameters[a]	Symbol	Variate type[b]	Value or distribution model[c]	Unit
Effective unit-conversion factor	c	C	2.419×10^{-3}	mSv kg Bq^{-1} y^{-1}
Radiological decay rate of ^{137}Cs	λ	C	0.0230	y^{-1}
External gamma exposure variability factor	X_γ	V	Tri(11/30, 1, 49/30)	unitless
Fraction input to slow compartment for ^{137}Cs	F	UV	$U(2\bar{F} - 1, 1)$	unitless
Variability expectation of F	$\bar{F}$	U	U(0.855, 0.945)	unitless
Biological half-life of slow compartment	H	V	$LN(\bar{H} - (h^2/2), h)$	y
Population-average value of H	$\bar{H}$	C	110/365	y
Uncertainty associated with H	β	U	U(0.9, 1.107)	unitless
SD of Ln(H)-variability	h	C	0.275	unitless
Annual dietary intake of ^{137}Cs	R	UV	$LN(\bar{R} - (r^2/2), r)$	Bq kg^{-1} y^{-1}
Population-average value of R	$\bar{R}$	U	$N(\langle R\rangle, \langle R\rangle g_R)$	Bq kg^{-1} y^{-1}
Expected 1999 values of $\bar{R}$/365 d	$\langle R\rangle$/365 d	C	50.1/70 (IA diet), 196.7/70 (IUA diet)	Bq kg^{-1} y^{-1}
SD of Ln(R) variability	r	C	0.8217	unitless
CV of $\langle R\rangle$ variability	g_R	C	0.9821	unitless
CV in R due to annual diet sample uncertainty	γ_R	C	0.039	unitless
Cumulative dose due to ^{90}Sr ingestion by time t	$D_{Sr}(t)$	UV	(see text)	mSv
Factor for variability in adult ^{90}Sr GI absorption	G	V	U(0.50, 1.5)	unitless
Uncertainty (model bias) associated with R	B	U	LN(−0.04309, 0.2936)	unitless
Uncertainty risk per unit dose	Z	U	LN(−4.828, 0.5064)	mSv^{-1}

[a] IA = imports available, IUA = imports unavailable, SD = standard deviation, CV = SD/mean.
[b] C = constant, U = uncertainty, V = interindividually variable (i.e., heterogeneous), UV = both uncertain and heterogeneous.
[c] U (a,b) = uniformly distributed between a and b, LN (a,b) = lognormally distributed with a geometric mean of exp(a) and a geometric SD (SD_g) of exp(b), N(a,b) = normally distributed with mean a and SD b, Tri(a,b,c) = triangularly distributed with bounds a and c and mode b.

Metabolic factors for ^{90}Sr. Cumulative dose, $D_{Sra}(t)$, by age t due to ^{90}Sr ingestion by adults, was obtained using the ICRP (1990) adults-only model for ^{90}Sr employed by Robison et al. (1995, 1997). In contrast to the situation for potential Rongelap Island resettlers, for whom ingested ^{90}Sr would be a relatively negligible source of radiation exposure (Robison et al. 1994b), ^{90}Sr would contribute a nonnegligible fraction of total dose for potential Bikini resettlers, albeit a relatively small one compared to that due to ^{137}Cs (Robison et al. 1995). Data are available from which models of uncertainty and interindividual variability in ^{90}Sr uptake and distribution and consequent dosimetry could be constructed (e.g., Rivera 1967; Bennett 1973, 1977, 1978; Papworth and Vennet 1973, 1984; Klusek 1979; Leggett et al. 1982; Christy et al. 1984; Christy and Eckerman 1987a,b). Because ^{90}Sr would contribute a relatively minor dose to Bikini resettlers, cumulative lifetime ^{90}Sr dose, $D_{Sr}(t)$, by age t was instead modeled first as $D_{Sr}(t) = G \times W(t) \times D_{Sra}(t)$, where the factors G and $W(t)$ are explained as follows.

The factor G was used to model variability in ^{90}Sr uptake about its population-average value and was assumed to be uniformly distributed between 0.5 and 1.5 based on measured ranges reported in ICRP (1990). The (deterministic) factor $W(t)$ was used to adjust for the fact that $D_{Sra}(t)$ underestimates $D_{Sr}(t)$, due to increased ^{90}Sr uptake in infancy/childhood and other factors (ICRP 1990). This factor was calculated as $W(t) = \int_0^t d_{Sr}(u)du / [t d_{Sr}(70)]$, where $d_{Sr}(t)$ refers to a linear interpolation of the age-specific effective ^{90}Sr dose equivalent values listed ICRP (1990, Table 3-2). For example, $W(1) = 3.41$ and $W(70) = 1.17$. Additional metabolic uncertainty and variability in $D_{Sr}(t)$ was assumed to be proportional to and (as a conservative assumption) completely correlated with that associated with dietary ^{137}Cs intake (see Appendix). All maximum 1-y effective doses were calculated (conservatively) assuming a resettling cohort arriving at age 0 (thus incurring a maximal ^{90}Sr dose).

Dietary intake of ^{137}Cs and ^{90}Sr. The population-average value of expected annual intake, $\langle R\rangle$, of total ^{137}Cs activity in the LLNL model diet for hypothetical Bikini residents as of 1999 (assuming imports are available) was taken to be (365 d) × (0.716 Bq kg^{-1} d^{-1}) for a reference adult, based on the analysis of food consumption survey data for 34 adult Ujelang females discussed in Robison et al. (1994b). Interindividual variability in corresponding expected daily intakes, $\langle R_{ij}\rangle$ was modeled using the empirical distribution of average daily uptakes in Bq kg^{-1} calculated from the food-survey data for these same 34 adult Ujelang females, which was multiplicatively scaled to have expected daily population average values equal to 100% of the total mean daily ^{137}Cs intakes corresponding to each of the two dietary scenarios considered. For potential Bikini resettlers, these expected values of food-specific ^{137}Cs activities and intakes are summarized in Table 2 for the 11 major local-food items likely to be consumed. The scaled empirical distribution of ^{137}Cs intake does not significantly differ from a lognormal distribution with a shape parameter of $SD_g = 0.8217$ (Fig. 1); $p > 0.15$ using Stephen's modified Kolmolgorov-Smirnov, Cramer-von-Mises, or Watson tests (Stephens 1970; Pearson and Hartley 1972). We used this lognormal distribution as the

Table 2. Diet model-bikini island for adults for ^{137}Cs ingestion.[a,b]

Local foods	Intake: Local foods only (g d^{-1}) L	Intake: Local + imported (g d^{-1}) I	^{137}Cs activity Mean (Bq g^{-1}) C	^{137}Cs activity SD/Mean γ_C	^{137}Cs intake Local only Mean (Bq d^{-1}) $A = LC$	Local only Var (Bq d^{-1})^2 σ^2	Local only SD/Mean γ	Imports available Mean (Bq d^{-1}) $B = IC$	Imports available Var (Bq d^{-1})^2 σ^2	Imports available SD/Mean γ
Coconut										
Milk[d]	122	51.9	0.268	0.644	32.6	442		13.9	80.2	
Meat	181	31.7	0.147	0.739	26.6	386		4.66	11.9	
Copra	71.4	12.2	0.268	0.644	19.1	152		3.27	4.43	
Juice	334	99.1	0.0577	0.777	19.3	224		5.72	19.7	
Total[c]	708		0.138		97.6		0.355			
Total[c]		195	0.141					27.6		0.391
Pork										
Heart	0.620	0.310	0.980	1.10	0.608	0.447		0.304	0.112	
Muscle	13.9	5.67	1.57	0.635	21.9	193		8.90	32.0	
Liver	6.70	2.60	0.812	0.912	5.44	24.6		2.11	3.71	
Total[c]	21.24		1.31		27.9		0.529			
Total[c]		8.58	1.32					11.3		0.528
Chicken										
Muscle	31.2	8.36	0.0213	0.635[e]	0.665	0.178		0.178	0.0128	
Liver	17.7	4.50	0.0213[f]	0.912[g]	0.377	0.118		0.0959	0.00764	
Gizzard	3.32	1.66	0.0213[f]	0.912[g]	0.0707	0.00416		0.0354	0.00104	
Total[c]	52.2		0.0213		1.11		0.493			
Total[c]		14.52	0.0213					0.309		0.474
Breadfruit	186	27.2	0.0190	0.584	3.54	4.27	0.584	0.517	0.0911	0.584
Pandanus	65.0	9.16	0.194	0.848	12.6	114	0.848	1.78	2.27	0.848
Sprouting coconut[d]	122	7.79	0.268	0.644	32.8	446	0.644	2.09	1.81	0.644
Papaya	27.0	6.59	0.110	1.34	2.97	15.8	1.34	0.725	0.944	1.34
Arrowroot	94.8	3.93	0.0543	0.413	5.15	4.52	0.413	0.213	0.00777	0.413
Pumpkin	5.44	1.24	0.0587	1.18	0.319	0.142	1.18	0.0728	0.00738	1.18
Marsh. Cake[d]	0.00	11.7	0.268	0.644	0.00	0.00	0.00	3.14	4.08	0.644
Coconut crabs	25.0	3.13	0.366	0.604	9.15	30.5	0.604	1.15	0.479	0.604
Subtotal	1,307		0.148		193		0.0274[h]			
Subtotal		289	0.169					48.9		0.0392[h]
% of Total	42	22			98.2			97.5		

[a] Three significant figures are shown for the purpose of calculating corresponding mean, standard deviation (SD), variance (σ^2) and coefficient-of-variation (γ) values.

[b] Local-foods-only, local + imported foods intakes and ^{137}Cs activities for specific foods decay corrected to 1999, are from Robison et al. (1995).

[c] Mean and SD values for totals listed under coconut, pork and chicken were calculated using subitem-specific intake weights. For example, for a given food item (e.g., coconut, consisting of m = 4 constituents) with the local foods only diet,

$$A_i = L_i C_i, \quad \sigma_i = A_i \gamma_{ci}, \quad \gamma = \left(\sum_{i=1}^{m} \sigma_i^2 \right)^{1/2} \left(\sum_{i=1}^{m} A_i \right)^{-1}.$$

[d] Assumed to equal copra meat.

[e] Assumed to equal pork muscle.

[f] Assumed to equal chicken muscle.

[g] Assumed to equal pork liver.

[h] The γ value given for the subtotal of all 14 items listed, e.g., from a local-foods-diet, is the annual value calculated as

$$g = \left(\sum_{j=1}^{11} A_j \right)^{-1} \left(\sum_{j=1}^{11} A_j^2 g_j^2 n_j^{-1} \right)^{1/2}$$

where n_j is the number of samples of food type j eaten per year, assumed to be 12, 52, and 182.5 for pork-related, chicken-related and other items, respectively (see Appendix).

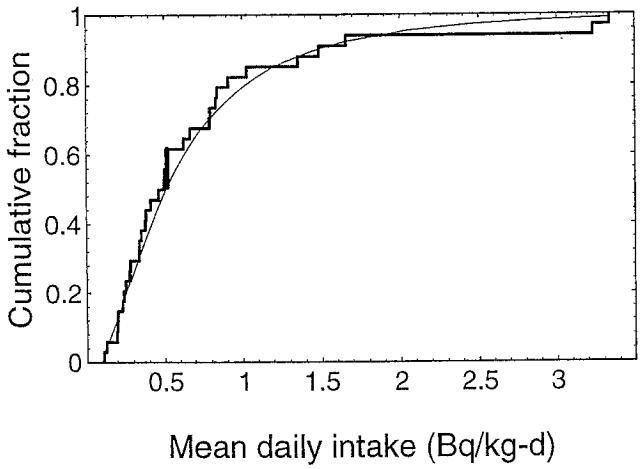

Fig. 1. Sample distribution of interindividual variability in daily intake of ^{137}Cs per unit body weight based on survey data for 34 adult Ujelang females (bold), shown here fit to a lognormal distribution (light) with SDg = 2.274 and a mean value scaled to equal 0.7157 Bq kg^{-1} d^{-1}, the expected value for 1999 Bikini resettlers assuming imported foods are available. The Ujelang survey data are discussed further in Robison et al. (1994b).

basis of our model of variability in $\langle R\rangle=\langle R_{ij}\rangle$ for the hypothetical Bikini resettlement population. By the method of moments (Aitchison and Brown 1957), this distribution has a corresponding coefficient of variation (CV) [i.e., standard deviation (SD) divided by expected value] with respect to modeled variability equal to g_R = 0.9821.

The distributional form and g_R value assumed for ^{137}Cs intake discussed above was assumed also to pertain to interindividual variability in lifetime-average daily ingestion of ^{90}Sr. Food-specific ^{90}Sr activities and intakes for potential Bikini resettlers, under the two dietary scenarios considered, are summarized in Table 3. A comparison of Tables 2 and 3 reveals that ^{137}Cs and ^{90}Sr intakes for the 11 major local-food items considered are uncorrelated under both dietary scenarios ($|r| < 0.16$, $p \approx 1$). Persons who might consume relatively large amounts of relatively ^{137}Cs-rich items therefore would not be expected to consume large ^{90}Sr doses relative to others. It follows that the simplifying assumption made in the present analysis, that interindividual variabilities in lifetime average rates of ^{137}Cs and ^{90}Sr ingestion are completely correlated, is conservative.

Uncertainty will arise from random dietary sampling associated with daily ^{137}Cs intake for any given individual about that individual's mean daily level. This uncertainty was estimated under diet-model assumptions stated above, such that local foods of type j are randomly and independently sampled n_j times per year from among Bikini sources. Table 2 lists predicted amounts and measured inter-sample variability of ^{137}Cs in 11 major food items local to Bikini. Activities associated with these 11 items were scaled to correspond to an assumption that they comprise 100% of local foods in either dietary scenario. Each corresponding CV, $\gamma_{R_{ij}} = \sigma_{R_{ij}}/\langle R_{ij}\rangle$, with respect to presumed dietary sampling error (Table 2) was assumed to pertain to all individuals. For this purpose, the local food items appearing in Table 2 were divided into three types (and the corresponding indicated annual sample sizes were assumed): pork-related items (n_1 = 12), chicken-related items (n_2 = 52), and other items (n_3 = 182.5). It follows that uncertainty due to random daily dietary sampling associated with annual ^{137}Cs intake is expected to be approximately normally distributed about its expected value, with an SD value inversely proportional to the square root of the total exposure time considered (see Appendix).

The Gaussian uncertainty model for random dietary sampling associated with daily ^{137}Cs intake also pertains to ingested ^{90}Sr. Because the CV values for approximate total dietary ^{137}Cs (Table 2) and ^{90}Sr (Table 3) are similar, the distribution for uncertainty in ^{90}Sr intake due to dietary sampling was taken to be that of ^{137}Cs, after scaling for the relative difference between the population-average values assumed for dietary ^{137}Cs and ^{90}Sr intakes (see Appendix). Measured concentrations of ^{137}Cs and ^{90}Sr in samples of drinking-coconut meat (a major local-food item) obtained from 70 different coconuts on Bikini Island were found to be uncorrelated (r = 0.15, p = 0.15) (Bogen et al. 1995). Thus, uncertainty in ^{90}Sr ingestion due to dietary sampling of different activities present on Bikini was assumed to be statistically independent of that pertaining to ^{137}Cs.

Model uncertainty (misspecification error) was estimated directly from data shown in Figure 3 of Robison et al. (1994b) relating LLNL model-diet predictions assuming imported foods are available, and corresponding Brookhaven National Laboratory measurements of whole-body ^{137}Cs dose among different samples of Marshallese people tested during the period 1977–1983. The mean of the six measured- to predicted-burden ratios shown is 1.25 ± 0.37 (differing insignificantly from 1, $p > 0.16$ by t-test). Based on these data, an uncertainty-CV of 30% was assumed, and model uncertainty for the LLNL model diet assuming imported foods are available was characterized as a corresponding lognormally distributed factor B with expectation 1 and SD$_g$ = 1.34. This factor was assumed also to apply to estimated ^{90}Sr dose.

Population risk

Predicted population risk I (the number of fallout-induced cancer fatalities) necessarily depends on the size, N, and age distribution of the population involved. To estimate I under both dietary models considered, it was assumed that resettlement occurs in 1999 and (a) that n = 200 or (b) that n = 2,000 but (due to the carrying capacity of a resettled Bikini) that only 200 resettlers would be eating non-imported foods (under either dietary scenario). The uncertainty distribution of I was used to calculate Prob(I=0), the probability of zero cases. This distribution was approximated by the method of Bogen and Spear (1987), treating I as compound-Poisson-distributed with an uncertain (population-average-dose)

Table 3. Diet model-bikini island for adults for ^{90}Sr ingestion.[a,b]

Local foods	Intake: Local foods only (g d^{-1}) L	Intake: Local + imported (g d^{-1}) I	^{90}Sr activity Mean (Bq g^{-1}) C	^{90}Sr activity SD/Mean γ_C	^{90}Sr intake Local only Mean (Bq d^{-1}) $A = LC$	Local only Var (Bq d^{-1})^2 σ^2	Local only SD/Mean γ	Imports available Mean (Bq d^{-1}) $B = IC$	Imports available Var (Bq d^{-1})^2 σ^2	Imports available SD/Mean γ
Coconut										
Milk[d]	122	51.9	0.00321	0.915	0.391	0.128		0.167	0.0232	
Meat	181	31.7	0.00586	0.612	1.06	0.420		0.186	0.0129	
Copra	71.4	12.2	0.00321	0.915	0.229	0.0440		0.0392	0.00128	
Juice	334	99.1	0.000452	0.682	0.151	0.0106		0.0448	0.000933	
Total[c]	708		0.00259		1.83		0.424			
Total[c]		195	0.00224					0.436		0.449
Pork										
Heart	0.620	0.310	0.00150	0.512	0.000930	2.27×10^{-7}		0.000465	5.67×10^{-8}	
Muscle	13.9	5.67	0.00152	0.500	0.0212	0.000112		0.00862	1.86×10^{-5}	
Liver	6.70	2.60	0.00292	1.06	0.0196	0.000430		0.00759	6.48×10^{-5}	
Total[c]	21.2		0.00196		0.0417		0.559			
Total[c]		8.58	0.00194					0.0167		0.548
Chicken										
Muscle[e]	31.2	8.36	0.00152	0.500	0.0474	0.000562		0.0127	4.04×10^{-5}	
Liver[e]	17.7	4.50	0.00152	0.500	0.0269	0.000181		0.00684	1.17×10^{-5}	
Gizzard[e]	3.32	1.66	0.00152	0.500	0.00505	6.37×10^{-6}		0.00252	1.59×10^{-6}	
Total[c]	52.2		0.00152		0.0793		0.345			
Total[c]		14.5	0.00152					0.0221		0.332
Breadfruit	186	27.2	0.0690	0.898	12.8	133	0.898	1.88	2.84	0.898
Pandanus	65.0	9.16	0.120	1.10	7.80	73.6	1.10	1.10	1.46	1.100
Sprouting coconut[d]	122	7.79	0.00321	0.915	0.393	0.129	0.915	0.0250	0.000524	0.915
Papaya	27.0	6.59	0.0486	0.580	1.31	0.579	0.580	0.320	0.0345	0.580
Arrowroot	94.8	3.93	0.0676	0.563	6.41	13.0	0.563	0.266	0.0224	0.563
Pumpkin	5.44	1.24	0.0676	0.563	0.368	0.0429	0.563	0.0838	0.00223	0.563
Marsh. Cake[d]	0.00	11.7	0.00321	0.915	0.00	0.00	0.00	0.0376	0.00118	0.915
Coconut crabs	25.0	3.13	0.0518	0.534	1.30	0.478	0.534	0.162	0.00750	0.534
Subtotal	1,307		0.0248		32.4		0.0340[f]			
Subtotal		289	0.0150					4.35		0.0358[f]
% of Total	42	22			97.8			92.5		

[a] Three significant figures are shown for the purpose of calculating corresponding mean, standard deviation (SD), variance (σ^2) and coefficient-of-variation (γ) values.
[b] Local-foods-only, local + imported foods intakes and ^{90}Sr activities for specific foods decay corrected to 1999, are from Robison et al. (1995).
[c] Mean and SD values for totals listed under Coconut, Pork and Chicken were calculated using subitem-specific intake weights (see Table 2, note [c]).
[d] Assumed to equal copra meat.
[e] Assumed to equal pork muscle.
[f] The γ value given for the subtotal of all 14 items listed (see Table 2, note [h]).

parameter here taken to be $NZ\overline{D(70)}$, where Z is an uncertain "risk" factor specifying total cancer (leukemia + nonleukemia) mortality risk-per-unit dose. Based on the BEIR V (NRC 1990) prediction of total cancer (leukemia + nonleukemia) fatalities for males and females likely to be caused by chronic low-LET radiation exposure, and associated analysis of statistical and model-related errors, a risk factor Z_b was taken to be approximately lognormally distributed, with expectation 0.008 mSv^{-1} and $SD_g = 0.5064$, for a cohort resettling Bikini at birth. The value of 0.008 mSv^{-1} is the BEIR V (NRC 1990) recommended population-weighted average value of 0.008 mSv^{-1} for acute low-LET radiation exposure, divided by the approximate factor of two recommended as an adjustment for estimating risk due to cumulative chronic exposure, and multiplied by a second approximate factor of two recommended as an adjustment for estimating risk associated with exposures specifically during childhood (given that a disproportionate amount of cumulative dose to Bikini resettlers would occur during the earlier years post resettlement, due to radiological decay of ^{137}Cs and ^{90}Sr).

Because the latter factor of two would not apply to adults accompanying resettling infants and youth, Z_b was assumed to pertain to a fraction f of the resettling population, and $Z_b/2$ was assumed to pertain to $100(1-f)\%$ of the resettling population. The SD_g value was estimated by the method of moments, given that, from the BEIR V analysis, the 90% upper confidence limit on Z_b is ~2.3 times its median value. Based on the likelihood that there would be a high proportion of infants and children among potential Bikini resettlers, the fraction f was assumed to be 0.5. Thus, the overall risk factor Z was taken to be equal to Z_b and $Z_b/2$ with equal likelihood. The factor Z (conservatively) does not reflect the possibility, given current fundamental radiobiological uncertainties, that the true fallout-related risk on Bikini may be zero.

RESULTS

The results of the JUV analysis of estimated dose to potential Bikini resettlers are summarized in Table 4 and Figs. 2–4. Specifically, Figs. 2a and 2c plot the calculated distributions for $\langle D(70)\rangle$ (characterizing interindividual variability in expected 70-y effective integral dose) and $\overline{D(70)}$ (characterizing uncertainty in population-average 70-y effective integral dose), and their corresponding Monte-Carlo sampling errors, under the assumption that imported foods will be available. Figs. 2b and 2d plot the calculated distributions for $\langle D(70)\rangle$ and $\overline{D(70)}$, and their corresponding Monte-Carlo sampling errors, under the assumption that imported foods will not be available (i.e., for a local-foods-only diet). Note that the 99.5th percentile values of $\overline{D(70)}$ listed in Table 4 are the maximum-likelihood values of dose to corresponding persons receiving the maximum 70-y doses among all persons exposed under the IA and IUA diet assumptions, assuming an exposed population size of 200 (NRC 1994; Bogen 1995).

Figs. 3a and 3c plot the calculated distribution for $\mathrm{Max}(\langle D(1)\rangle)$ (characterizing interindividual variability in the maximum value of expected 1-y effective integral doses, regardless of occurrence year, and its corresponding Monte-Carlo sampling error), assuming that imported foods will be available. Figs. 3b and 3d plot the corresponding distribution and sampling error assuming that imported foods will not be available. The estimated maximum 1-y doses are predicted to fall in years 1999, 2000, 2001, and 2002 for ~0.1%, 38.5%, 59.5% and 1.9% of residents (imports available), or for ~0%, 4.5%, 88.1% and 7.4% of residents (imports not available). The year of each individual's predicted maximum 1-y dose is primarily a function of the corresponding value of H (the half-life for the dominant ^{137}Cs metabolic compartment). Note that the 99.5th percentile values of $\mathrm{Max}(\langle D(1)\rangle)$ listed in Table 4 represent the maximum-likelihood values of dose to the corresponding persons receiving the maximum 1-y doses among all persons exposed under the IA and IUA diet assumptions, assuming an exposed population size of 200 (NRC 1994; Bogen 1995).

Figs. 4a–d plot the population-average values of $\langle D(1)\rangle$ (expected effective integral dose as a function of time t) and their 95% confidence limits (95%CL) with respect to interindividual variability for the imports-available and local-foods-only diets, both in absolute terms as well as values relative to the population-average at time t. Figs. 4e–h plot the expected values of $\overline{D(t)}$ (population-average effective integral dose over time t) and their 95%CL with respect to uncertainty for the imports-available and local-foods-only diets, both in absolute terms and as values relative to the expected value at time t.

Table 4. Summary of uncertainty and interindividual variability in estimated integral effective doses for hypothetical Bikini island residents, assuming 1999 resettlement after soil removal/K treatment and availability and nonavailability of imported foods.

		Dose and exposure scenario[a]			
Dietary model ⟶		IA	IUA	IA	IUA
Exposure duration ⟶		Max 1-y	Max 1-y	70 y	70 y
Distributed characteristic	Estimator[b]	(mSv)	(mSv)	(mSv)	(mSv)
Interindividual variability	Q(0.025)	0.17	0.31	6.5	1.2
	Q(0.50)	0.36	1.0	13	3.8
	EV	0.45	1.4	16	5.2
	Q(0.975)	1.3	4.9	45	18
	Q(0.995)	2.0	8.2	73	31
Uncertainty	Q(0.025)	—	—	11	30
	Q(0.50)	—	—	16	50
	EV	—	—	16	52
	Q(0.975)	—	—	24	87
	Q(0.995)	—	—	28	100

[a] IA = model diet assuming that "imported foods are available"; IUA = model diet assuming availability of "local foods only," i.e., that "imported foods are unavailable." Values listed are rounded to two significant digits; — = not calculated.
[b] $Q(p)$ = the pth quantile or fractile = 100pth percentile; EV = expected value. The Monte Carlo coefficients of variation of the mean (or standard error of the mean divided by the mean) of all listed fractile estimates are <2%, and those of listed EV values are <0.2%.

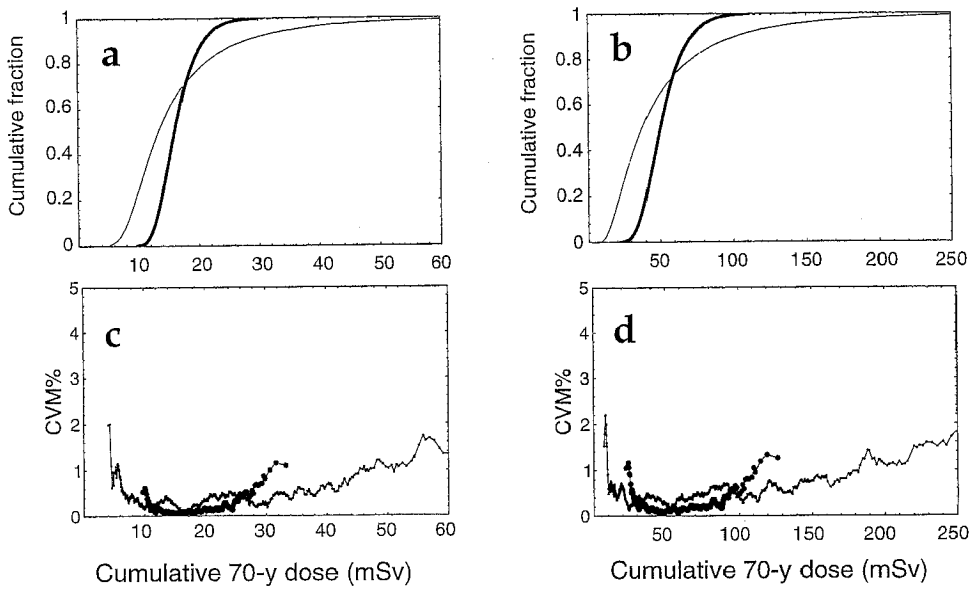

Fig. 2. (a–b): Cumulative relative-frequency distributions for ⟨D(70)⟩ (light curves, characterizing interindividual variability in expected 70-y effective integral dose) and $\overline{D(70)}$ (bold curves, characterizing uncertainty in population-average 70-y effective integral dose). (c–d): Corresponding Monte-Carlo sampling errors, defined as the standard error of the mean divided by the mean of the ith ordered value of 10 samples of 2,000 simulated variate values, for $i = 1, 2, \ldots, 2{,}000$ (light and bold point sets refer to ⟨D(70)⟩ and $\overline{D(70)}$, respectively). Plots pairs (a, c) and (b, d) correspond to imports-available and local-foods-only diet assumptions, respectively.

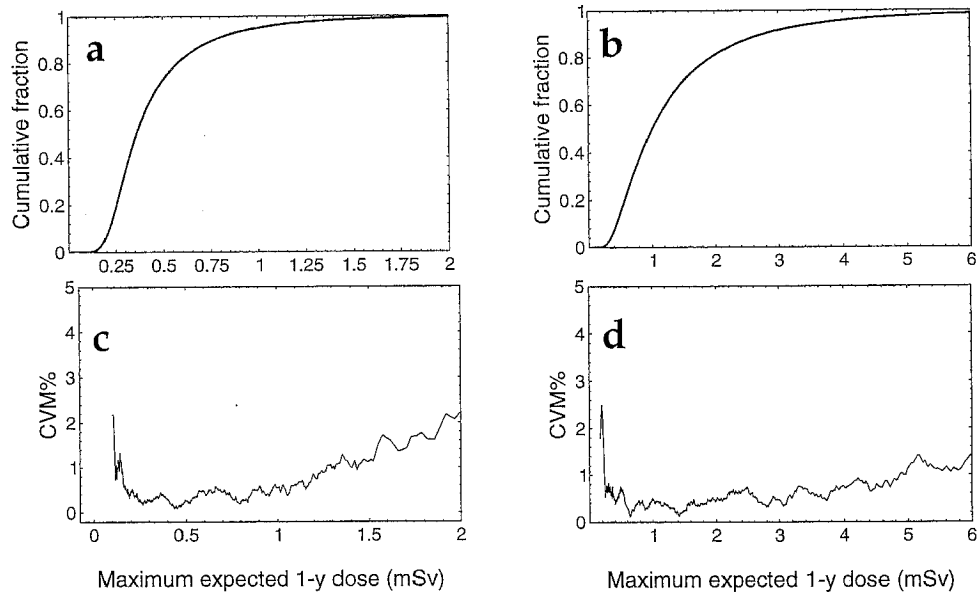

Fig. 3. (a–b): Cumulative relative-frequency distributions for Max(⟨$D(1)$⟩) characterizing interindividual variability in the maximum value of expected 1-y effective integral doses, regardless of occurrence year. (c-d): Corresponding Monte-Carlo sampling errors (see Fig. 4). Plot pairs (a, c) and (b, d) correspond to imports-available and local-foods-only diet assumptions, respectively.

Based on the hypothetical Bikini-remediation/resettlement scenario described above starting in 1999, population risk was estimated as described above from the characterizations of uncertainty in population-average lifetime dose $\overline{D(70)}$ obtained under the different dietary (IA, IUA) and population-size ($N = 200$, $N = 2{,}000$) assumptions considered. Each scenario implies a population-risk expectation, $\langle I \rangle$, and a corresponding probability of zero cases, $p_0 = \text{Prob}(I=0)$. Under the {IA, $N = 200$} scenario, $\langle I \rangle \approx 0.20$ cases

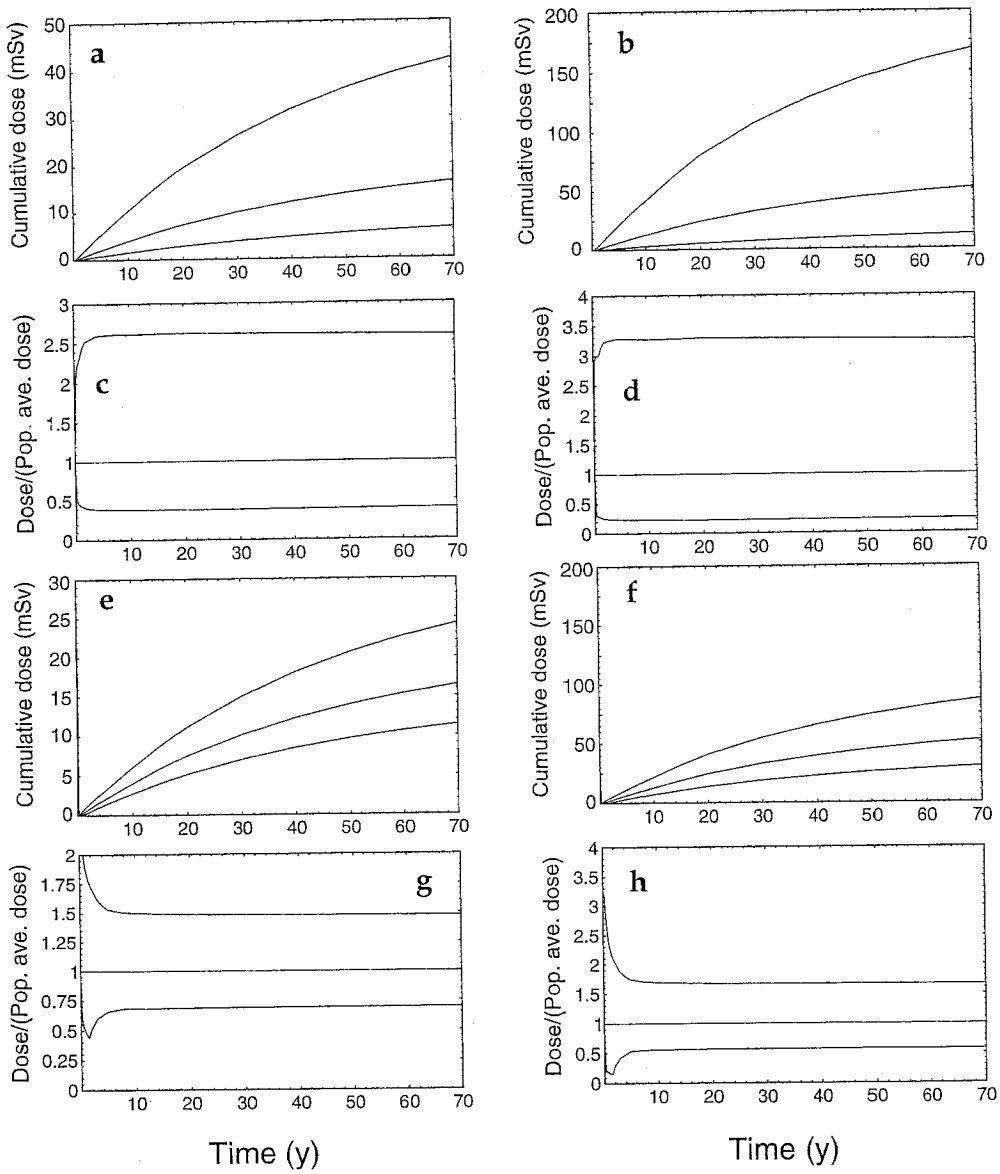

Fig. 4. (a-b): Values of $\langle D(t) \rangle$ (expected effective integral dose over time t); in each plot: middle curve = population-average value, upper and lower curve = corresponding 95% confidence limits (95%CL) with respect to interindividual variability. (e-f): Values of $\overline{D(t)}$ (population-average effect integral dose over time t); in each plot: middle curve = expected value function, upper and lower curves = corresponding 95%CL with respect to uncertainty. (c, d, g, h): Plots a, b, e and f shown as values relative middle curve of each, respectively. Plots a, c, e and g correspond to an imports-available diet, and the other plots to a local-foods-only diet.

and $p_0 = 83\%$; i.e., under this scenario it is rather more likely than not that zero cancer deaths will arise as a result of fallout-related exposures on Bikini. Under the {IA, $N = 2{,}000$} scenario, $\langle I \rangle \approx 0.86$ cases and $p_0 = 43\%$. Under the {IUA, $N = 200$} scenario, $\langle I \rangle \approx 0.63$ cases and $p_0 = 58\%$; i.e., even under this scenario it is more likely than not that zero cancer deaths will arise as a result of fallout-related exposures on Bikini. Under the {IUA, $N = 2{,}000$} scenario, $\langle I \rangle \approx 1.3$ cases and $p_0 = 30\%$.

DISCUSSION AND CONCLUSION

A detailed analysis of uncertainty and interindividual variability in estimated doses was conducted for a rehabilitation scenario for Bikini Island at Bikini Atoll, in which the top 40 cm of soil would be removed in the housing and village area, and the rest of the island is treated with potassium fertilizer, prior to an assumed resettlement date of 1999. Predicted doses were considered for fallout-related exposure by inhalation and inges-

tion pathways, and two dietary scenarios were considered. Corresponding calculations of uncertainty and variability in estimated dose showed that after ~5 y of residence on Bikini under either IA or IUA assumptions, the upper and lower 95% confidence limits on uncertainty in calculated dose are estimated to lie within a ~twofold factor of its expected value; the upper and lower 95% confidence limits on interindividual variability in calculated dose are estimated to lie within a ~threefold factor of its population-average value. For reference, the expected values of population-average dose at age 70 y are estimated to be 16 and 52 mSv under the IA and IUA dietary assumptions, respectively (Robison et al. 1995, 1997). Assuming that 200 Bikini resettlers would be exposed to local foods, the maximum 1-y dose received by any Bikini resident is most likely to be approximately 2 and 8 mSv under the IA and IUA assumptions, respectively. Under the most likely dietary scenario, involving access to imported foods, this analysis indicates that it is most likely that no additional cancer fatalities (above those normally expected) would arise from the increased radiation exposures considered.

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APPENDIX

Analytic and Monte-Carlo methods used to characterize uncertainty and interindividual variability in estimated doses to hypothetical Bikini residents

Define annual intake R_j of ^{137}Cs in Bq kg^{-1} y^{-1} from local foods of type j as $R_j = \Sigma_i^{n_j} 365/n_j R_{ij}$ and corresponding total annual ^{137}Cs intake as $R = \Sigma_j R_j$. From eqns (1)–(3) and the notation, assumptions and definitions given in the text, integrated whole-body dose, $Q_{ij}(t-t_i)$ after t years due to ingestion of ^{137}Cs in a food item of type j at time $t_i \le t$ is given by

$$Q_{ij}(t - t_i) = \int_{t_i}^{t} cq_{ij}(u)\, du \tag{A1}$$

$$= cFBR_{ij}\left\{\frac{e^{-\lambda t_i}\left(1 - e^{-(\beta K+\lambda)(t-t_i)}\right)}{\beta K + \lambda}\right\} \equiv cFBR_{ij}S. \tag{A2}$$

For large n_j and for t_i distributed randomly throughout each year, it follows that total integrated whole-body dose $Q(t)$ in Bq kg^{-1} after time t (y) is approximately

$$FB\left\{c\sum^{t} RS\right\} \equiv FBX, \tag{A3}$$

where X is defined here as the braced quantity in eqn (A3), for uniformly distributed t_i between 0 and t. Thus, e.g., $\langle Q(t)\rangle = \langle X\rangle = \langle R\rangle\langle S\rangle$, where $\langle S\rangle$, the expectation of S with respect to both t_i and β, is given by

$$\langle S\rangle = 1 + \frac{\{\Delta\beta + e^{-\lambda t}[\mathrm{Ei}(b_1) - \mathrm{Ei}(b_0)] - \mathrm{Ei}(c_1) + \mathrm{Ei}(c_0) + \mathrm{Ln}(c_1/c_0)\}}{\Delta\beta \mathrm{K}\lambda \mathrm{t}},$$

$$b_i = -\beta_i K t, \qquad i = 0,\ 1, \tag{A4}$$

$$c_i = b_i - \lambda t, \qquad i = 0,\ 1, \quad \text{and}$$

$$\Delta\beta = (\beta_1 - \beta_0) = (1.107 - 0.9) = 0.207,$$

in which $\mathrm{Ei}(z)$ is the exponential integral $\int_{-z}^{\infty} x^{-1}e^{-x}dx$. The (unsubscripted) constant c was estimated to be 2.419×10^{-3} mSv kg Bq^{-1} y^{-1} from values of cumulative whole-body-equivalent ^{137}Cs dose for adults predicted from the equivalent ICRP (1990) model.

From eqns (A3)–(A4) and corresponding assumptions (see text), interindividual variability in expected dose $\langle D(t)\rangle$ by time t was characterized by evaluating

$$\langle D(t)\rangle = D_{\mathrm{AmPu}}(t) + X_\gamma \overline{D_x(t)} + \langle B\rangle\left[\langle F\rangle\langle X\rangle + G\left(\frac{\langle X\rangle/F}{\overline{\langle X\rangle/F}}\right) W(t)\, D_{\mathrm{Sr}}(t)\right]. \tag{A5}$$

Variability in $\langle D(t)\rangle$ thus arises from uniform variability in F and G (taken to be 100% rank-correlated) and from lognormal variability in both $\langle R\rangle$ and H (see text). Uncertainty in population-average dose $\overline{D(t)}$ was characterized by evaluating

$$\overline{D(t)} = D_{\mathrm{AmPu}}(t) + \overline{X_\gamma D_x(t)} + B\bar{F}\left[\bar{X} + \bar{G}\left(\frac{\bar{X}'}{\overline{\langle F\rangle}}\right) W(t)\, D_{\mathrm{Sr}}(t)\right], \tag{A6}$$

in which the prime symbol denotes an independent random sample from the subscripted variate (for reasons discussed in the text). Uncertainty in eqn (A6) arises from the uniform and lognormal uncertainties assumed for $\bar{F}$ and B, respectively (see text), in addition to uncertainty associated with the variate $\bar{X}$ arising from X defined in eqn (A3). Let the subscript p on a variate denote a value pertaining to a particular individual in the exposed population, such that $X_p = X|\{R = R_p, H = H_p\}$ and $(X_p|\beta)$ is the sum of a presumed large number of identical independently distributed random variates. From the Lindeberg and Central Limit theorems, it follows that $(X_p|\beta)$ is approximately normally distributed with mean and variance given by

$$\langle X_p|\beta\rangle = ct\langle R_p\rangle\langle S_p|\beta\rangle \quad \text{and}$$

$$\sigma^2_{X_p|\beta} = c^2 t\langle R_p\rangle^2[(1 + \gamma_R^2)\langle S_p^2|\beta\rangle - \langle S_p|\beta\rangle^2],$$

respectively, in which

$$\gamma_R = \overline{\langle R\rangle}^{-1}\left(\sum_{j=1}^{11}\langle R_j\rangle^2\gamma^2_{R_{ij}} n_j^{-1}\right)^{1/2} = 0.039 \tag{A7}$$

is the CV for uncertainty in any individual's modeled lifetime, time-weighted average ^{137}Cs intake, based on the assumptions stated in the text and the food-type-specific CV values listed in Table 2. Assuming the exposed population size is sufficiently large to ensure that differences between first and second sample moments with respect to variability and their corresponding population moments are negligible, it follows from the definition of variability expectation that uncertainty in $\overline{X|\beta}$ is approximately normally distributed with mean and variance given by

$$\overline{\langle X|\beta\rangle} = \frac{1}{N}\sum_{p=1}^{N}\langle X_p|\beta\rangle \approx ct\overline{\langle R\rangle}\,\overline{\langle S|\beta\rangle} \quad \text{and} \tag{A8}$$

$$\sigma^2_{\overline{X|\beta}} = \frac{1}{N^2}\sum_{p=1}^{N}\sigma^2_{X_p|\beta} \approx c^2t\overline{\langle R\rangle}^2(1+g_R^2)[(1+\gamma_R^2)\overline{\langle S^2\beta\rangle}-\overline{\langle S\beta\rangle}^2, \tag{A9}$$

respectively, where

$$\langle S|\beta\rangle = [(\beta K+\lambda)t]^{-1}[(1-e^{-\lambda t})\lambda^{-1} - (e^{-(\beta K+\lambda)t}-e^{-\lambda t})(\beta K)^{-1}], \quad \text{and}$$

$$\langle S^2|\beta\rangle = (\beta K+\lambda)^{-2}t^{-1}\{(2\lambda)^{-1} + e^{-\lambda t}[(1-2e^{-\beta Kt})(2\beta Kt)^{-1} + 2(e^{-(\beta K-\lambda)t}-1)(\beta K-\lambda)^{-1}-(2\lambda)^{-1}]\}.$$

The averages $\overline{\langle S|\beta\rangle}$ and $\overline{\langle S^2|\beta\rangle}$ with respect to H were each evaluated numerically for different β values equally spaced over the range of β. Following this procedure, it turns out that $\sigma_{\overline{X|\beta}}t^{-1/2}$ is for each given t, $0 < t \leq 70$ y, a virtually linear function of $\overline{\langle X|\beta\rangle}t^{-1}$ over a β- and t-dependent range of the latter, and that corresponding $\overline{\langle X|\beta\rangle}t^{-1}$ values are virtually uniformly distributed over these linear ranges (Bogen et al. 1995). The linear coefficients $\{a,b\}|t$ and corresponding $\overline{\langle X|\beta\rangle}t^{-1}$-range boundaries $\{x_{lo}, x_{hi}\}|t$ were therefore determined for representative values of t, and this information was then used to evaluate uncertainty in $\bar{X}$, for $\bar{X}$ modeled as a compound normal distribution with mean = Ut and SD = $t^{1/2}(a + bU)$, where U is uniformly distributed between x_{lo} and x_{hi}.

Except where the use of 100% rank-correlated variates was indicated, all variate simulations were conducted using 10 sets of virtually uncorrelated vectors of 2,000 values for each variate involved, generated using systematic Latin-Hypercube sampling procedures. Each ith output fractile (and the first moment) was estimated as the mean of 10 ith ordered values (and first moments) of the 10 corresponding sets of 2,000 evaluations of eqn (A5) or (A6), for $i = 1, 2, \ldots, 2000$. Corresponding Monte-Carlo sampling errors, defined for each estimate as the coefficient of variation of the mean (CVM, equal to the standard error of the mean divided by the mean). Calculations were done on a PowerPC™† workstation using the programs *Mathematica*™ 2.2.2 (Wolfram 1991) and *RiskQ* (Bogen 1992). Analyses of quantile convergence indicate that fractile estimates obtained are generally accurate to within ⟨2% (see Figs. 2 and 3), and that mean values obtained are accurate to within <0.2%.

■■

† Power PC, Apple Computers, Inc., Capistino, CA