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A METHOD OF BENDING VIBRATION ANALYSIS FOR A VEHICLE WITH FLEXIBLY SUSPENDED COMPONENTS

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ABSTRACT

A method is developed for finding the natural modes and frequencies of a vehicle with flexibly suspended components. The vehicle is idealized as a system of one nonuniform beam and any number of flexibly suspended masses. A vibration analysis is first made of the beam, omitting the masses, by a modified Stodola method. The equations for the connected system are then derived using Lagrange's equations.

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RESEARCH AND DEVELOPMENT OPERATIONS

DEFINITION OF SYMBOLS

Symbol	Definition
A	coefficient matrix
a_{ef}	element of coefficient matrix
B	coefficient matrix
b_{ef}	element of coefficient matrix
C	base of vehicle
D	forward end of vehicle
$i, \underline{i}$	index of beam bending modes
$I_o'(x)$	rotary inertia of beam per unit length
I_b	moment of inertia of beam
I_j	added moment of inertia of jth mass
$j, \underline{j}$	index of added mass
k_{rj}	rotational spring constant between main beam and jth attached mass
k_{tj}	translational spring constant between main beam and jth attached mass
m_j	mass of jth component
m	number of suspended masses
$m'(x)$	mass of beam per unit length
m_b	total mass of beam
M_i	generalized mass of ith main beam mode
n	number of single beam modes used in analysis
$\bar{q}_s, q_s$	general variable

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
T	kinetic energy of system
V	potential energy of system
X	longitudinal station coordinate
X_{mc}	center of mass location of beam
$\bar{X}$	$X - X_{mc}$
X_{mcj}	center of gravity location of jth component
X_j	longitudinal station coordinate of attachment of the jth mass
$\hat{X}$	$X_j - X_{mc}$
$\bar{X}_j$	$X_{mcj} - X_j$
y	beam rigid body displacement
$Y_i(x)$	displacement of beam in ith mode
$Y_i'(x)$	slope of displacement in ith mode
Y_{ij}	displacement of beam in ith mode at attachment point of jth component
$\bar{Y}(x)$	total displacement of beam
$\bar{Y}'(x)$	slope of total displacement of beam
z_j	displacement of jth component at attachment point
$\bar{z}_j$	displacement of jth component at center of gravity of the component
θ_j	rotation of jth mass
η_i	amplitude of ith bending mode
φ	main beam rotation

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
ω_i	natural frequency of main beam in ith mode
ω	natural frequency of total system
$\delta_{i\underline{i}}$	$\delta_{i\underline{i}} = 1$ if $i = \underline{i}$, $\delta_{i\underline{i}} = 0$ if $i \neq \underline{i}$

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A METHOD OF BENDING VIBRATION ANALYSIS FOR A VEHICLE WITH FLEXIBLY SUSPENDED COMPONENTS

SUMMARY

A method is developed for finding the natural modes and frequencies of a vehicle with flexibly suspended components. The vehicle is idealized as a system of one nonuniform beam and any number of flexibly suspended masses. A vibration analysis is first made of the beam, omitting the masses, by a modified Stodola method. The equations for the connected system are then derived using Lagrange's equations.

I. INTRODUCTION

Space vehicles often have components of rather large mass that are attached to the main structure by flexible mounts. (In Apollo spacecraft, the lunar module is a prime example.) The natural frequency of the component may be in the frequency range of the lower natural frequencies of the system, in which case, the effect of these components on the natural mode shapes and frequencies of the vehicle cannot be adequately determined using a single beam model for analysis. Consequently, a method of analysis which allows masses to be attached to the main beam by flexible springs is presented.

The vehicle is modeled as a single, continuous, nonuniform beam having masses attached by springs in both translation and rotation. An "attached mass" is assumed to be rigid.

II. EQUATIONS OF MOTION

The motion of the main structure is represented by a number of flexible body modes and two rigid body modes in one plane. (The number of flexible body modes is determined by the difficulty of calculation.) The attached components are represented as rigid bodies with flexible attachment, in translation and rotation, in the plane of the main body motion.

The total displacement of the vehicle is given

$$\bar{Y}(x) = \sum_{i=1}^{n} \eta_i \; Y_i(x) + y + \bar{x}\varphi,$$

and the slope of the displacement is denoted by

$$\bar{Y}'(x) = \sum_{i=1}^{n} \eta_i \; Y_i'(x) + \varphi.$$

The motion of the center of gravity of the attached masses is represented by

$$\bar{z}_j = z_j + \bar{X}_j \theta_j.$$

Lagrange's equation

$$\frac{d}{dt} (\partial T/\partial \dot{\bar{q}}_s) + \frac{\partial V}{\partial \bar{q}_s} = 0, \qquad s = 1,2, \ldots, (n + 2 + 2n)$$

results in the following equations of motion:

$$A\{\ddot{\bar{q}}_s\} + B\{\bar{q}_s\} = \{0\},$$

where A is the inertia matrix, B is the stiffness matrix, and

$$\{\bar{q}_s\} = \begin{Bmatrix} \eta_1(t) \\ \vdots \\ \eta_n(t) \\ y(t) \\ \varphi(t) \\ z_1(t) \\ \vdots \\ z_m(t) \\ \theta_1(t) \\ \vdots \\ \theta_m(t) \end{Bmatrix}$$

Assuming solutions of the above equations of the form

$$\{\bar{q}_s\} = \{q_s\} \sin \omega t,$$

the following equations are obtained:

$$(\omega^2 A - B) \; \{q_s\} = \{0\}.$$

The total kinetic energy of the system is expressed by

$$T = \frac{1}{2} \int_C^D m' \left[\sum_{i=1}^{n} \dot{\eta}_i \; Y_i(x) + \dot{y} + \bar{X} \; \dot{\varphi} \right]^2 dx$$

$$+ \frac{1}{2} \int_C^D I'_o \left[\sum_{i=1}^{n} \dot{\eta}_i \; Y'_i(x) + \dot{\varphi} \right]^2 dx$$

$$+ \frac{1}{2} \sum_{j=1}^{m} m_j (\dot{z}_j + \bar{X}_j \dot{\theta}_j)^2 + \frac{1}{2} \sum_{j=1}^{m} I_j \; \dot{\theta}_j^2$$

and the total potential energy is given by

$$V = \frac{1}{2} \sum_{i=1}^{n} \eta_i^2 \; \omega_i^2 \; M_i + \frac{1}{2} \sum_{j=1}^{m} k_{tj} \left[\sum_{i=1}^{n} (\eta_i \; Y_{ij} + y + \hat{X}_j \varphi) - z_j \right]^2$$

$$+ \frac{1}{2} \sum_{j=1}^{m} k_{rj} \left[\sum_{i=1}^{n} (\eta_i \; Y'_{ij} + \varphi) - \theta_j \right]^2 .$$

The required derivatives with time dependency eliminated follows:

$$\frac{\partial T}{\partial \dot{\eta}_i} = \dot{\eta}_i \int_C^D m'(x)\ Y_i^2(x)\ dx + \dot{\eta} \int_C^D I_o(x)\ Y_i'^2(x)\ dx$$

$$= \dot{\eta}_i\ M_i$$

$$\frac{d}{dt}\ (\partial T/\partial \dot{\eta}_i) = \ddot{\eta}\ M_i = \omega^2\ M_i\ \eta_i$$

$$\frac{\partial T}{\partial \dot{y}} = \dot{y} \int_C^D m'(x)\ dx = m_b\ \dot{y}$$

$$\frac{d}{dt}\ (\partial T/\partial \dot{y}) = m_b\ \ddot{y} = -\omega^2\ m_b\ y$$

$$\frac{\partial T}{\partial \dot{\varphi}} = \dot{\varphi} \int_C^D \bar{X}^2\ m'(x)\ dx = \dot{\varphi} I_b$$

$$\frac{d}{dt}\ (\partial T/\partial \dot{\varphi}) = I_b \ddot{\varphi} = -\omega^2\ I_b \varphi$$

$$\frac{\partial T}{\partial \dot{z}_j} = m_j\ \dot{z}_j + m_j\ \bar{X}_j\ \dot{\theta}_j$$

$$\frac{d}{dt}\ (\partial T/\partial \dot{z}_j) = m_j \ddot{z}_j + m_j\ \bar{X}_j \ddot{\theta} = -\omega^2\ m_j z_j - \omega^2\ m_j \bar{X}_j \theta_j$$

$$\frac{\partial T}{\partial \dot{\theta}_j} = m_j \bar{X}_j \dot{z}_j + (m_j \bar{X}_j + I_j)\ \dot{\theta}_j$$

$$\frac{d}{dt}\ (\partial T/\partial \dot{\theta}_j) = m_j \bar{X}_j \ddot{z}_j + (m_j \bar{X}_j^2 + I_j)\ \ddot{\theta}_j$$

$$= -\omega^2\ m_j \bar{X}_j z_j - \omega^2 (m_j \bar{X}_j^2 + I_j)\ \theta_j$$

$$\frac{\partial V}{\partial \eta_i} = \eta_i\ \omega_i^2\ M_i + \sum_{j=1}^{m} k_{tj} \left[\sum_{i=1}^{n} (\eta_i Y_{ij} + y + \hat{X}_j \varphi) - z_j \right] Y_{ij}$$

$$+ \sum_{j=1}^{m} k_{rj} \left[\sum_{i=1}^{n} (\eta_i\ Y'_{ij} + \varphi) - \theta_j \right] Y'_{ij}$$

$$\frac{\partial V}{\partial y} = \sum_{j=1}^{m} k_{tj} \left[\sum_{i=1}^{n} (\eta_i Y_{ij} + y + \hat{X}_j \varphi) - z_j \right]$$

$$\frac{\partial V}{\partial \varphi} = \sum_{j=1}^{m} k_{tj} \left[\sum_{i=1}^{n} (\eta_i Y_{ij} + y + \hat{X}_j \varphi) - z_j \right] \hat{X}_j$$

$$+ \sum_{j=1}^{m} k_{rj} \left[\sum_{i=1}^{n} (\eta_i Y'_{ij} + \varphi) - \theta_j \right]$$

$$\frac{\partial V}{\partial z_j} = - \sum_{j=1}^{m} k_{tj} \left[\sum_{i=1}^{n} (\eta_i Y_{ij} + y + \hat{X}_i \varphi) - z_i \right]$$

$$\frac{\partial V}{\partial \theta_j} = - \sum_{j=1}^{m} k_{rj} \left[\sum_{i=1}^{n} (\eta_i Y'_{ij} + \varphi) - \theta_j \right].$$

The elements of the A and B matrix are given in general form. There is an element for each i and each j as i varies from 1 to n and j from 1 to m.

$a_{ef} = M_i \quad \text{if} \quad i = \underline{i}$ $\qquad e = i, \quad f = \underline{i}$

$a_{ef} = m_b$ $\qquad e = n + 1, \quad f = n + 1$

$a_{ef} = I_b$ $\qquad e = n + 2, \quad f = n + 2$

$a_{ef} = m_j \quad \text{if} \quad j = \underline{j}$ $\qquad e = n + 2 + j, \quad f = n + 2 + \underline{j}$

$a_{ef} = \bar{X}_j m_j \quad \text{if} \quad j = \underline{j}$ $\qquad e = n + 2 + j, \quad f = n + 2 + m + \underline{j}$

$a_{ef} = \bar{X}_j m_j \quad \text{if} \quad j = \underline{j}$ $\qquad e = n + 2 + m + j, \quad f = n + 2 + \underline{j}$

$a_{ef} = (m_j \bar{X}_j^2 + I_j) \quad \text{if} \quad j = \underline{j}$ $\qquad e = n + 2 + m + j, \quad f = n + 2 + m + \underline{j}$

The remaining elements of the A matrix are zero.

$$b_{ef} = \delta_{i\underline{i}} M_i \omega_i^2 + \sum_{j=1}^{m} k_{tj} Y_{ij} Y_{\underline{i}j} + \sum_{j=1}^{m} k_{rj} Y'_{ij} Y'_{\underline{i}j} \qquad e = i, \quad f = \underline{i}$$

$$b_{ef} = \sum_{j=1}^{m} k_{tj} Y_{ij} \qquad e = i, \quad f = n + 1$$

$$b_{ef} = \sum_{j=1}^{m} \hat{X}_j \, k_{tj} \, Y_{ij} + \sum_{j=1}^{m} k_{rj} \, Y'_{ij}$$ $e = i$, $f = n + 2$

$$b_{ef} = - k_{tj} \, Y_{ij}$$ $e = i$, $f = n + 2 + j$

$$b_{ef} = - k_{rj} \, Y'_{ij}$$ $e = i$, $f + n + 2 + m + j$

$$b_{ef} = \sum_{j=1}^{m} k_{tj} \, Y_{ij}$$ $e = n + 1$, $f = i$

$$b_{ef} = \sum_{j=1}^{m} k_{tj}$$ $e = n + 1$, $f = n + 1$

$$b_{ef} = \sum_{j=1}^{m} \hat{X}_j \, k_{tj}$$ $e = n + 1$, $f = n + 2$

$$b_{ef} = - k_{tj}$$ $e = n + 1$, $f = n + 2 + j$

$$b_{ef} = 0$$ $e = n + 1$, $f = n + 2 + m + j$

$$b_{ef} = \sum_{j=1}^{m} \hat{X}_j \, k_{tj} \, Y_{ij} + \sum_{j=1}^{m} k_{ij} \, Y'_{ij}$$ $e = n + 2$, $f = i$

$$b_{ef} = \sum_{j=1}^{m} \hat{X}_j \, k_{tj}$$ $e = n + 2$, $f = n + 1$

$$b_{ef} = \sum_{j=1}^{m} \hat{X}_j^2 \, k_{tj} + \sum_{j=1}^{m} k_{rj}$$ $e = n + 2$, $f = n + 2$

$$b_{ef} = -\hat{X}_j \, k_{tj}$$ $e = n + 2$, $f = n + 2 + j$

$$b_{ef} = -k_{rj}$$ $e = n + 2$, $f = n + 2 + m + j$

$$b_{ef} = -k_{tj} \, Y_{ij}$$ $e = n + 2 + i$, $f = i$

$$b_{ef} = -k_{tj}$$ $e = n + 2 + j$, $f = n + 1$

$$b_{ef} = -\hat{X}_j \, k_{tj}$$ $e = n + 2 + j$, $f = n + 2$

$$b_{ef} = \delta_{j\underline{j}} \, k_{tj}$$ $e = n + 2 + j$, $f = n + 2 + \underline{j}$

$$b_{ef} = 0$$ $e = n + 2 + j$, $f = n + 2 + m + \underline{j}$

$$b_{ef} = -k_{rj} \, Y'_{ij}$$ $e = n + 2 + m + j$, $f = i$

$$b_{ef} = 0$$ $e = n + 2 + m + j$, $f = n + 1$

$b_{ef} = -k_{rj}$ $\quad\quad e = n + 2 + m + j$
$\quad\quad f = n + 2$

$b_{ef} = 0$ $\quad\quad e = n + 2 + m + j$
$\quad\quad f = n + 2 + \underline{j}$

$b_{ef} = \delta_{j\underline{j}} \, k_{rj}$ $\quad\quad e = n + 2 + m + j$
$\quad\quad f = n + 2 + m + j$

The solution of the equation

$$(\omega^2 A - B) \; \{q_s\} = \{0\}$$

is acquired by transforming the equation into the standard eigenvalue problem.

III. DISCUSSION OF RESULTS

To evaluate the model, an analysis was made of the third stage (S-IVB-D, IU and payload) Saturn V dynamic test vehicle. The results were compared to dynamic test results recorded in references 2 and 3. The configuration of the vehicle is shown in figure 1.

In the analysis, the lunar module (LEM) was idealized as a mass with center of gravity at the attachment point. The remainder of the vehicle was considered to be a heterogeneous beam. Analyses were made for engine ignition and engine cutoff conditions.

The attachment spring constants used were $.19017 \times 10^8$ (newtons) and $.1367 \times 10^9$ (newton meters) for translation and rotation, respectively. These amounts of stiffness would give LEM frequencies of 6 Hz and 10 Hz in translation and rotation, respectively, if the springs were attached to rigid structure. Four free-free bending modes of the beam were computed for input to the coupled mode analysis. Six coupled modes were computed, but only four are shown in this report because of lack of test data for comparison.

Dynamic test results are compared with theoretical results in figures 2 through 11. The relative displacements obtained from the dynamic tests are shown for three different locations on the LEM. Only the relative displacement at the attachment location is shown for the theoretical analysis. The relative displacements are normalized to unity at the place of maximum amplitude of the beam. (The relative displacement of the LEM may be larger than unity.) The corresponding natural frequencies are given on each figure.

The correlation between dynamic test results and theoretical results is considered acceptable. A better correlation could be expected if the mass characteristics of the dynamic test vehicle and the spring constants of the LEM were better defined.

CONCLUSIONS

This method of analysis is particularly useful when the natural frequency of some component is in the frequency range of the lower natural frequencies of the system. Both the natural frequencies and the mode shapes are more adequately determined using this method of analysis.

The accuracy of results obtained from this method is highly dependent on a knowledge of the effective spring constants of the corrections between components and vehicle structure. These effective spring constants are often difficult to calculate by analytical means. Hence, early component testing to determine these spring rates would prove extremely advantageous when this type of analysis is needed.

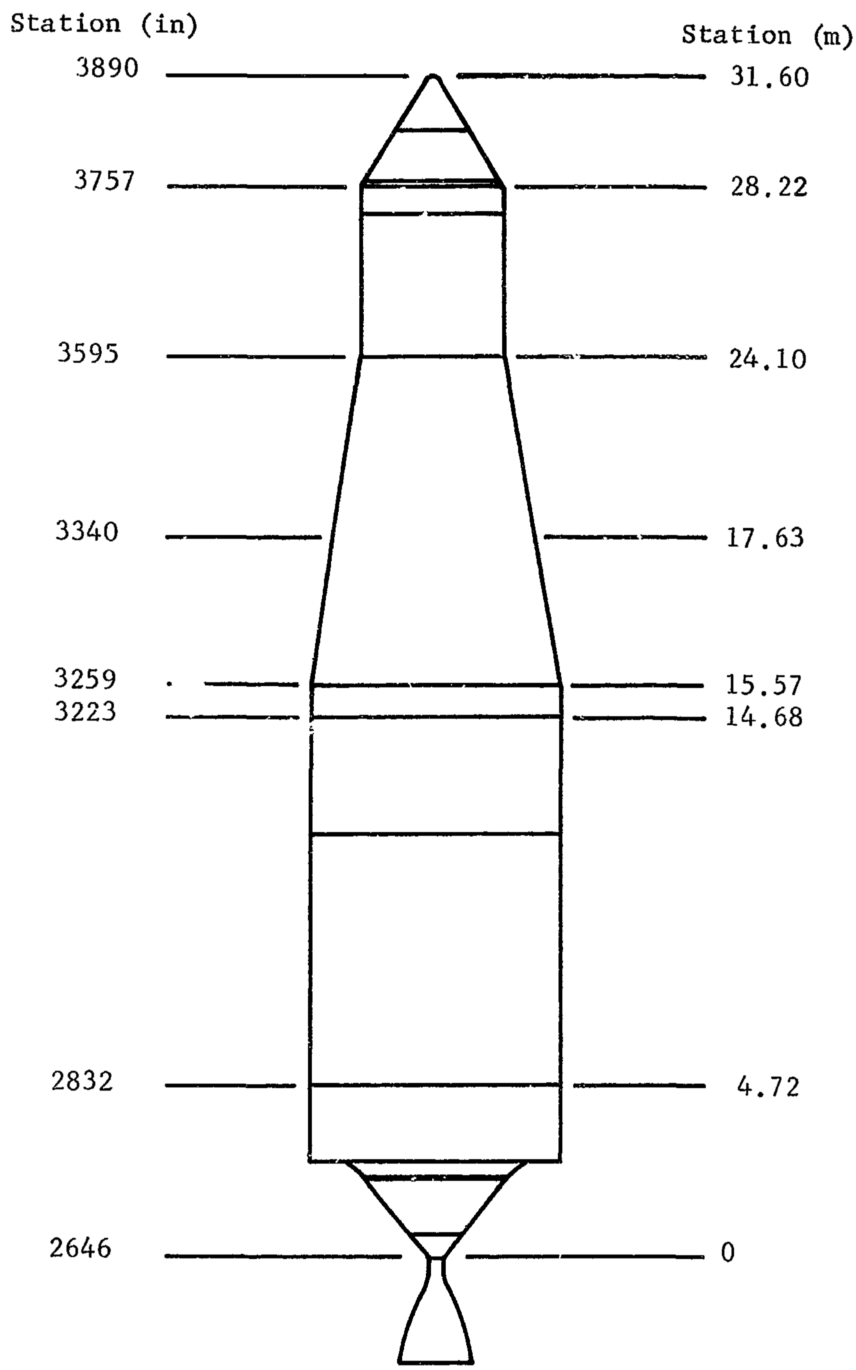

Figure 1. Vehicle Configuration

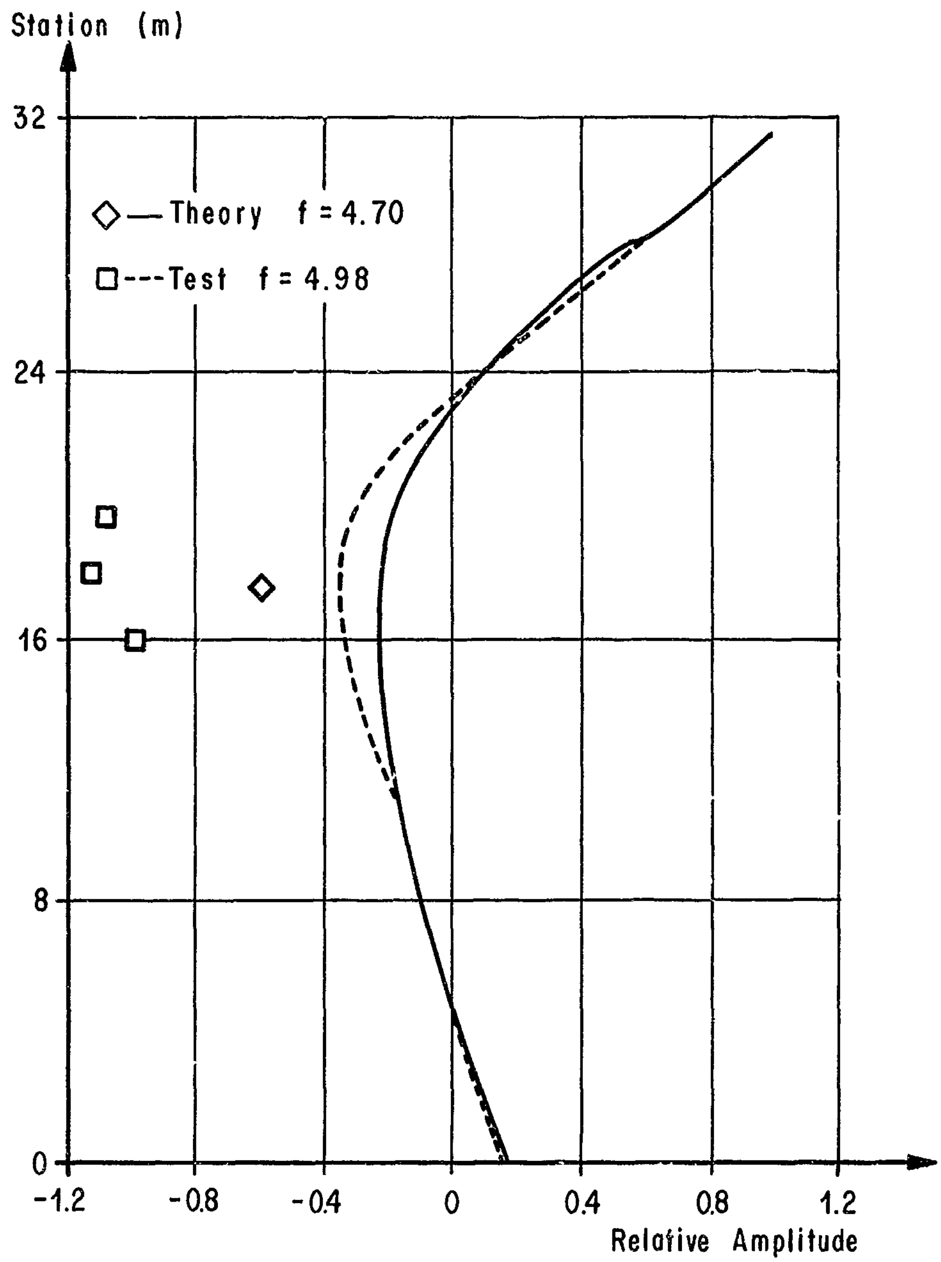

FIG. 2. DEFLECTION CURVE
(IGNITION FUEL CONDITION)

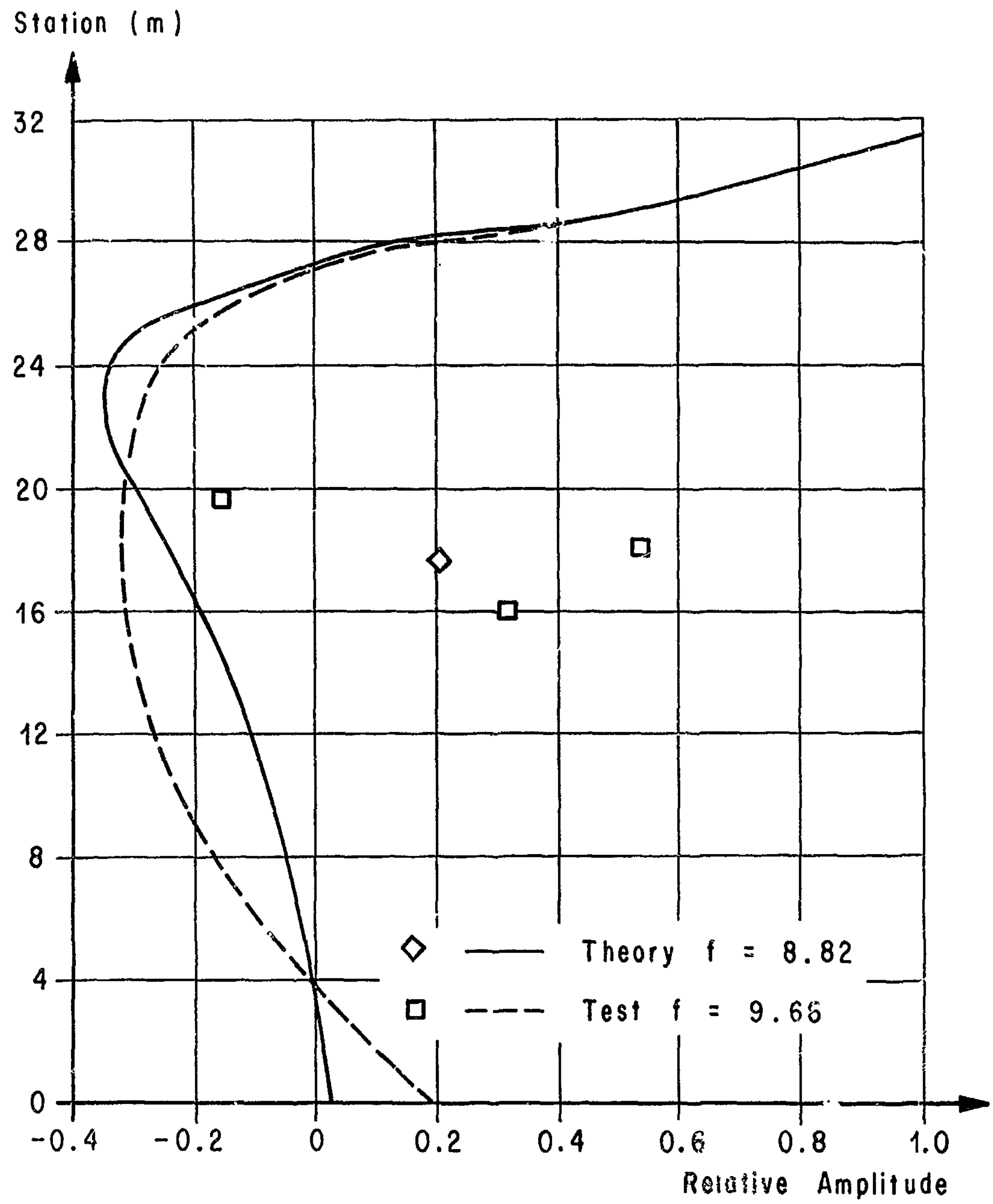

FIG. 3. DEFLECTION CURVE
(IGNITION FUEL CONDITION)

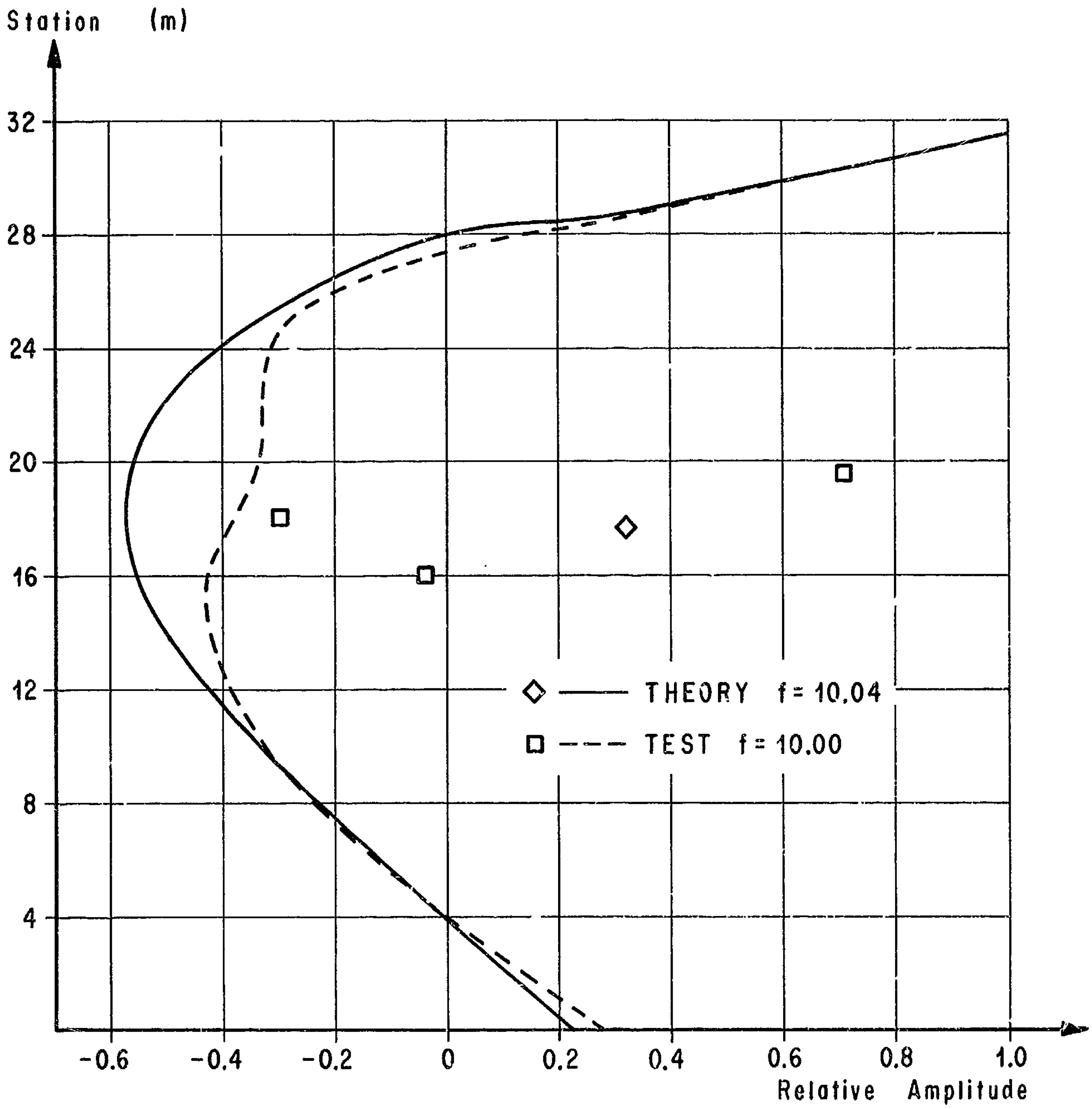

FIG. 4. DEFLECTION CURVE
(IGNITION FUEL CONDITION)

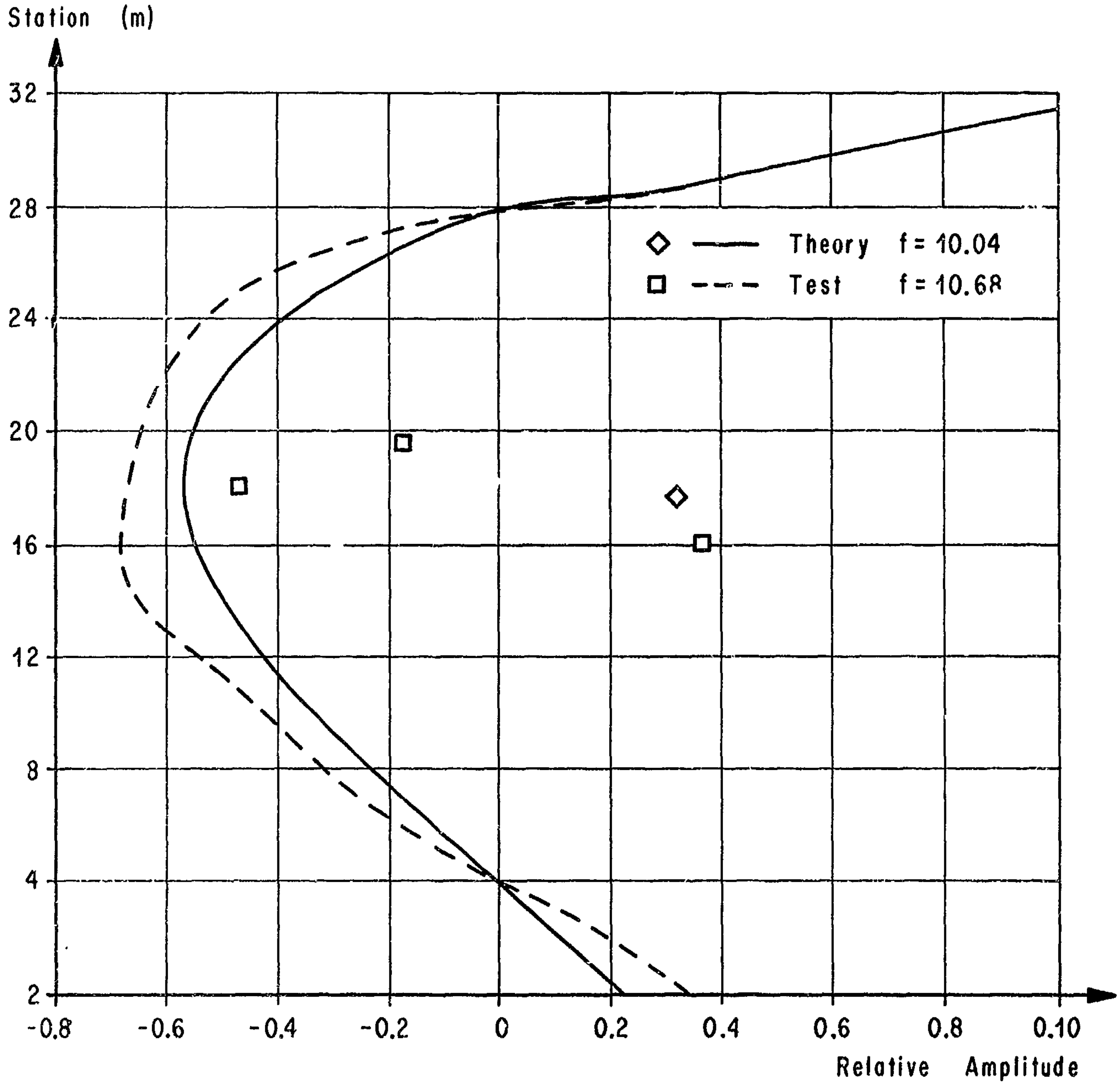

FIG. 5. DEFLECTION CURVE
(IGNITION FUEL CONDITION)

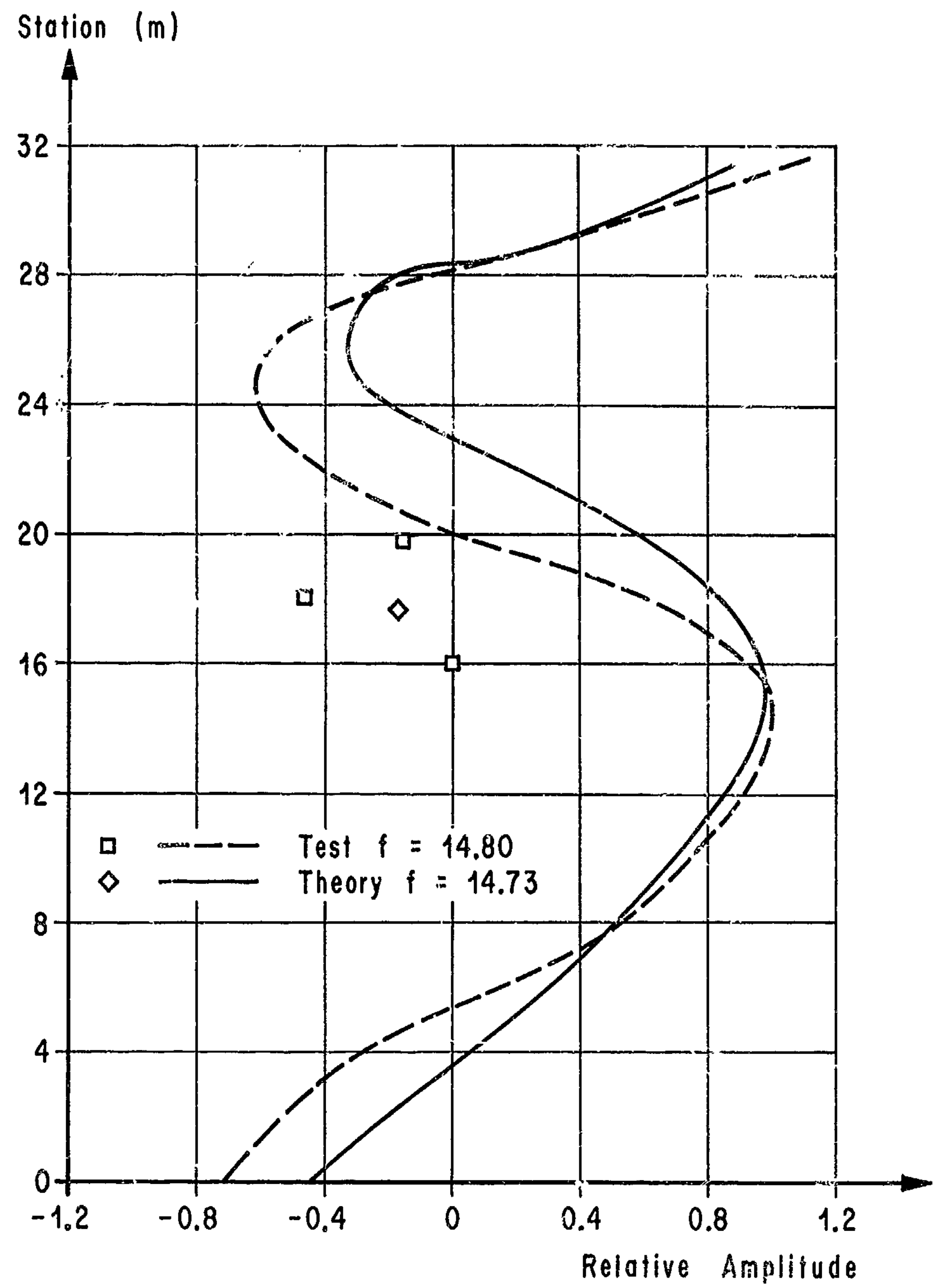

FIG. 6. DEFLECTION CURVE
(IGNITION FUEL CONDITION)

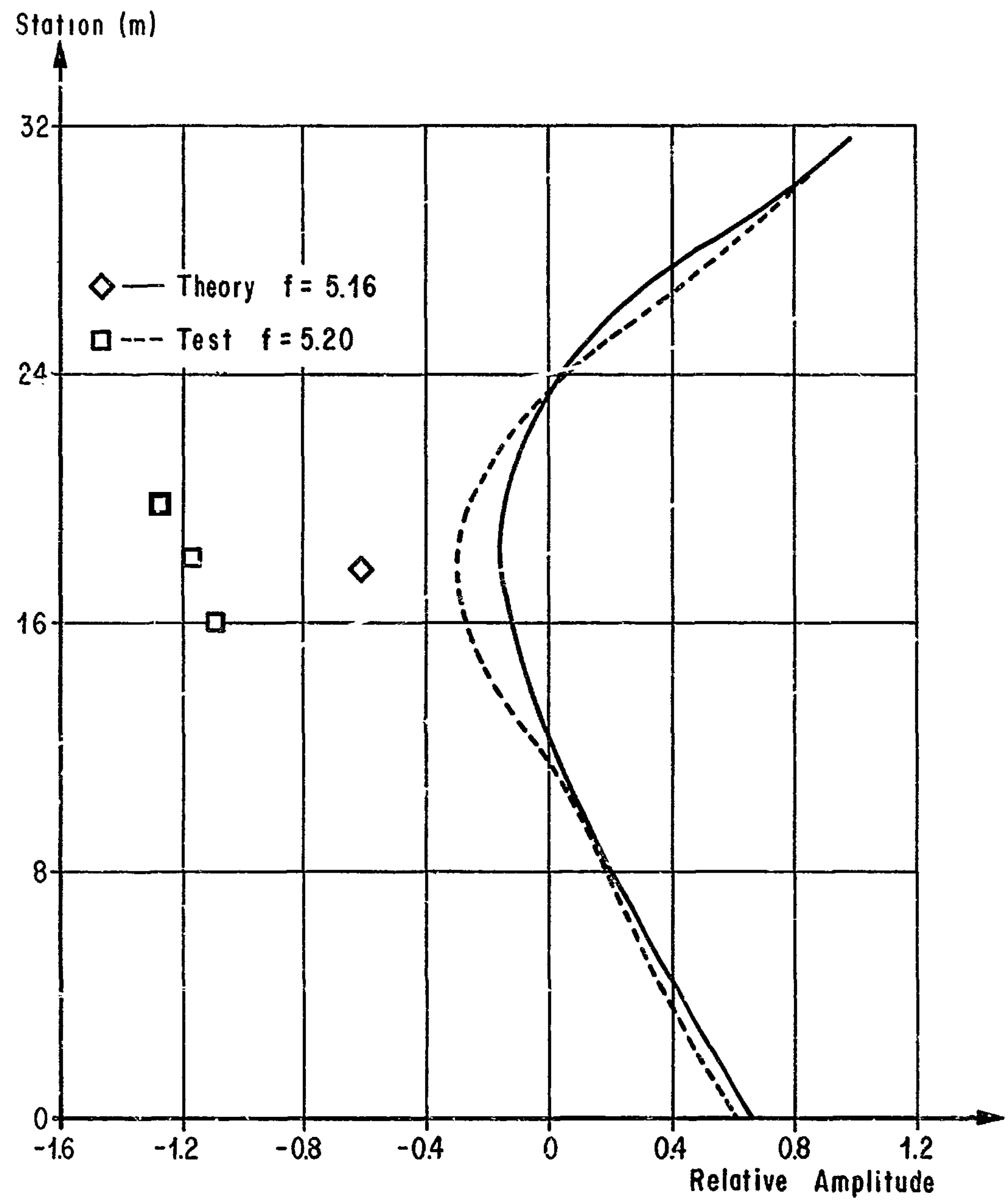

FIG. 7. DEFLECTION CURVE
(IGNITION FUEL CONDITION)

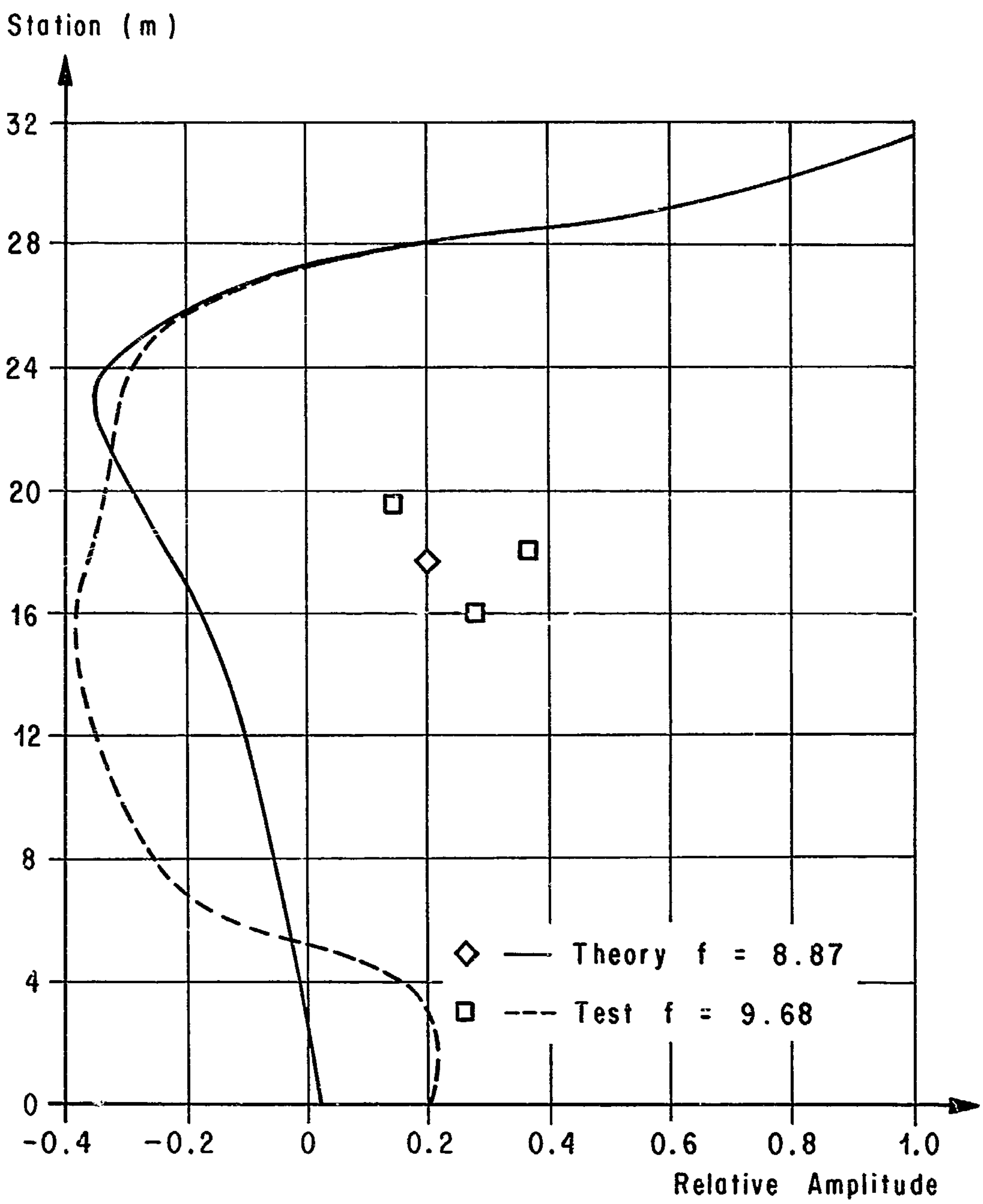

FIG. 8. DEFLECTION CURVE
(CUTOFF FUEL CONDITION)

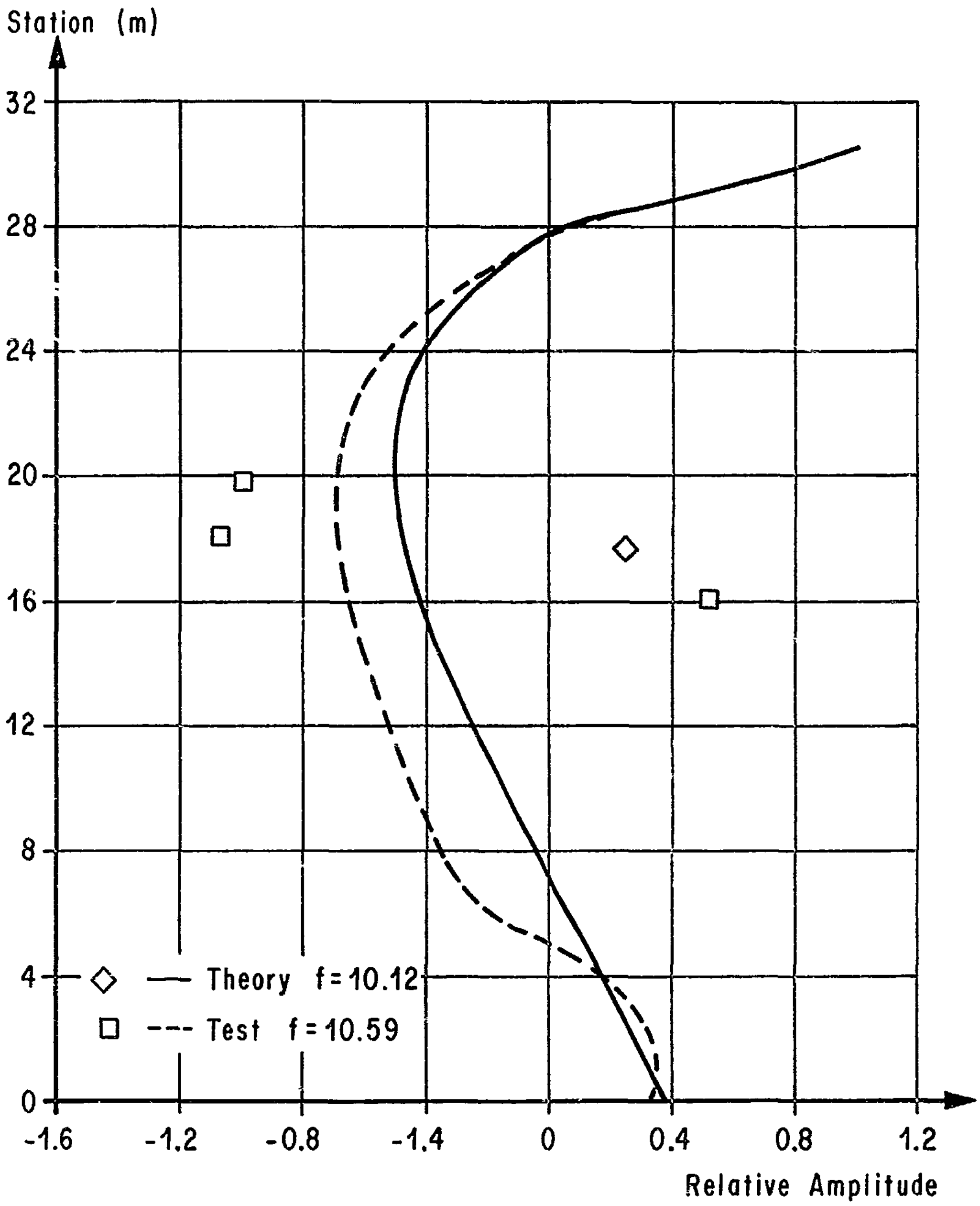

FIG. 9. DEFLECTION CURVE
(CUTOFF FUEL CONDITION)

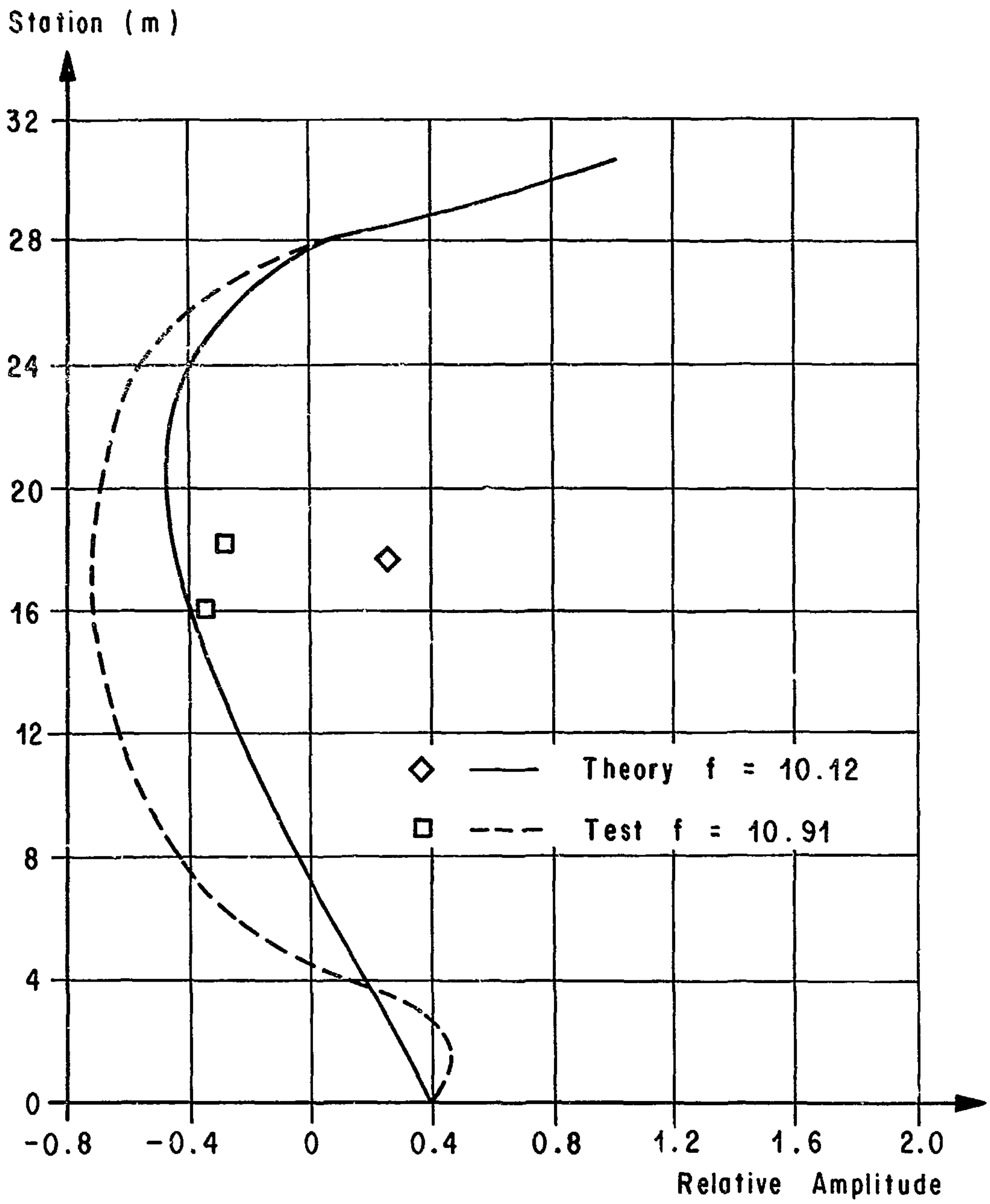

FIG. 10. DEFLECTION CURVE
(CUTOFF FUEL CONDITION)

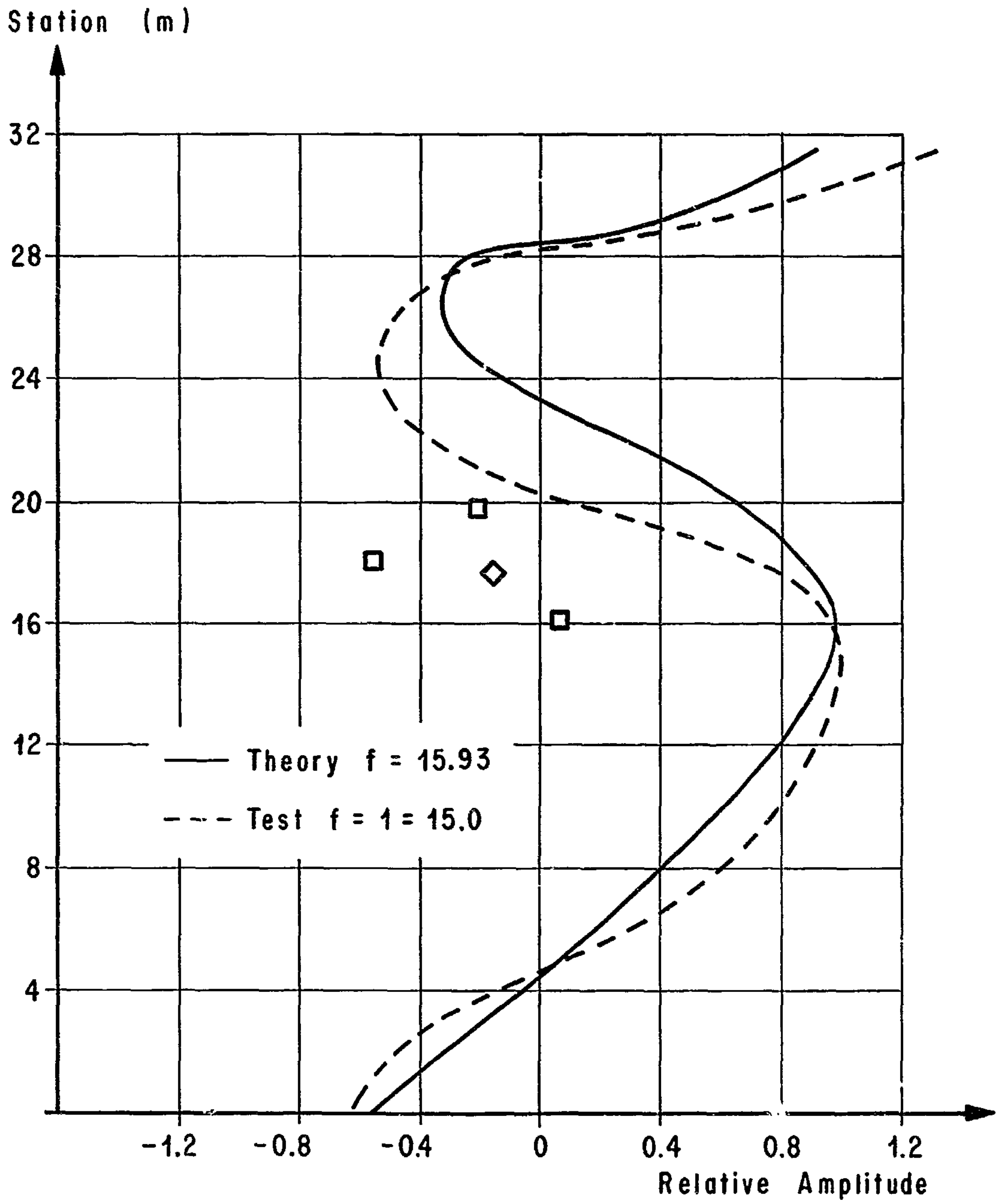

FIG. 11. DEFLECTION CURVE
(CUTOFF FUEL CONDITION)

REFERENCES

1. Kiefling, L. A., "Modified Stodola Method for Bending Vibration Analysis," R-AERO-IN-22-64, MSFC, Huntsville, Alabama, September 18, 1964.

2. Test Condition Summary Report (Second S-IVB-D, IU, and Payload Configuration; Pitch at Ignition), HSM-R764, Space Division, Chrysler Corporation, Huntsville Operations.

3. Test Condition Summary Report (Second S-IVB-D, IU, and Payload Configuration; Pitch at Cutoff), HSM-R767, Space Division, Chrysler Corporation, Huntsville Operations.

APPROVAL TM X-53733

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by Homer Pack

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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